

Geometry

Unit 1:

Congruence,

Angles, and Proofs

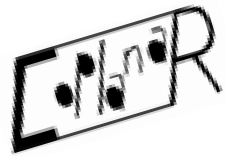
(Part 1)

			
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algebra



PERpendicular Lines



Co:llinear

Mathematics

MAtHew M. Winking

Define the following Geometric terms:

point:



LINE:



SEGMENT:



RAY:



Plane:



Collinear:



Concurrent:



ANGLE:



Parallel Lines:



Perpendicular Lines:





Circle:



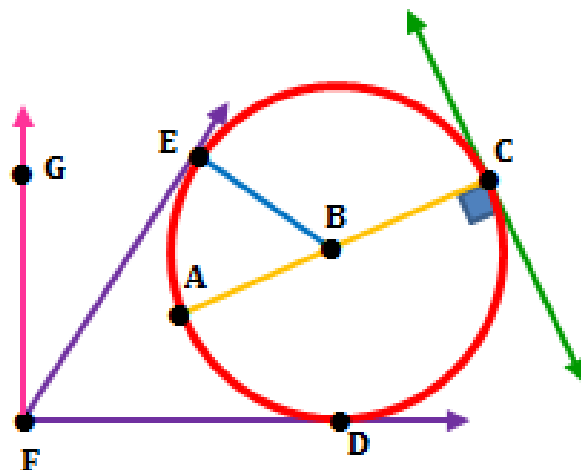
Use the word bank below to describe each object or set of objects.

Line	Point	Perpendicular	Segment	Plane
Circle	Parallel	Diameter	Ray	Radius
	Angle	Skew	Coplanar	Collinear

1. How would you best describe the **purple geometric shape** shown in the diagram at the right?

2. How would you best describe the **red geometric shape** shown in the diagram at the right?

3. How would you best describe the **center of the circle** shown in the diagram at the right?



4. How would you best describe the **pink geometric shape** shown in the diagram above?

5. How would you best describe the **blue geometric shape** shown in the diagram above?

6. How would you best describe the **green geometric shape** shown in the diagram above?

7. How would you best describe the **orange geometric shape** shown in the diagram above?

8. How would you best describe the relation between the set of geometric shapes **Point A, Point B, and Point C**?

9. How would you best describe the relation between the set of **green** and **orange** geometric shapes?

Use the word bank below to describe each object or set of objects.

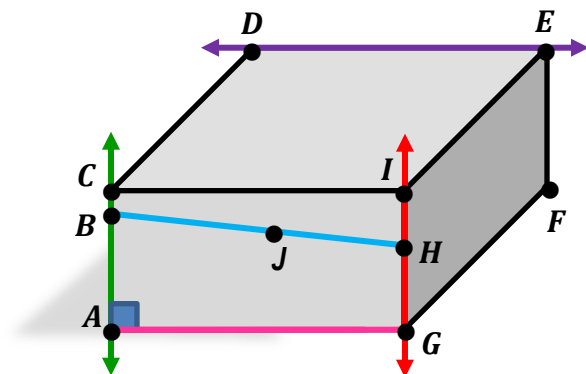
Line	Point	Perpendicular	Segment	Plane
Circle	Parallel	Diameter	Ray	Radius
	Angle	Skew	Coplanar	Collinear

The geometric shape shown in the diagram below is a 3-dimensional rectangular prism.

10. How would you best describe the relationship between the line \overleftrightarrow{AC} and the line \overleftrightarrow{GI} ?

11. How would you best describe the relationship between the line \overleftrightarrow{AC} and the segment \overline{AG} ?

12. How would you best describe the relationship between the line \overleftrightarrow{AC} and the line \overleftrightarrow{DE} ?



13. How would you best describe the set of **Point A, Point B, Point J and Point H**?

14. How would you best describe the set of **Point G, Point H, and Point I**?

15. (True or False) Any 3 distinct points are always coplanar.

16. (True or False) Any 2 distinct circles are always coplanar.

17. (True or False) If 2 lines intersect once then the lines are coplanar.

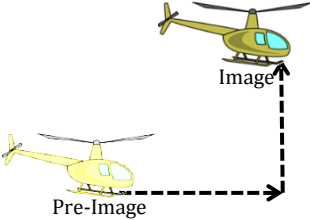
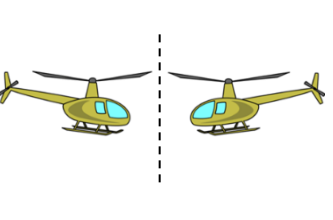
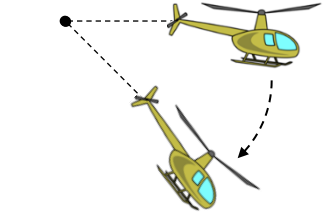
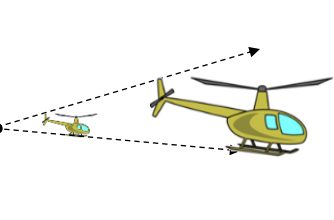
18. (True or False) Two lines that are skew can sometimes intersect.

19. (True or False) An angle and a circle can have more than 3 intersections.

20. (True or False) Two distinct circles can have more than 2 intersections.

21. (True or False) Any given line and point are always coplanar.

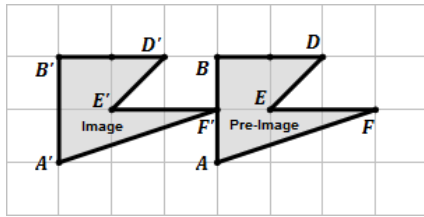
Transformation Types:

<p>Translation ↓ Translation</p>	<p>Reflection ↕ Reflection</p>	<p>Rotation ↻ Rotation</p>	<p>• Dilation → Dilation</p>
			

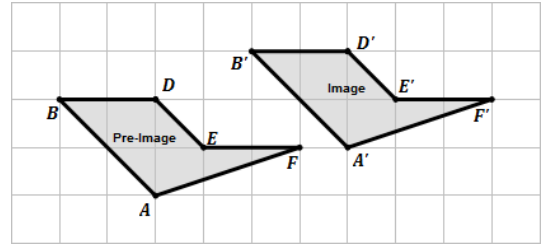
Translations

1. Describe the transformation in rectangular units from the Pre-Image to the Image for each of the following:

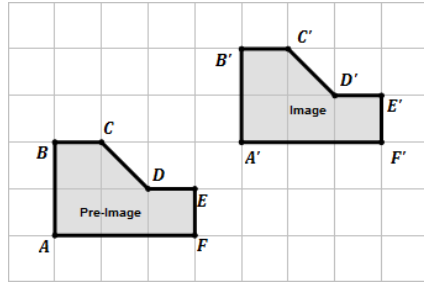
A.



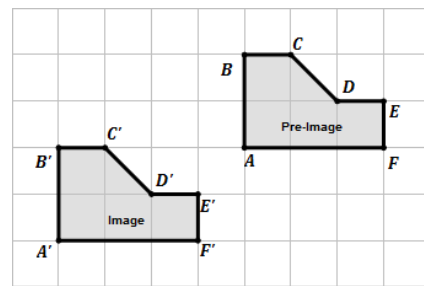
B.



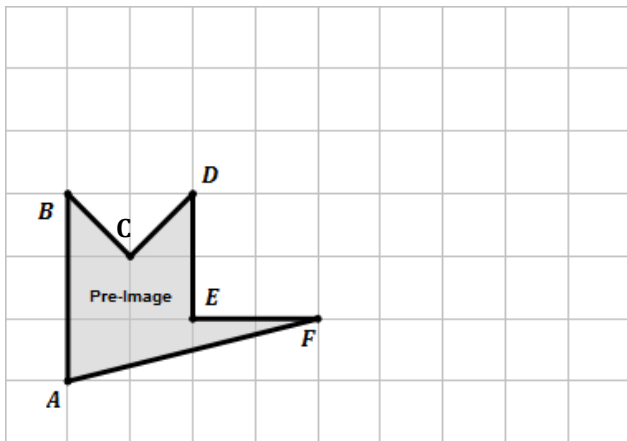
C.



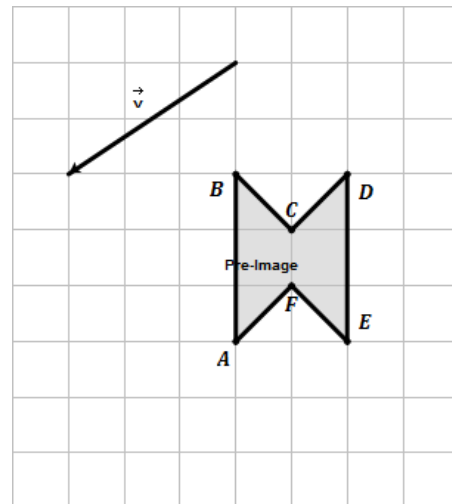
D.



