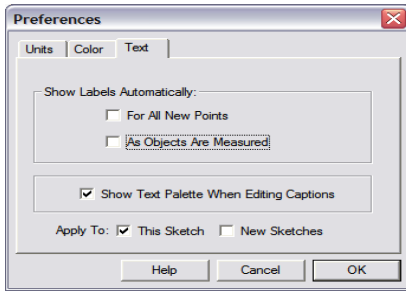
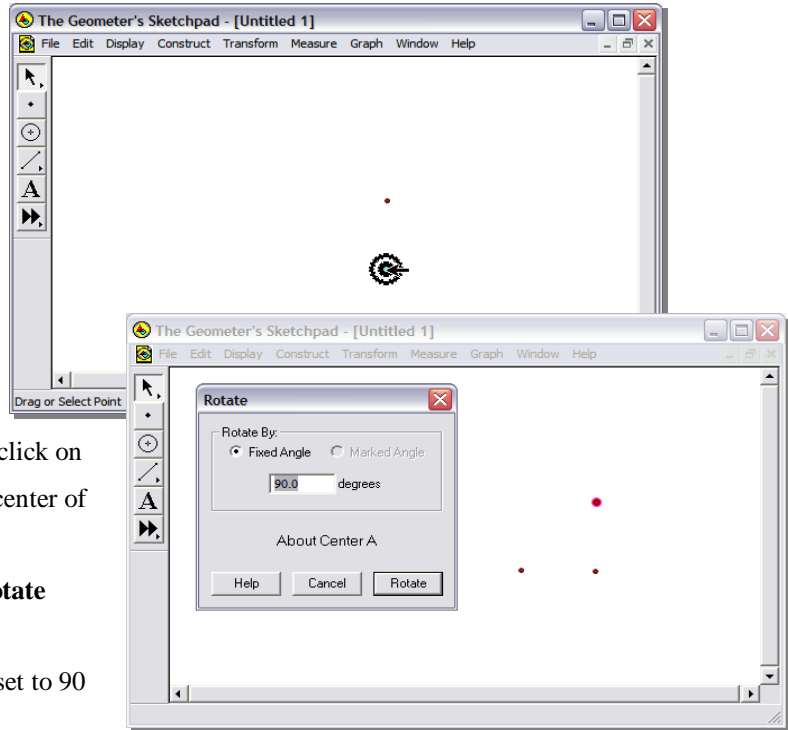


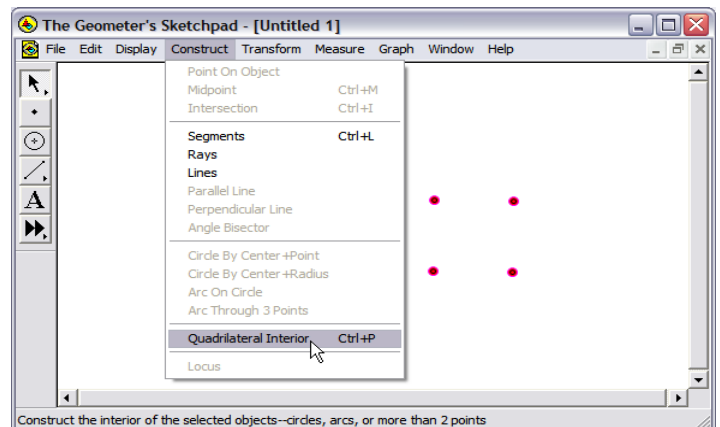
- Open a new sketch. Select **Preferences...** under the **Edit** menu. Select the Text Tab at the top. Uncheck both boxes under the title “Show Labels Automatically”




