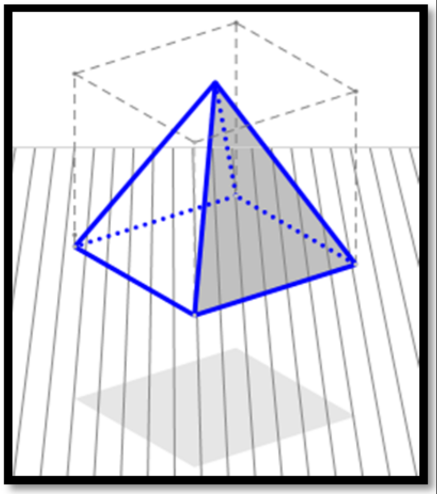
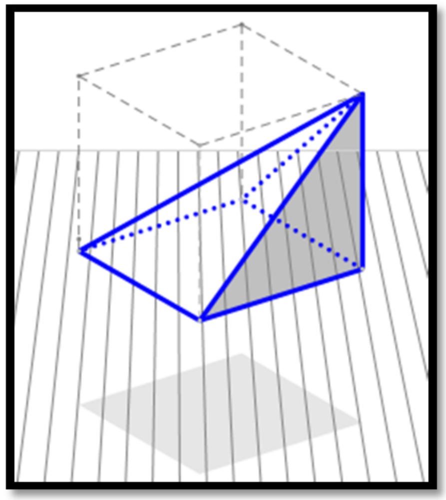
Sec 5.3 – Circles & Volume
Volume of Pyramids & Cones

Name:

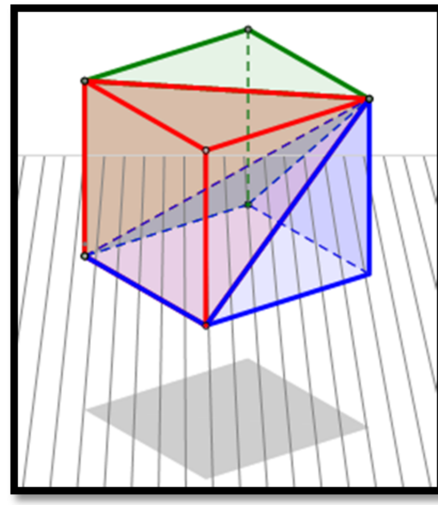
Using Cavalieri's Principle we can show that the volume of a pyramid is exactly ⅓ the volume of a prism with the same Base and height.



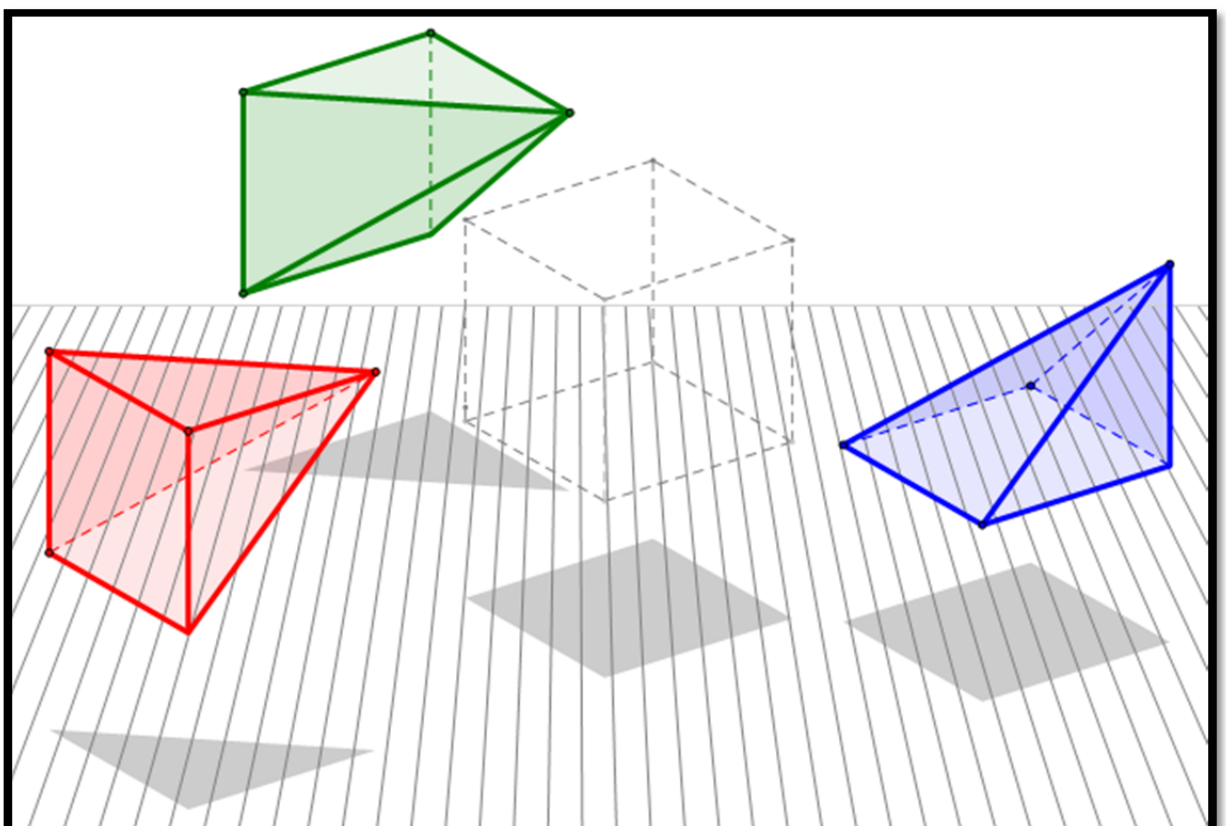
Consider a square based pyramid inscribed in cube.