2. Translate the following object right 4 units and up 1 unit. Label each vertex appropriately.

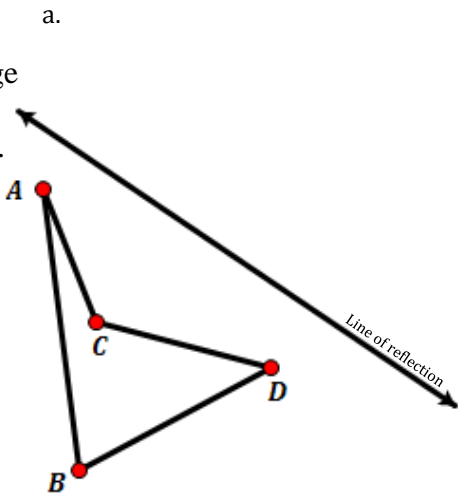


3. Translate the following object by the vector \vec{v} .

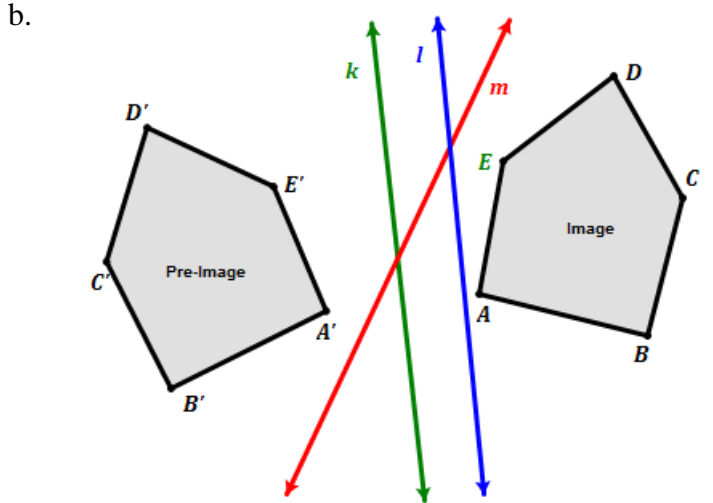
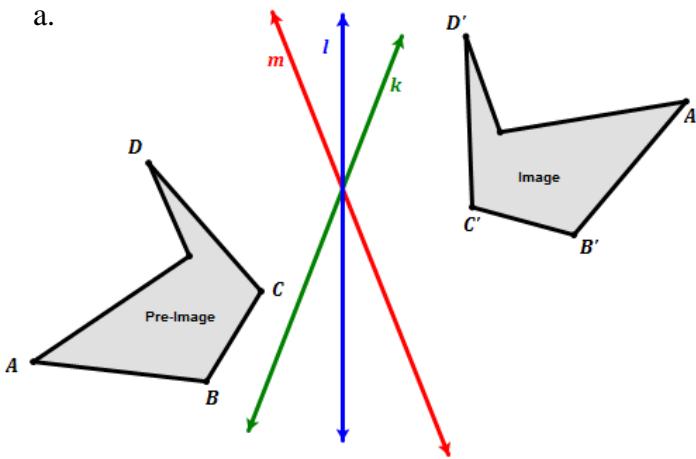