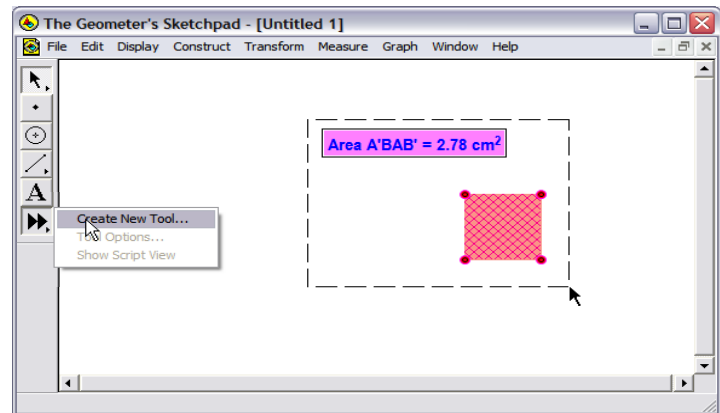
- Create two points in the sketch as shown at the right. Double click on the bottom point until it flashes (this will mark the point as a center of rotation).
- Then, single click on the top point to highlight it and select **Rotate** under the **Transform** menu.
- Make sure that “Fixed Angle” is checked and that the angle is set to 90 degrees. Finally press **Rotate**.



- Next, double click the original top point until it flashes.
- Then, single click the original bottom point to highlight it and select **Rotate** under the **Transform** menu
- Make sure that “Fixed Angle” is checked and that the angle is set to - 90 degrees. Finally press **Rotate**.
- This should create the four vertices of a square that is defined by the original two points. Highlight or select all four points in consecutive order and select **Quadrilateral Interior** from the **Construct** menu.



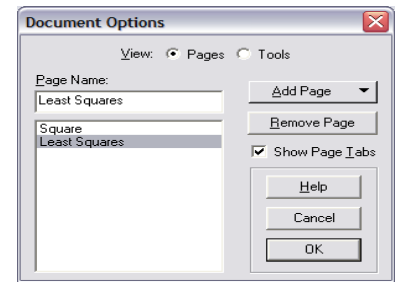
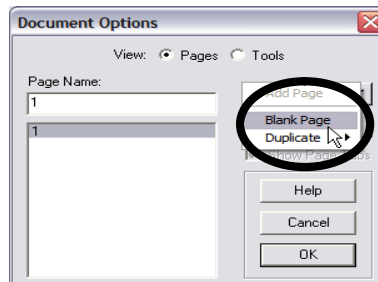
- With the square’s interior still selected, select Area under the Measure menu.
- Highlight everything on the sketch (the points, the square, and the area measure). Then click on the custom tool button, , and select **Create New Tool...**



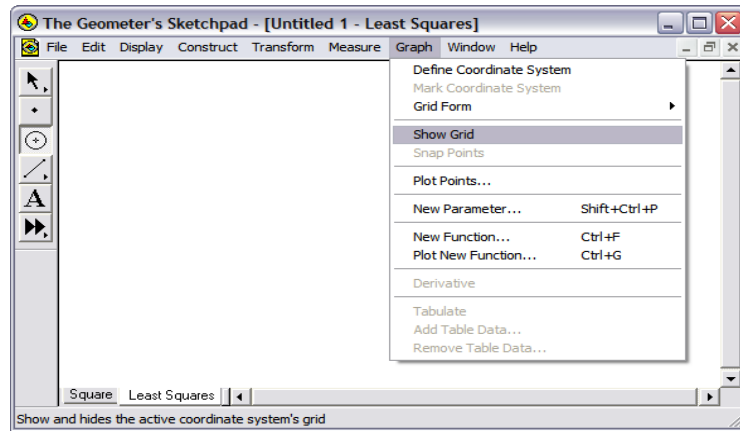
- Change the Tool Name to Square.
- Next, select **Document Options...** under the **File** menu.