Next, translate the peak of the pyramid. Cavalieri's Principle would suggest that the volume of the oblique pyramid is the same as the original pyramid.

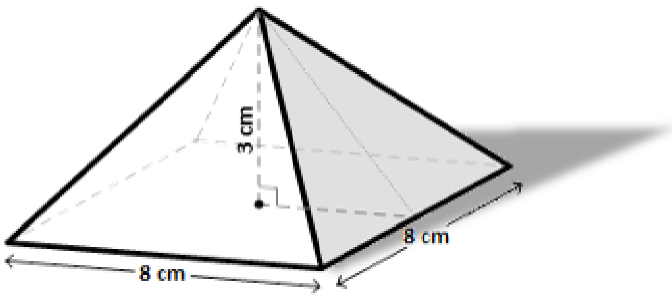


Next, we can create 2 more oblique pyramids with the same volume of the original with the remaining space in the cube.

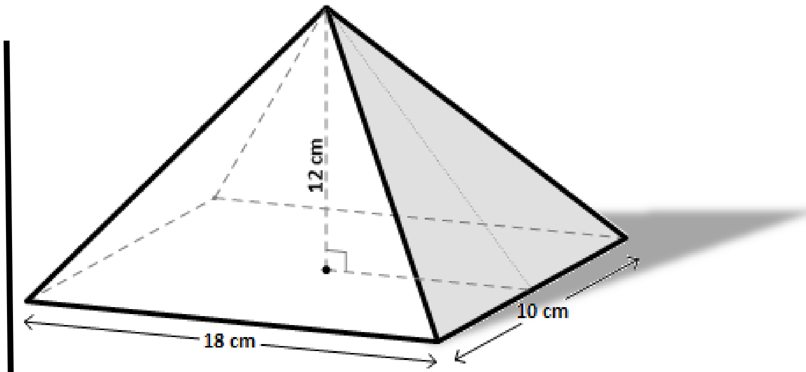


In this diagram, we can see the 3 oblique pyramids of equal volume pulled out from the cube. So, this demonstrates a pyramid inscribed in a cube has exactly ⅓ the volume the cube. This idea can be extended to any pyramid or cone.

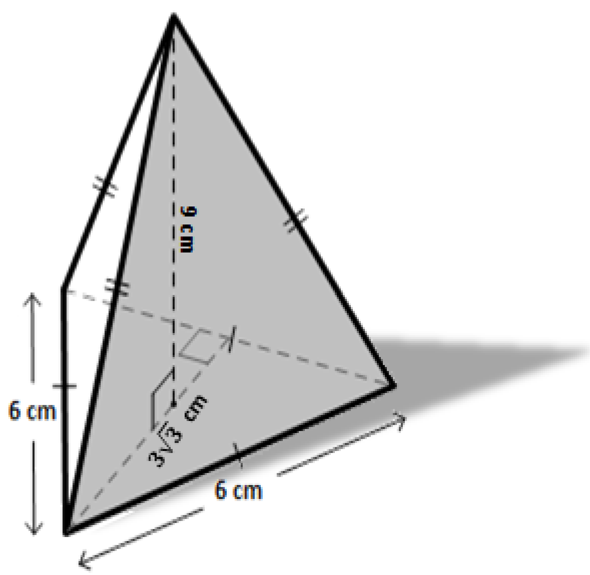
1. Find the Volume of the following solids (figures may not be drawn to scale).