Reflections

4. Create a reflection of the Pre-image over the line of reflection.

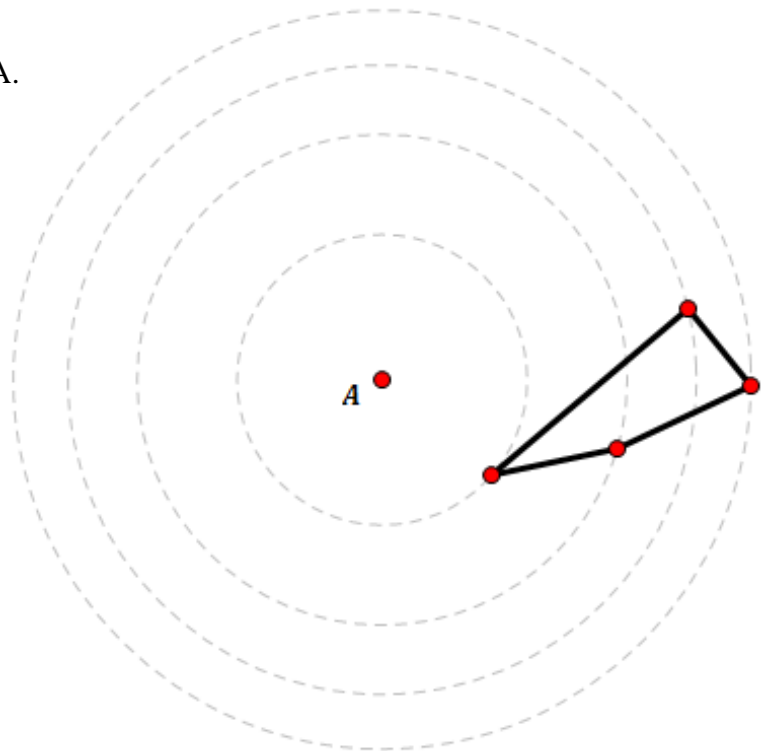


5. Determine which is the correct line of reflection in each diagram between the Image and Pre-Image?

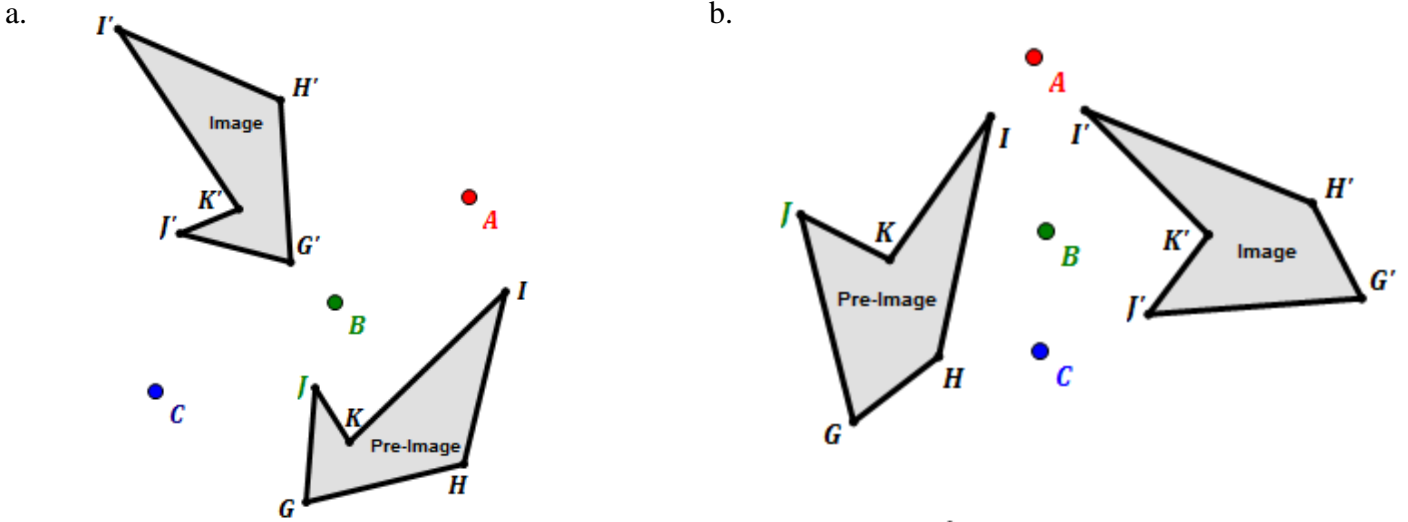


Rotations

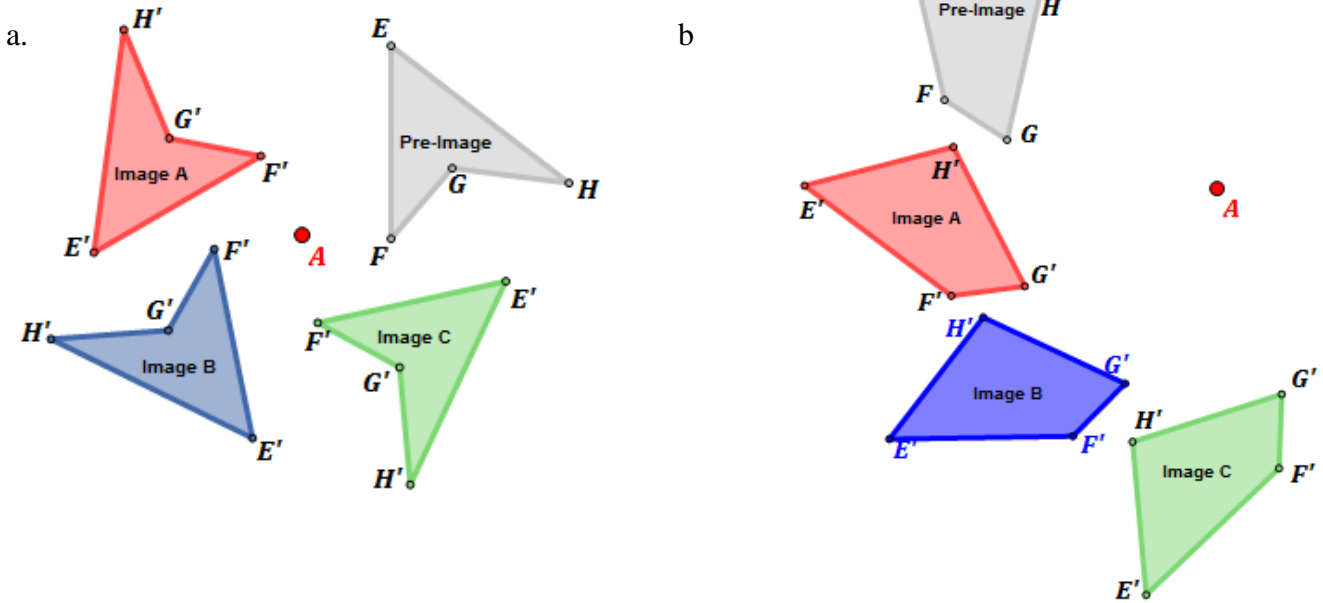
6. Rotate the following polygon 110° about the point A.



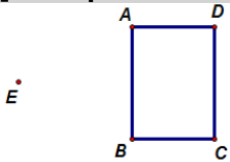
7. Determine which is the correct center if an 80° rotation was used?



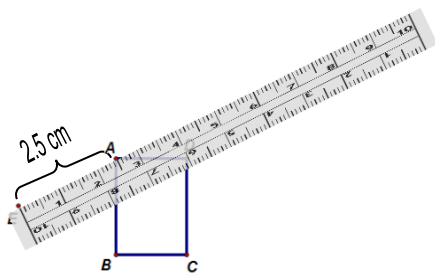
8. Which Image is a rotation of 120° about the point A?



[Example Dilation]: Dilate the $\square ABCD$ by a factor of 2.0 from point E.

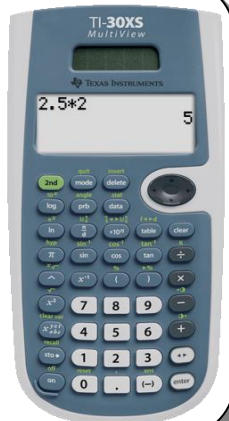


Step 1: Measure the distance from the point of dilation to a point to be dilated (preferably using centimeters).

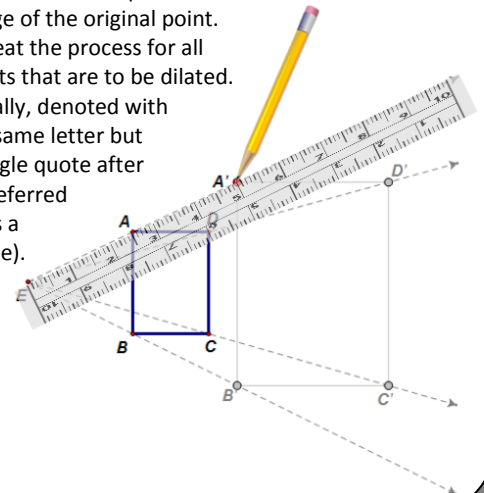


Step 2: Multiply the measured distance by the scale factor.

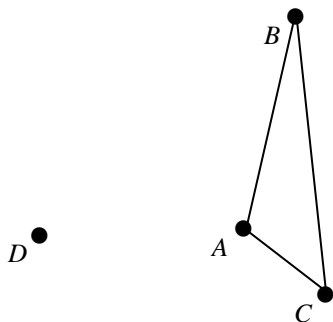
$$2.5 \text{ cm} \times 2.0 = 5 \text{ cm}$$



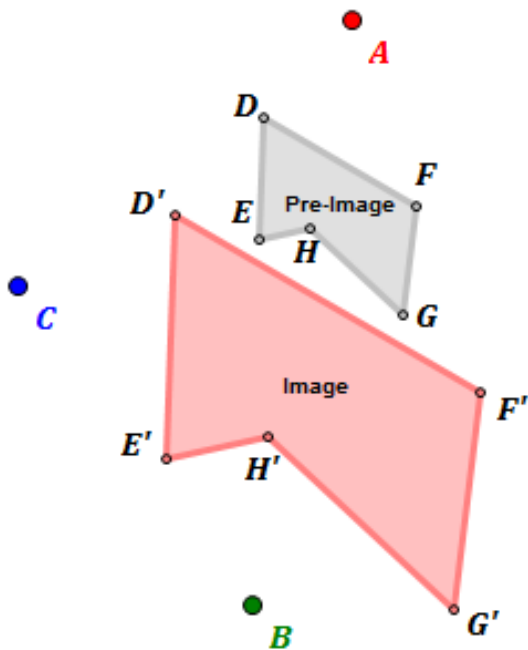
Step 1: With the ruler in the same place as it was in step #1, mark a point at the measured distance determined in step #2 as the image of the original point. Repeat the process for all points that are to be dilated. Usually, denoted with the same letter but a single quote after it (referred to as a prime).



9. Dilate the $\triangle ABC$ by a factor of $\frac{3}{2}$ from point D.

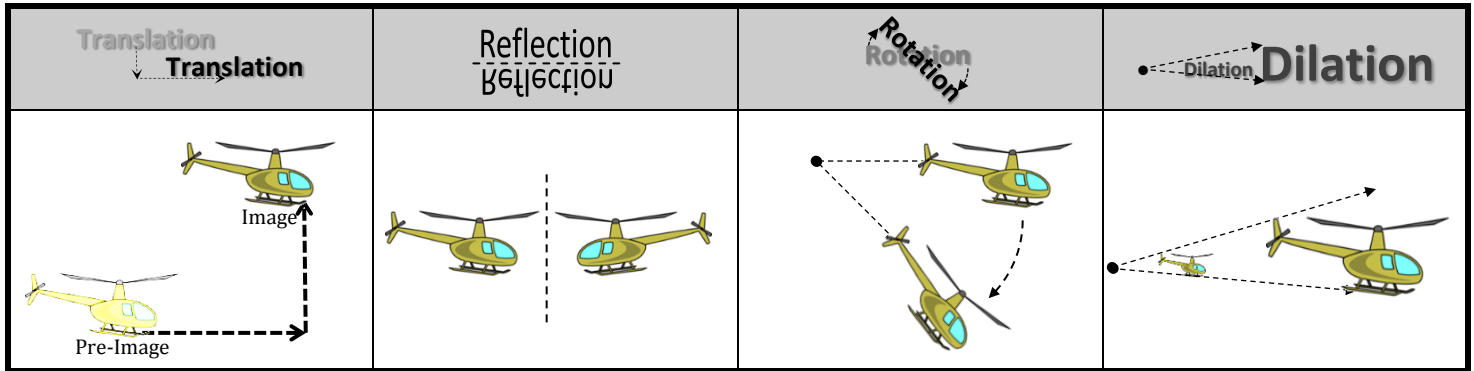


10. Which is the correct point of Dilation if the pre-image was dilated by a factor of 2?



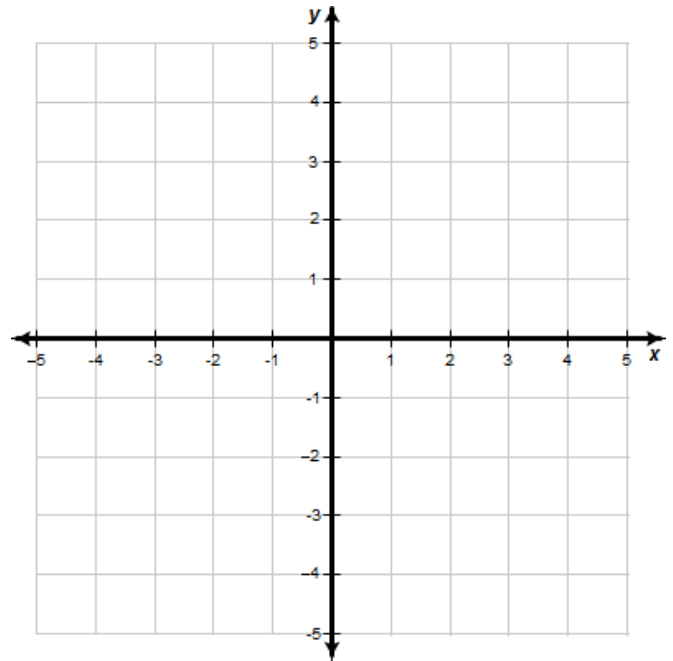
11. Which of the four transformations are isometries?