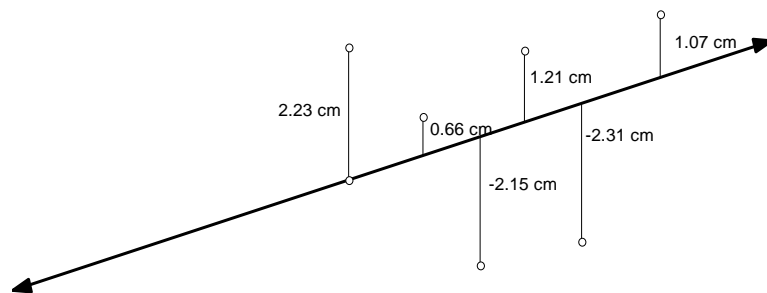
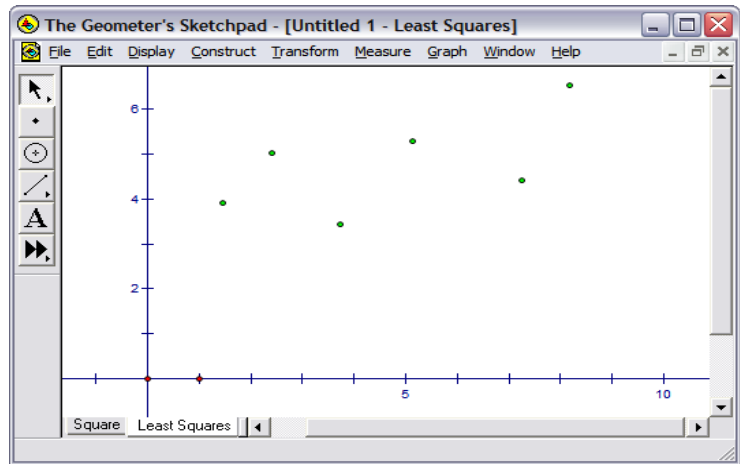
- Under the Document Options window, click on the **Add Page** Button and select **Blank Page**.
- Finally, change the Page Names to *Square* and *Least Squares* as shown in the window at the far right.



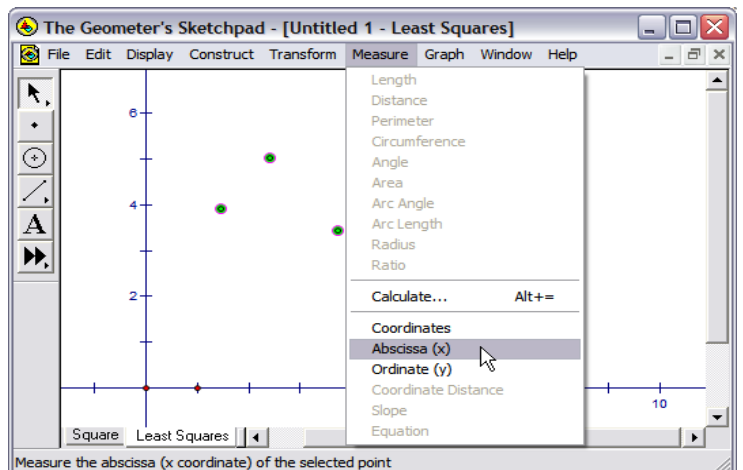
- After altering the document options, you should notice two tabs at the bottom of the sketch. If it is not already selected click on the “Least Squares” Tab.
- Next, select Show Grid under the Graph Menu.
- Then, select Hide Grid under the Graph Menu. This should leave just the x and y axes.
- Reposition the origin to the bottom left hand corner of the sketch (as shown in the second picture at the right).
- Then using the point tool, create 6 points in the first quadrant that will represent a scatter plot. For the purpose of this investigation it will be easier if you initially position the points showing a positive association and so that no two points have the same abscissa (x coordinate).



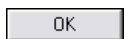
- One of the most commonly used lines of best fit for a scatter plot is a linear regression or least squares line. If asked most people agree that a line of best fit should be as close to all of the points at the same time as possible. This can be done in a couple of ways. One way might be to minimize the overall total vertical distance (known as residual) from the line. In other words, measure the vertical distance each point is away from the line (example shown at the right) and then move the line around until the sum of all the vertical distances has been minimized. Although this is close to a least squares line it is not (the difference can be a likened to the differences between mean deviation and standard deviation). Another hypothesis usually posed by students and mathematicians is that the line should be equally ‘weighted’ on both sides. In other words there should be an equal amount of vertical distances above and below the line. It turns out there are an infinite number of lines that can meet this property. Any line that passes through the ‘mean point’ will always have an equal residual (or total vertical distance) above and below the line of best fit. The mean point is simply the (average x, average y). The least squares line does pass through this point.