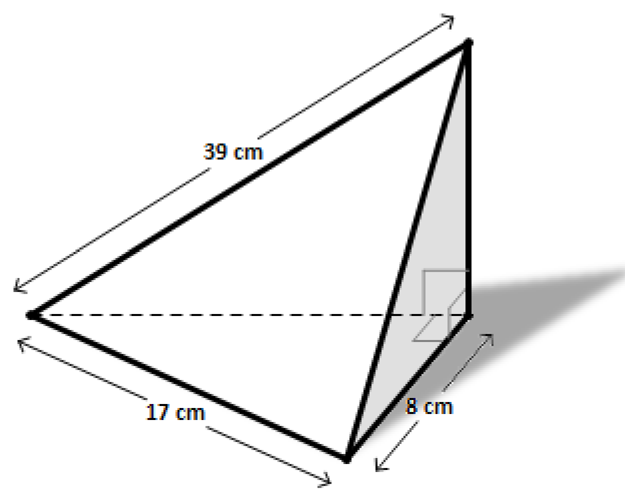
Volume:



Volume:

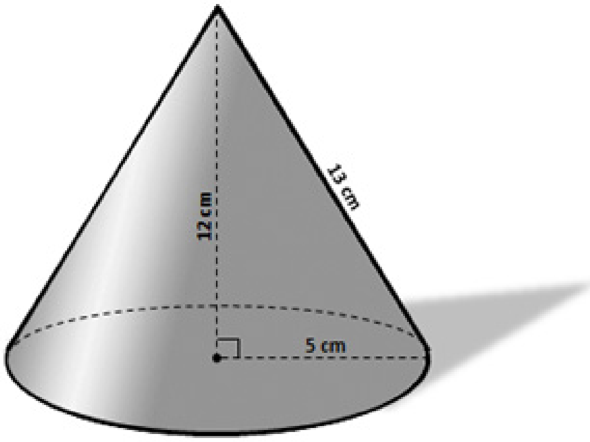


Volume:

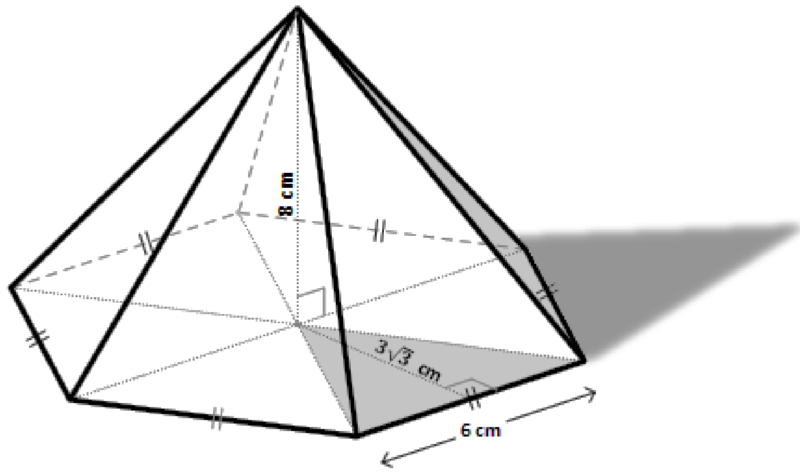


Volume:

2. Find the Volume of the following solids (figures may not be drawn to scale).

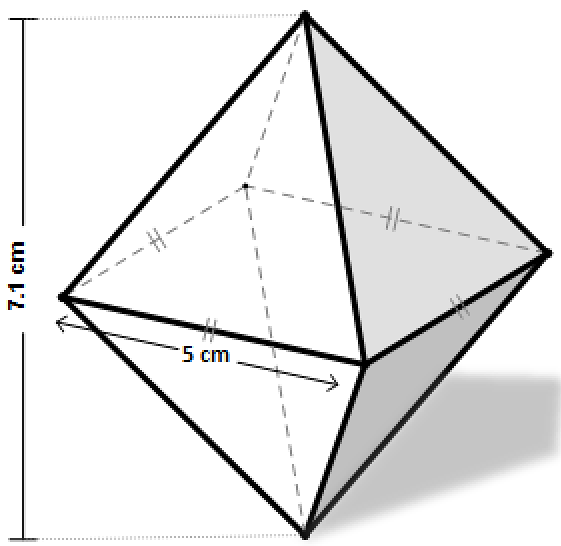


Volume:



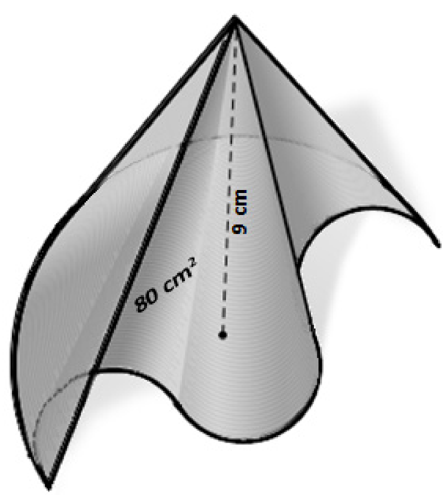
Volume:

Find the volume of the regular octahedron.