Transformation Types:



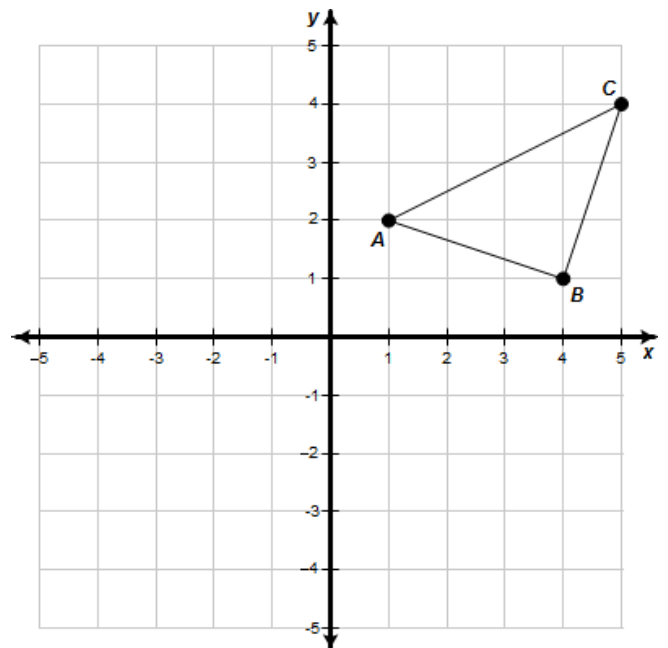
Translations

- Plot the points and create the pre-image triangle with vertices $M(-2, 3)$, $N(-3, 5)$, and $P(-5, 2)$. Then, determine the coordinates M' , N' , and P' of the image triangle that has been translated right 6 and down 3. Explain what you did to each of the coordinates.



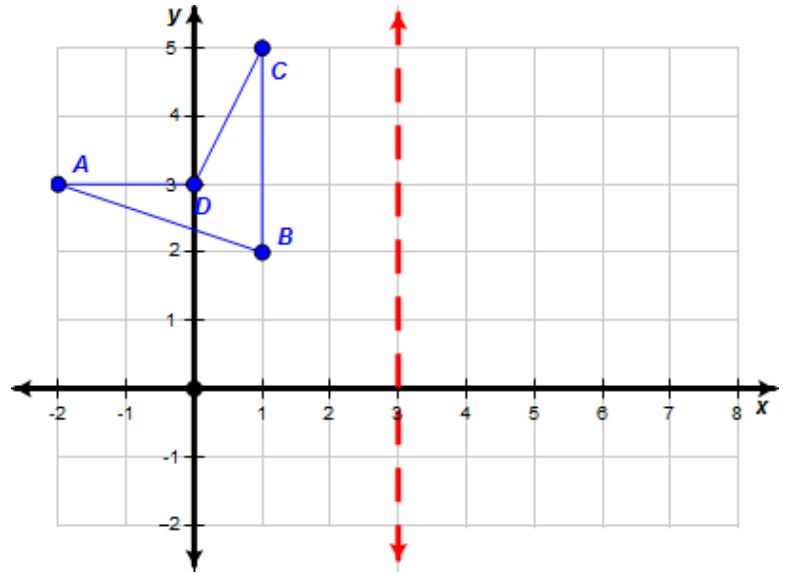
Reflections

- List the coordinates of the triangle ABC
- Reflect the triangle ABC over the **y-axis** and list the coordinates of the vertex A' , B' , and C' . Describe what happened to the coordinates from the pre-image to the image.
- Reflect the triangle ABC over the **x-axis** and list the coordinates of the vertex A' , B' , and C' . Describe what happened to the coordinates from the pre-image to the image.



Reflections

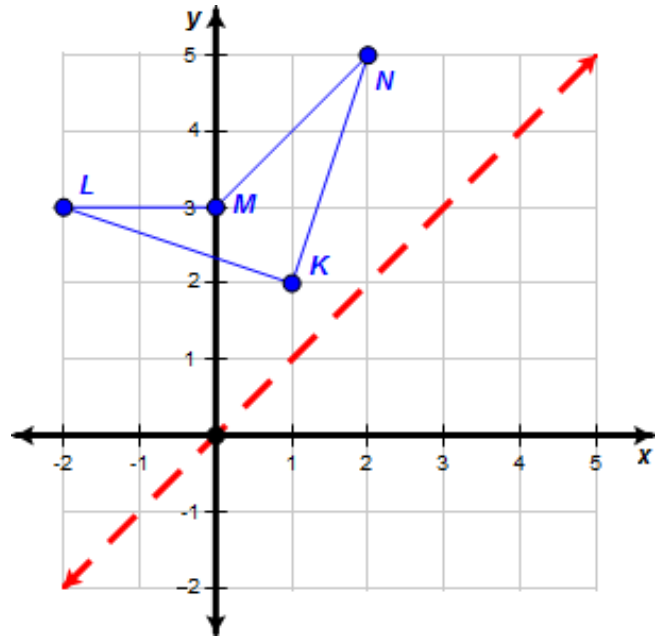
5. List the coordinates of the quadrilateral ABCD



6. Reflect the quadrilateral ABCD over the line $x = 3$ and list the coordinates of the vertex A' , B' , C' , and D' .

Reflections

7. List the coordinates of the quadrilateral KLMN



8. Reflect the quadrilateral KLMN over the line $y = x$ and list the coordinates of the vertex K' , L' , M' , and N' . Describe what happened to the coordinates from the pre-image to the image.

Reflections

9. If the point A is located at $(-3, 2)$ and A' is the image of A after being reflected over the x -axis, what are the coordinates of A' ?

10. If the point B is located at $(-4, -1)$ and B' is the image of B after being reflected over the y -axis, what are the coordinates of B' ?

11. If the point C is located at $(2, -3)$ and C' is the image of C after being reflected over the line $y = x$, what are the coordinates of C' ?