- Highlight all of the points that represent the scatter plot and select **Abscissa(x)** under the **Measure** menu.
- Highlight all of the same points again and this time select **Ordinate (y)** under the **Measure** menu.




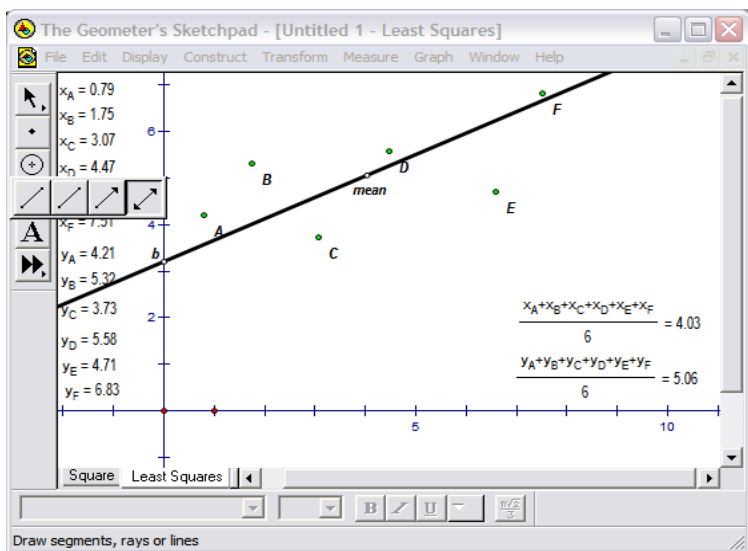
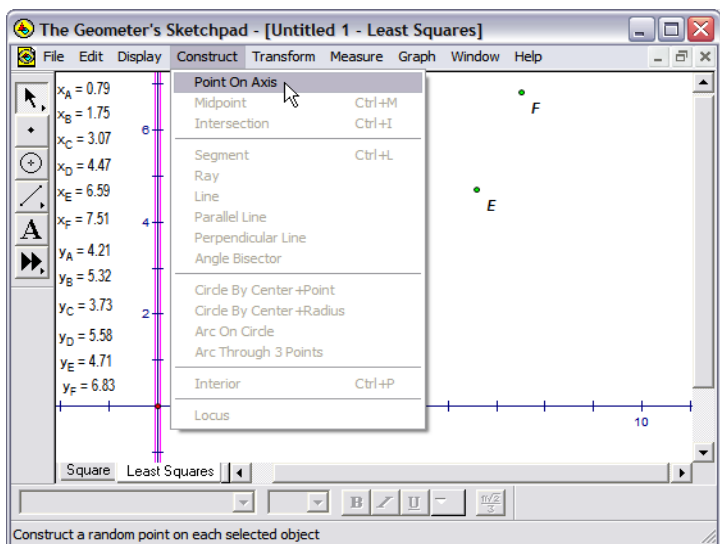
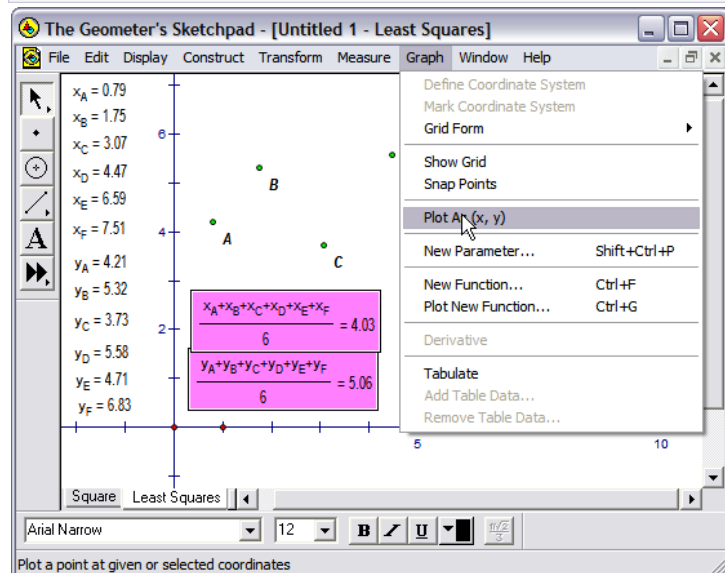
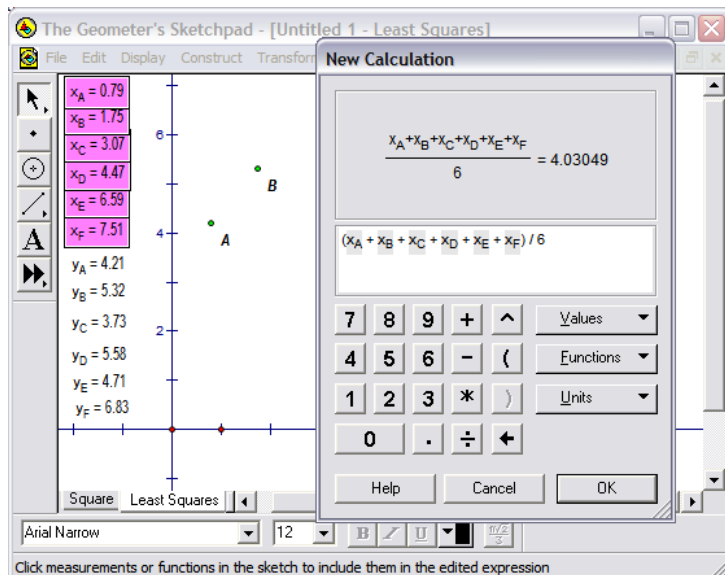
- Under the Measure menu select Calculate. This should bring up a calculator looking type of window (as shown at the right). Reposition the window so that you can see all of the abscissas. Start the calculation with an open parenthesis,  $($ , and then click on the measurement  $x_A$  in the actual sketch. Then click the addition button,  $+$ , and continue adding all 6 abscissas. Finally, close the parenthesis, divide by 6 to find the average x, and press



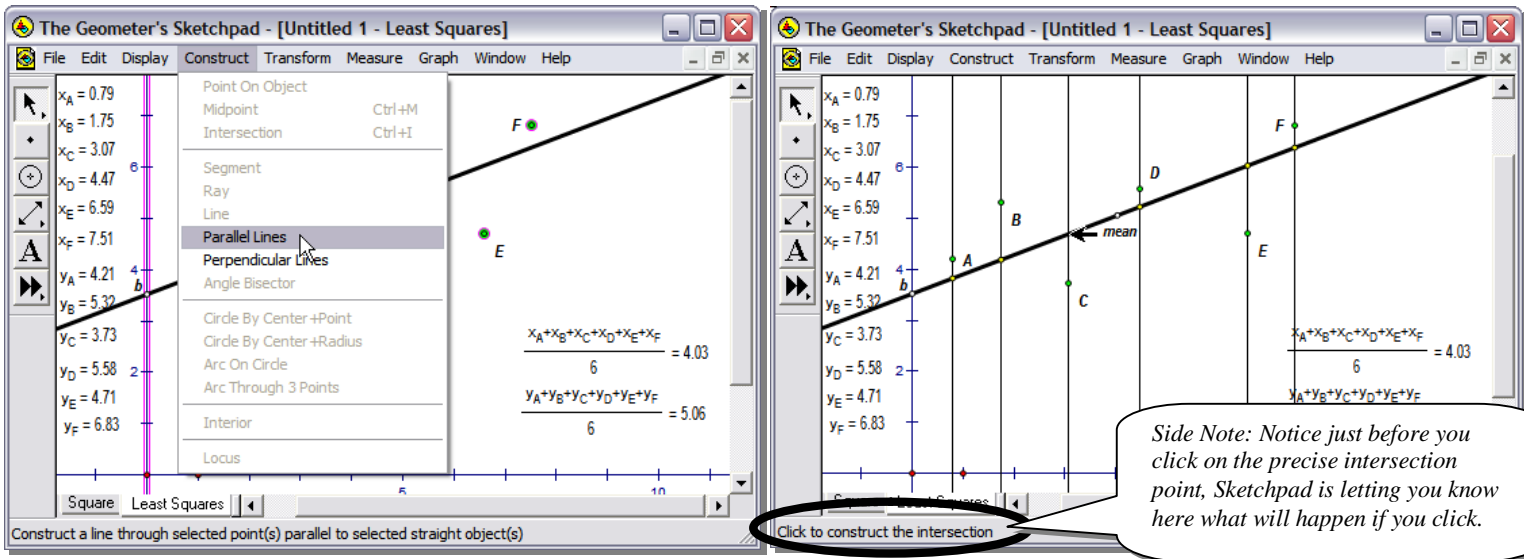
- Repeat the same steps for the Ordinate (y coordinates to find an average y.
- After the average x and average y have been determined. Deselect everything by clicking in a blank space and then highlight the average x,  $\left(\frac{x_A + x_B + x_C + x_D + x_E + x_F}{6}\right)$ , and then the average y,  $\left(\frac{y_A + y_B + y_C + y_D + y_E + y_F}{6}\right)$  in that order.