Rotations

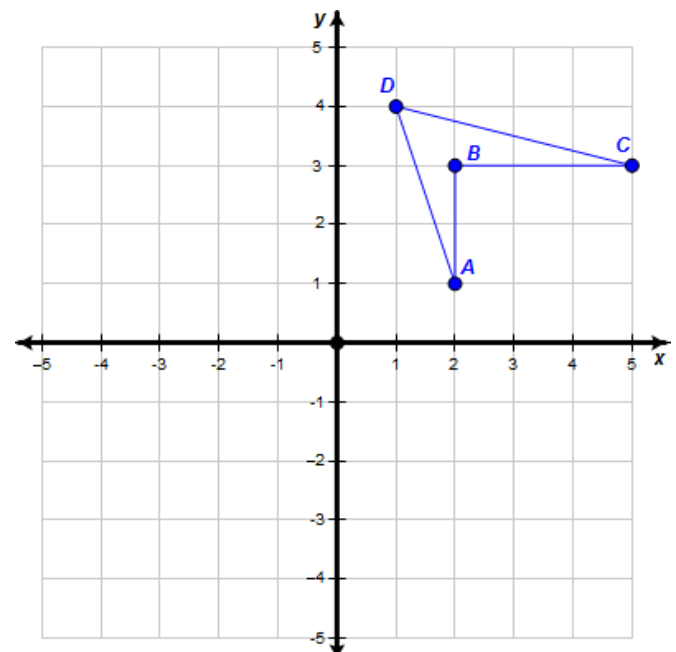
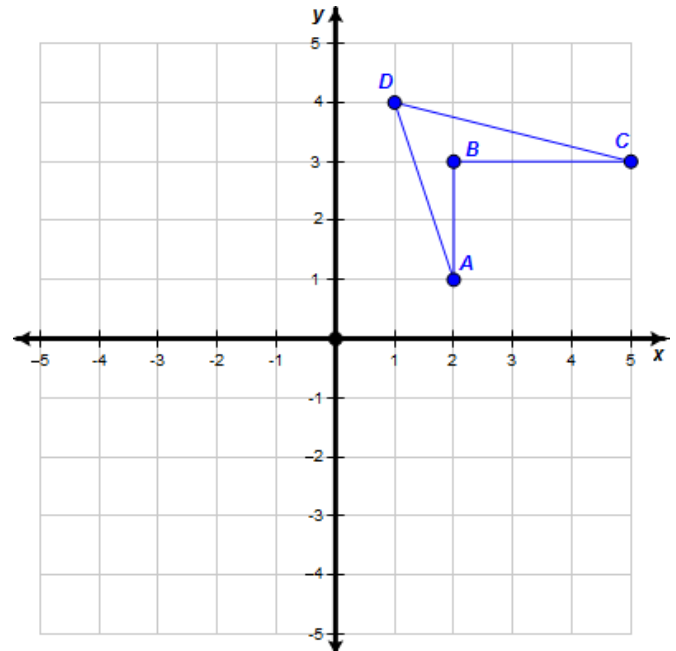
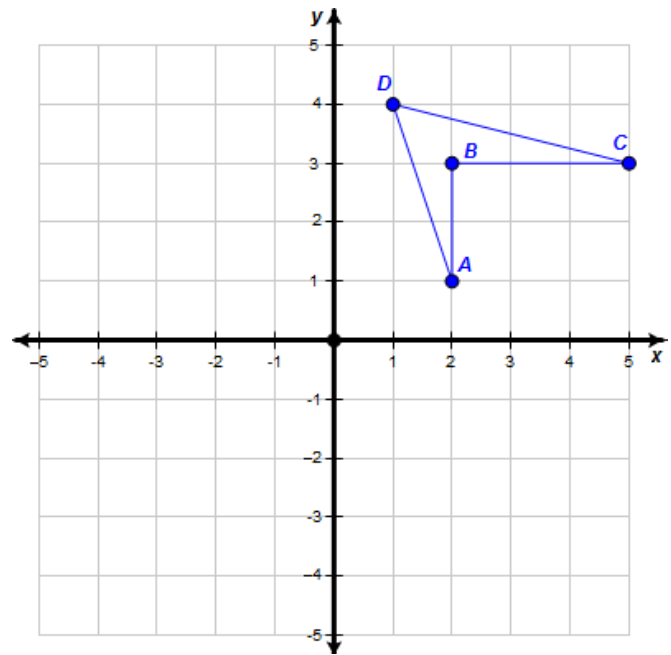
12. List the coordinates of the quadrilateral ABCD.

13. Rotate the quadrilateral **ABCD about the origin by 90°** and list the coordinates of the vertex A' , B' , C' , and D' . Describe what happened to the coordinates from the pre-image to the image.

14. Rotate the quadrilateral **ABCD about the origin by 180°** and list the coordinates of the vertex A' , B' , C' , and D' . Describe what happened to the coordinates from the pre-image to the image.

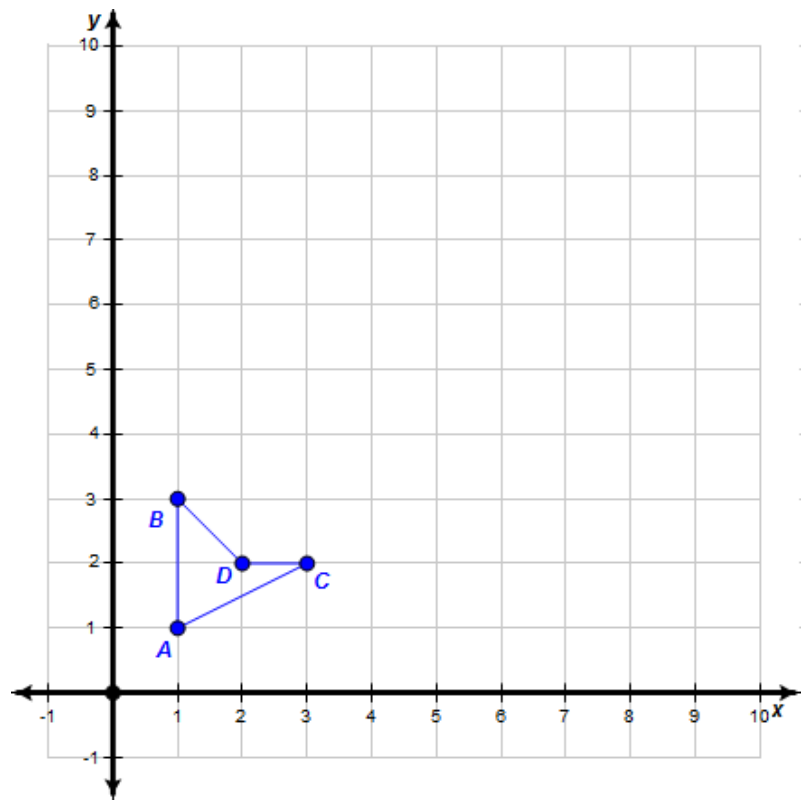
15. Rotate the quadrilateral **ABCD about the origin by 270°** and list the coordinates of the vertex A' , B' , C' , and D' . Describe what happened to the coordinates from the pre-image to the image.

16. If the point A is located at $(-3, 2)$ and A' is the image of A after being rotated about the origin by 270° . What are the coordinates of A' ?



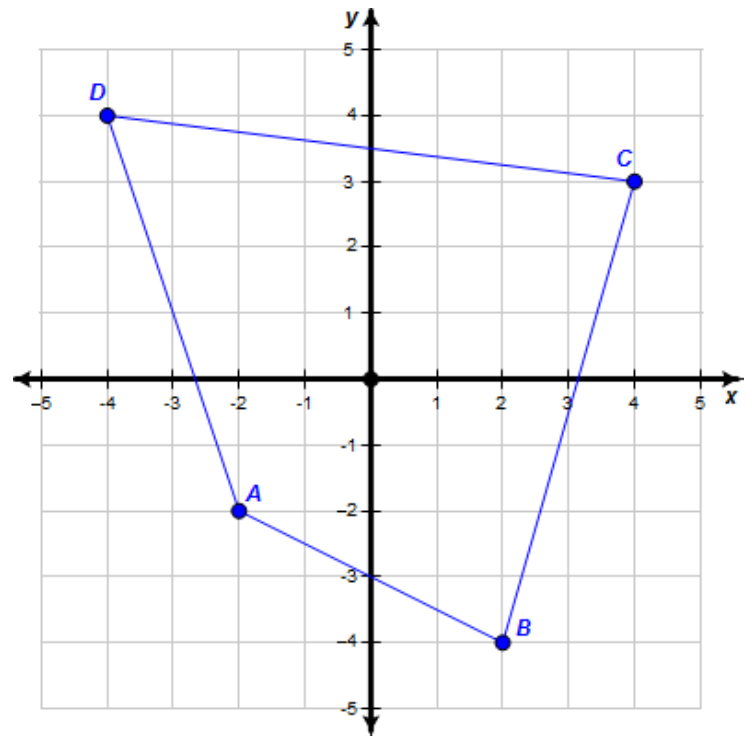
Dilations

17. List the coordinates of the quadrilateral ABCD.



18. Dilate the quadrilateral ABCD **by a scale factor of 3 from the origin** and list the coordinates of the vertex A', B', C', and D'. Describe what happened to the coordinates from the pre-image to the image.

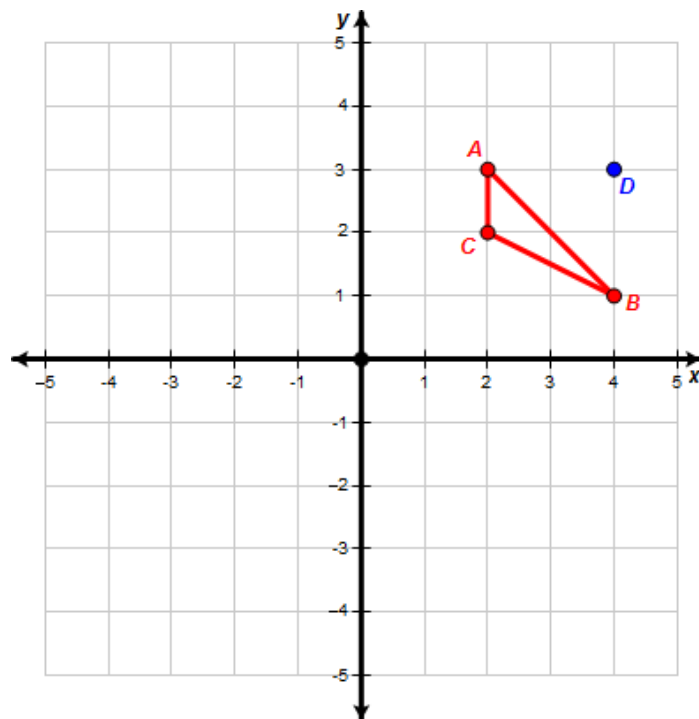
19. List the coordinates of the quadrilateral ABCD.