Finally, select **Plot As (x,y)** under the **Graph** menu as shown at the right.

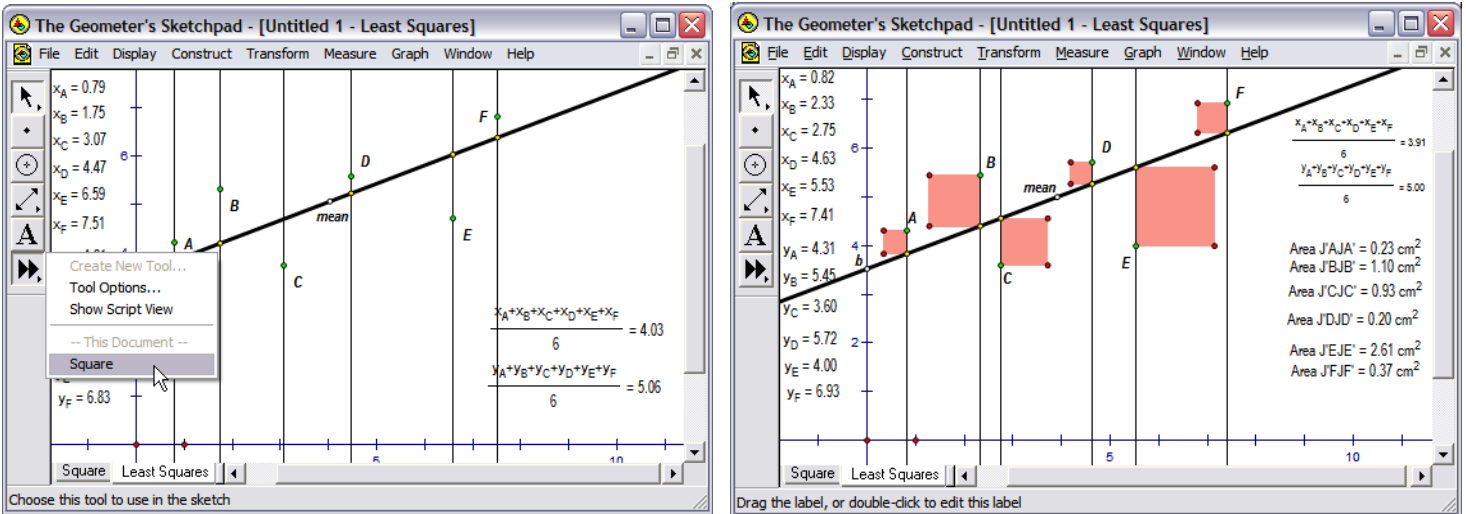
- After the mean point appears on the graph, you may want to change the color of the point and label it so that you don't mistake the point for one of the original scatter plot points.
- Highlight the y-axis and select Point On Axis (shown below at the bottom left).
- Label this point a lower case "b" it will represent the y-intercept for our least square line. Switch the Straight Edge tool to a line, , and create a line passing through the point "b" on the y-axis and the "mean point" (shown below at the bottom right).



- Highlight all of the original scatter plot points and the y-axis. Then, select **Parallel Lines** from the **Construct** menu (shown below at the left).
- Then, carefully click on the intersection of each newly created parallel line with the least squares line to create a point of intersection. (Side note: You should notice at the very bottom of the Sketchpad window a description of what Sketchpad will do if you click. This can be helpful when you are creating the intersection points by clicking on them.)

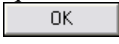


- Click and hold down on the Custom Tool Button, , and select the custom tool we made earlier, **Square**. Using this tool click on the scatter plot point first and then the intersection point of the parallel lines to create each square (You may need to experiment some to determine how best to create each square so that it is the most visible. You may also need to move some of the original points of the scatter plot around temporarily for better viewing.)

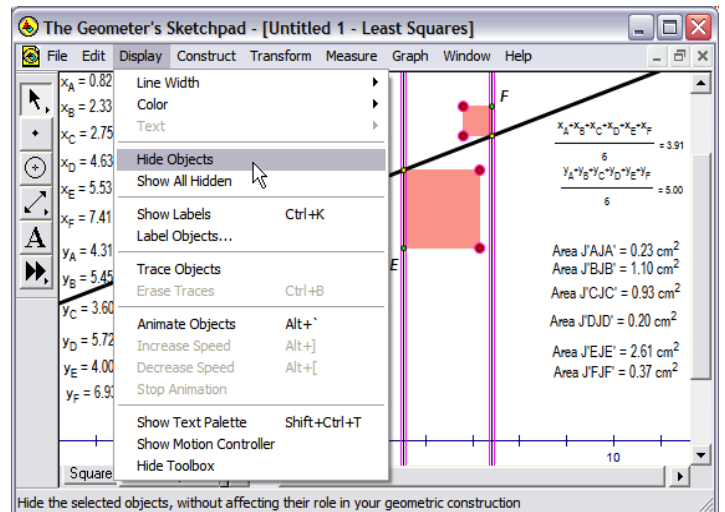


- Next for better viewing, highlight all of the parallel lines and the unnecessary points on the squares. Select **Hide Objects** under the **Display** menu.

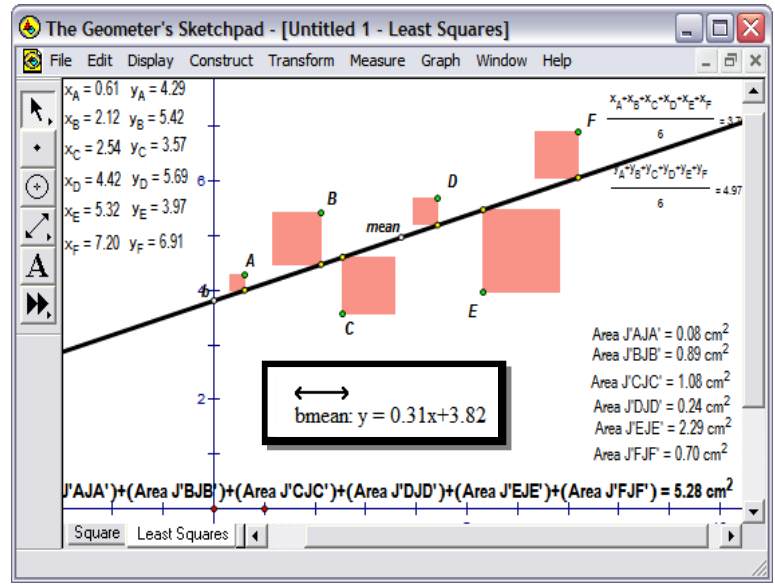
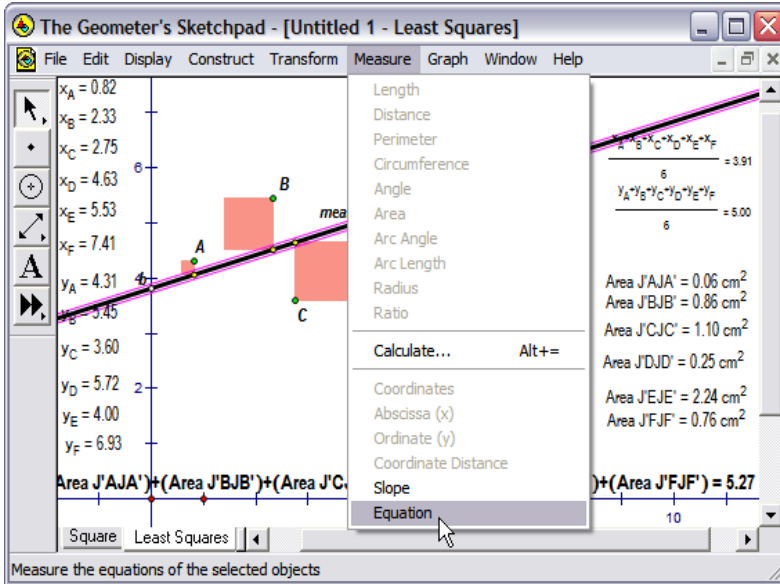
- Then, select **Calculate** under the **Measure** menu.