20. Dilate the quadrilateral ABCD **by a scale factor of $\frac{1}{2}$ from the origin** and list the coordinates of the vertex A', B', C', and D'. Describe what happened to the coordinates from the pre-image to the image.

21. If the point A is located at $(3, -2)$ and A' is the image of A after being dilated by a scale factor of 5 from the origin. What are the coordinates of A'?

22. Dilate the triangle ABC by a scale factor of 4 from the point D and list the coordinates of the vertex A', B', and C'.

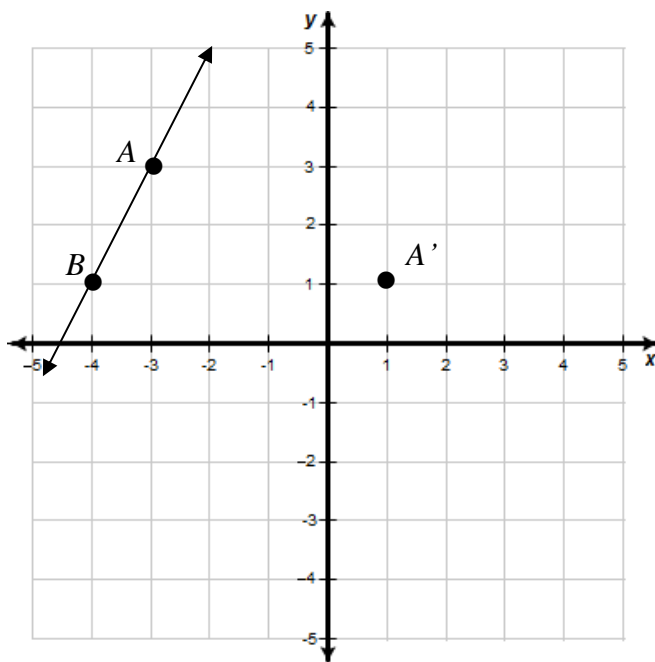


Create a list of all of the basic coordinate transformation rules:

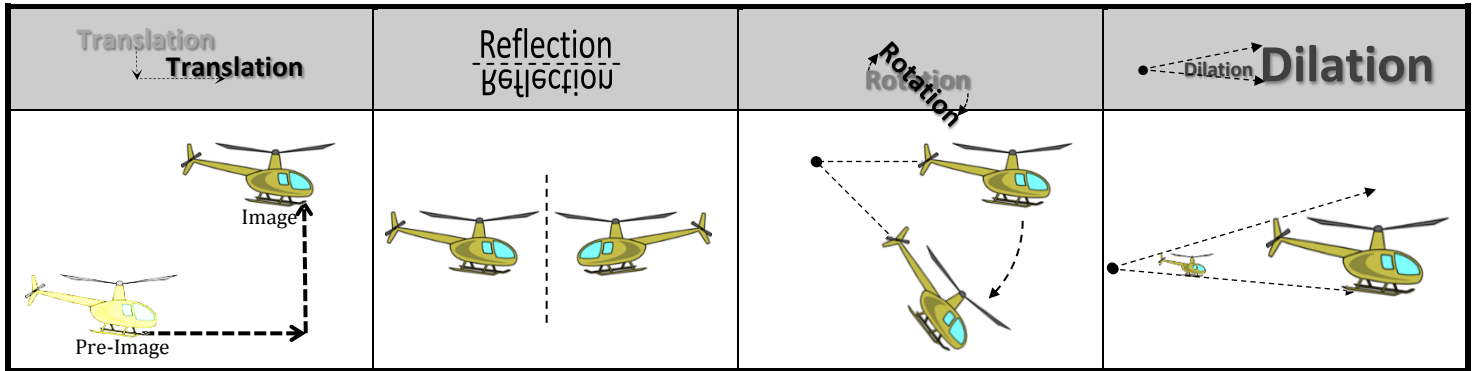
Translations

23. Point A $(-3, 3)$ is on \overleftrightarrow{AB} . A translation moves the point A to its image $A'(1,1)$.

What is the distance, in units, between any point on \overleftrightarrow{AB} and its image?



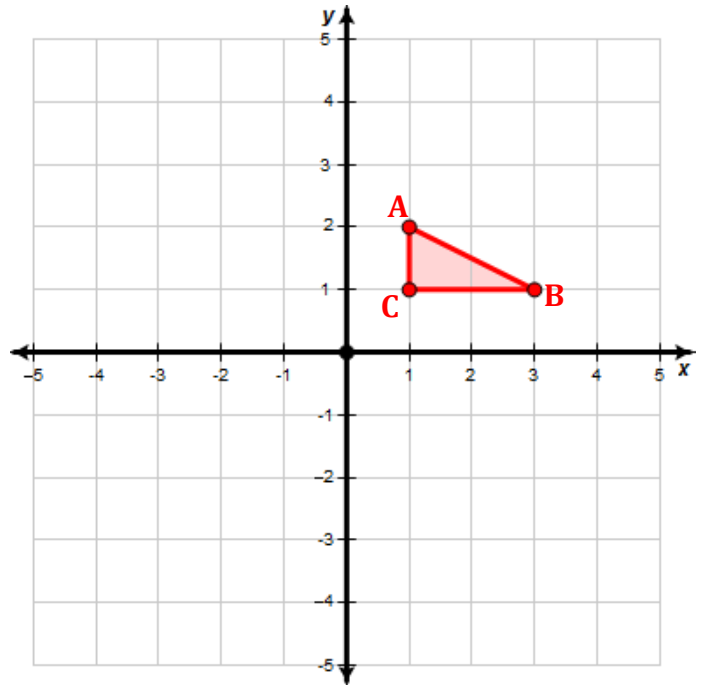
Transformation Types:



1. Consider the pre-image triangle with vertices $A(1,2)$, $B(3,1)$, and $C(1,1)$.

a. Rotate the pre-image triangle ABC 90° about the origin and label this triangle $A'B'C'$

b. Reflect the triangle $A'B'C'$ over the x -axis and label this triangle $A''B''C''$.