- As we did earlier when calculating the mean point, add together all of the square's areas together and press .

- Now Simply move the point "b" until the total sum of the areas is minimized. The result should be the least squares line.

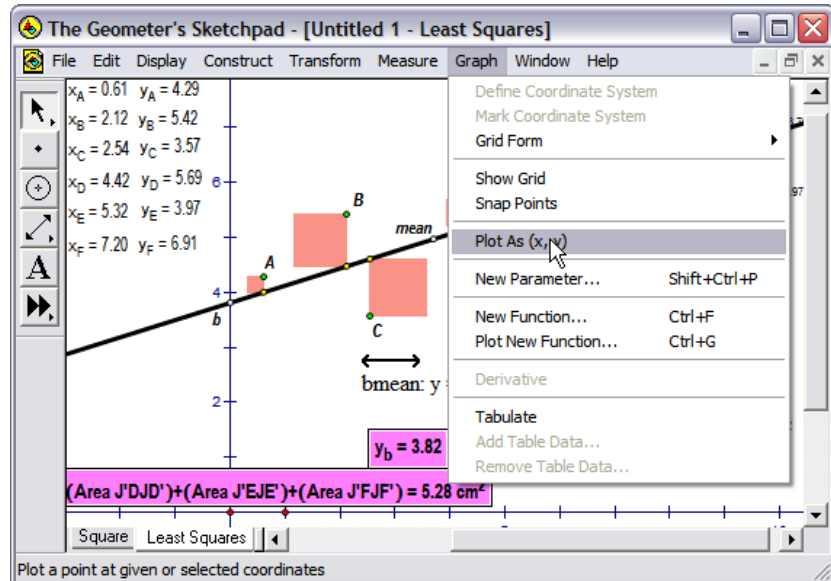


- Highlight the Least Squares line and select **Equation** under the **Measure** menu. You can verify the results with a Texas Instruments Calculator.



### Advanced Analysis:

- You can make an actual function plot of how the value of the y-intercept effects the overall sum of the squares. Highlight the “b” point on the least squares line. Select **Ordinate(y)** under the **Measure** menu.
- Highlight this the new ordinate measure and the combined sum of the areas measure (in that order) and select **Plot As (x,y)** under the **Graph** menu. (shown at the right)
- This should create a new point on the sketch. Look carefully at the measures you just plotted to determine its location. You may have to move the point “b” a little to see the newly plotted point.
- Once the new point is found, highlight the plotted point and the point “b”. Then select **Locus** under the Construct Menu.



- This should create a graph (a parabola) that represents the combined sum of the square's areas as a function of y-intercept “b”. In other words as you move the point “b” it shows you how the combined area of the squares is effected.