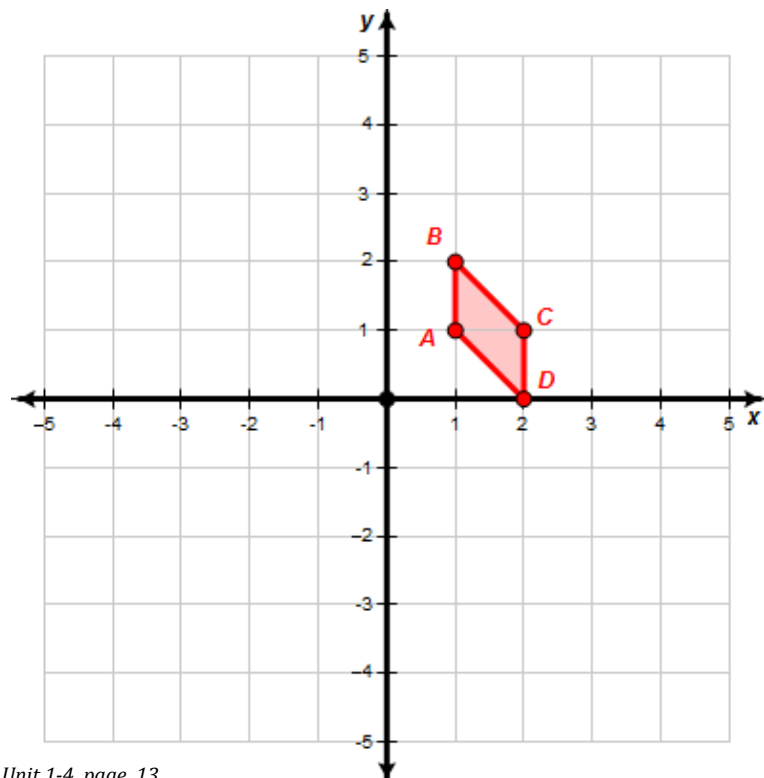


2. Consider the pre-image quadrilateral with vertices $A(1,1)$, $B(1,2)$, $C(2,1)$, and $D(2,0)$

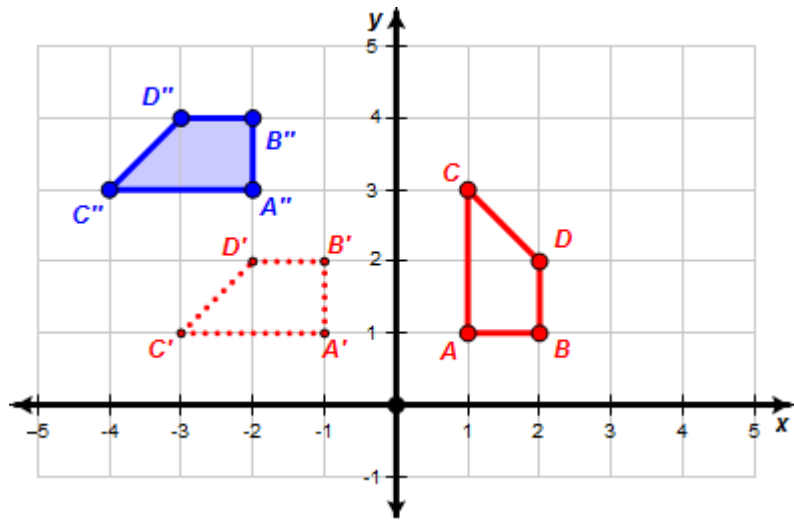
a. Dilate the quadrilateral by a factor of 2 from the origin and label the image quadrilateral $A'B'C'D'$.

b. Translate the quadrilateral image $A'B'C'D'$ down 5 units and left 1 unit. Label this new image $A''B''C''D''$.

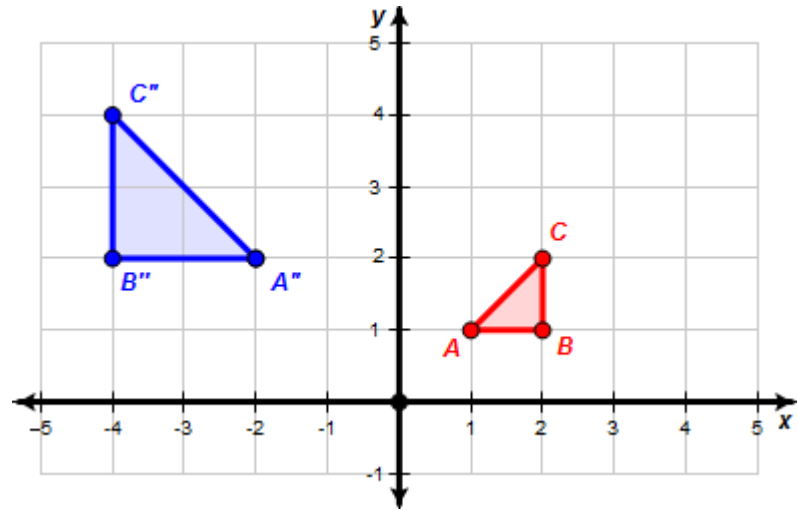
c. Reflect the quadrilateral image $A''B''C''D''$ over the y -axis and label this image $A'''B'''C'''D'''$



3. Describe two transformations that would map quadrilateral ABCD onto quadrilateral A''B''C''D''

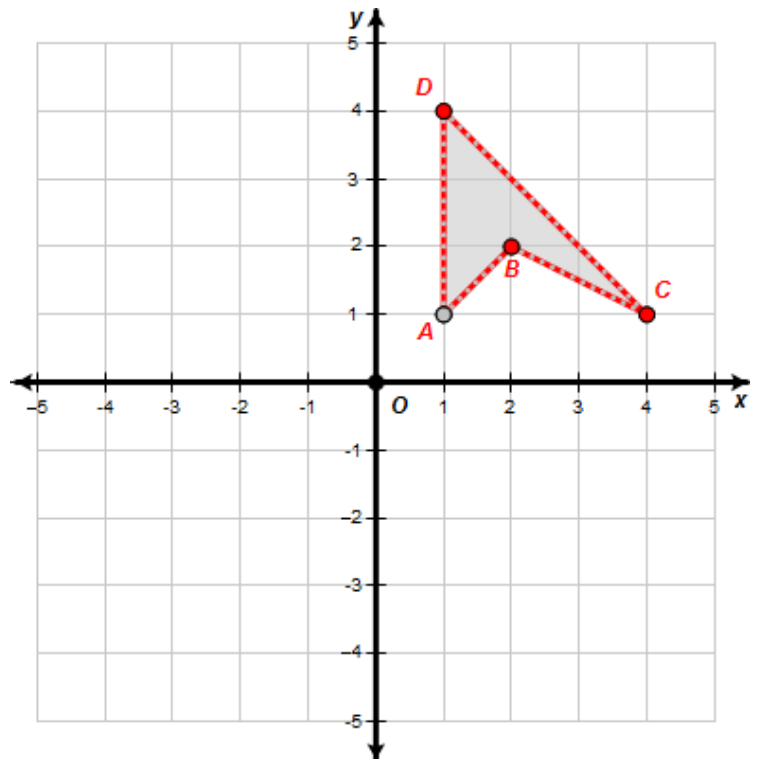


4. Describe two transformations that would map triangle ABC onto triangle A''B''C''

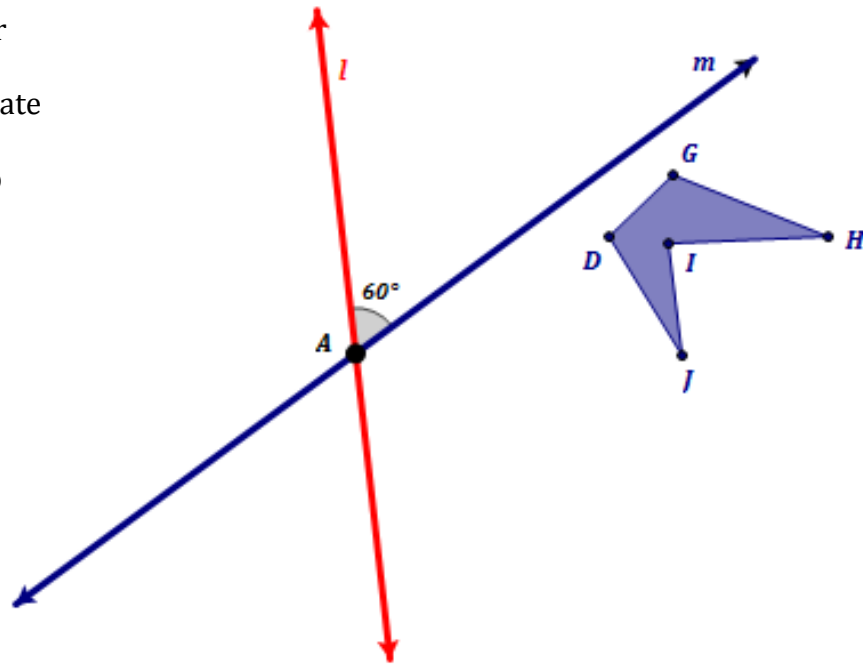


5. First reflect quadrilateral ABCD over the y-axis and label the triangle A'B'C'D'. Then, reflect quadrilateral A'B'C'D' over the x-axis and label this quadrilateral A''B''C''D''.

6. Anytime you use a double reflection, there should be a rotation about the intersection of the reflection lines that maps the pre-image onto the final image. In this example what is the amount of the rotation?

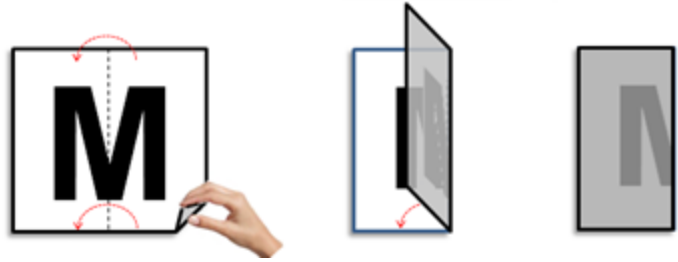


7. Given that pentagon $DGHIJ$ is first reflected over **line m** to create the image $D'G'H'I'J'$. Then, the image $D'G'H'I'J'$ is reflected over the **line l** to create the image $D''G''H''I''J''$. What is a different transformation that would also map $DGHIJ$ onto $D''G''H''I''J''$?

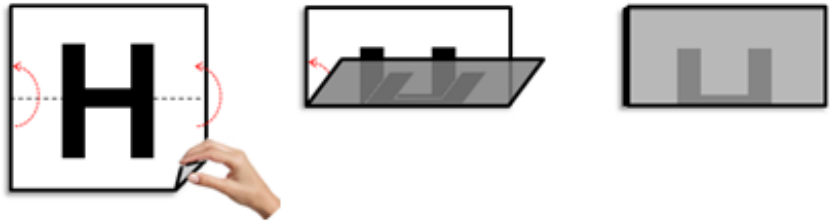


Symmetries

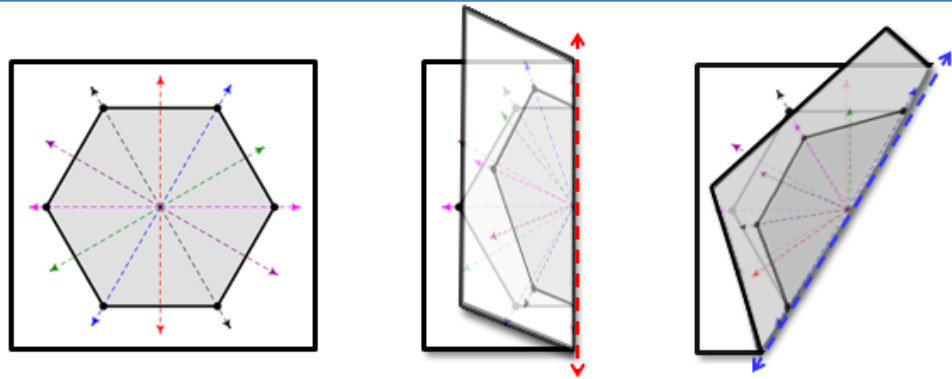
A shape has a **vertical line of symmetry** if you can fold it in half vertically and have the halves match up. On a scrap sheet of paper try folding the letter "M" in half.



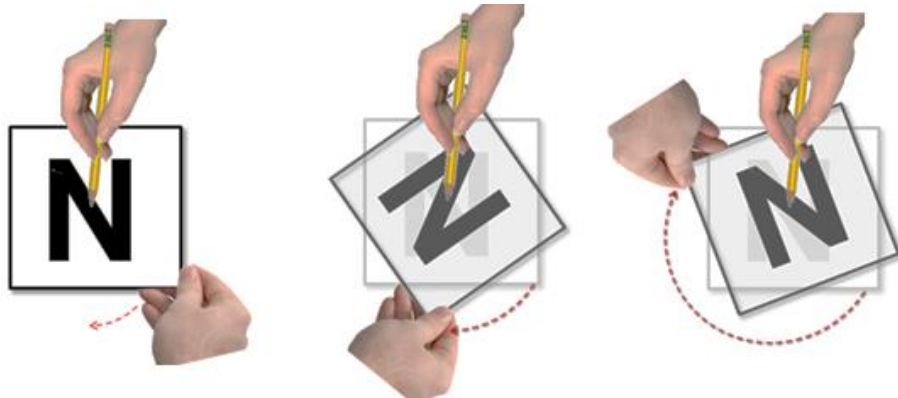
A shape has a **horizontal line of symmetry** if you can fold it in half horizontally and have the halves match up. On a scrap sheet of paper try folding the letter "H" in half.



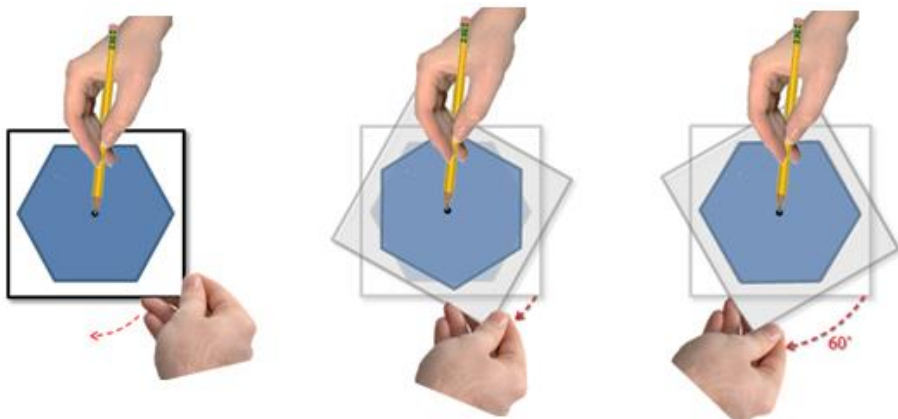
A general **line of symmetry** is any line in which you can fold the shape in half and it maps onto itself. Consider the hexagon at the right has 6 lines of symmetry.



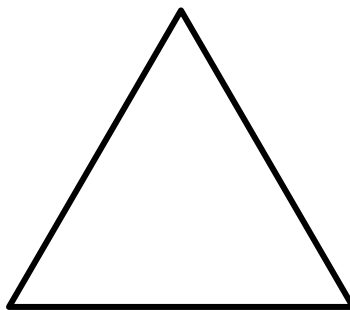
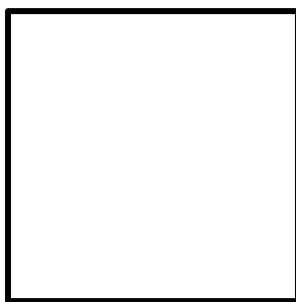
A shape has **point symmetry** if you can rotate the shape 180° from a center point and have it look the same as it did before you rotated it. On a scrap sheet of paper try rotating a letter "N" as shown at the right.



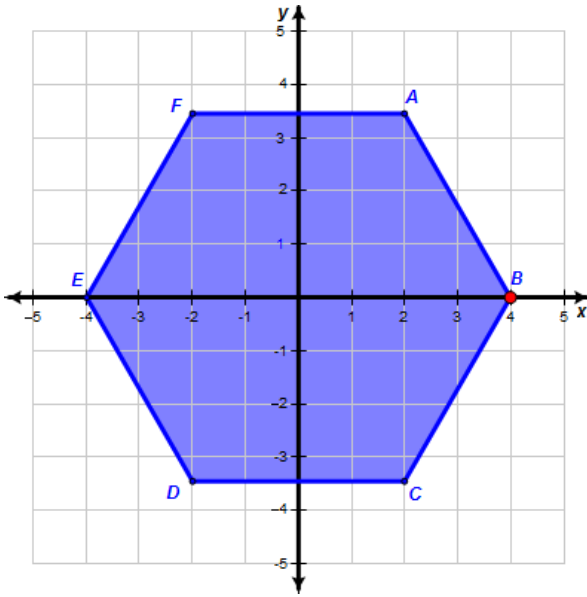
A shape has **rotational symmetry** if you can rotate the shape by a set degree from a center point and have it look the same as it did before you rotated it. On a scrap sheet of paper try rotating a hexagon (which has 60° rotational symmetry.)



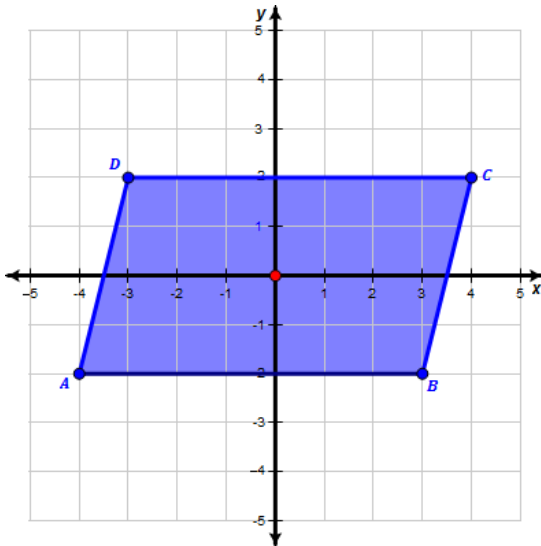
1. Provide 2 Letters of the Alphabet that have a **vertical line of symmetry**.
2. Provide 2 Letters of the Alphabet that have a **horizontal line of symmetry**.
3. Provide 2 Letters of the Alphabet that have both a **horizontal line of symmetry** and **vertical line of symmetry**.
4. Provide a letter of the Alphabet that has **point symmetry** but **NOT** a **vertical line of symmetry**.
5. Which letter depending on how it's written could have **infinite lines of symmetry**?
6. Draw all the lines of symmetry for the following regular shapes.



7. Describe in detail at least three transformations that would map hexagon ABCDEF onto itself.



8. Describe any symmetries that parallelogram ABCD might have.



9. Ambigrams are usually words created with point symmetry.