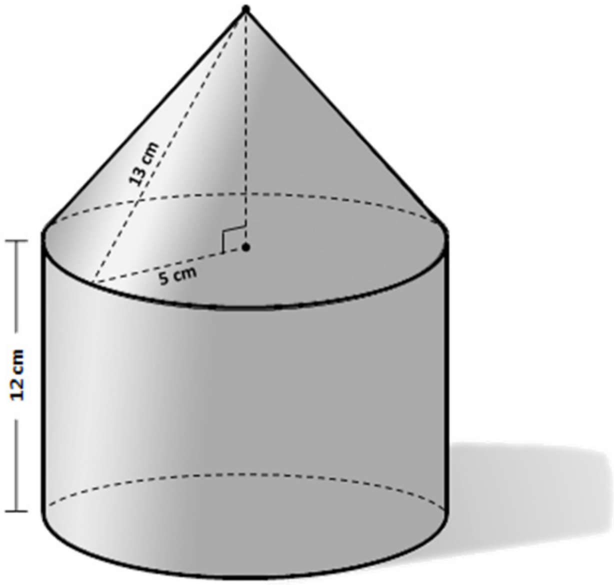
Volume:

Find the volume of the irregular solid. The base has an area of 80cm² and a height of 9 cm.

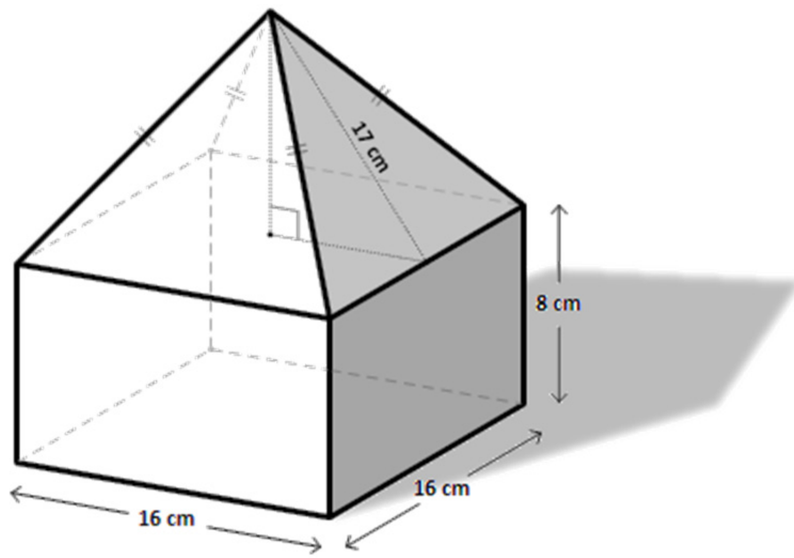


Volume:

3. Find the Volume of the following solids (figures may not be drawn to scale).

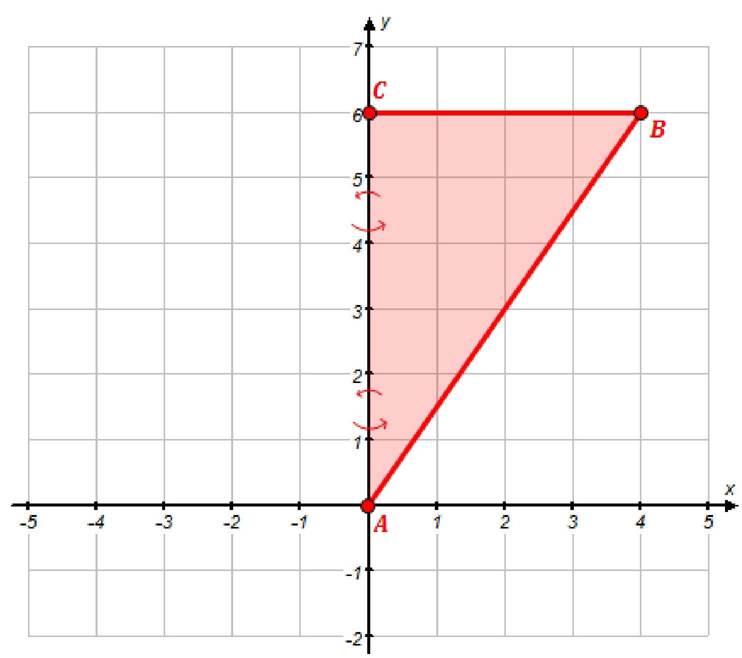


Volume:



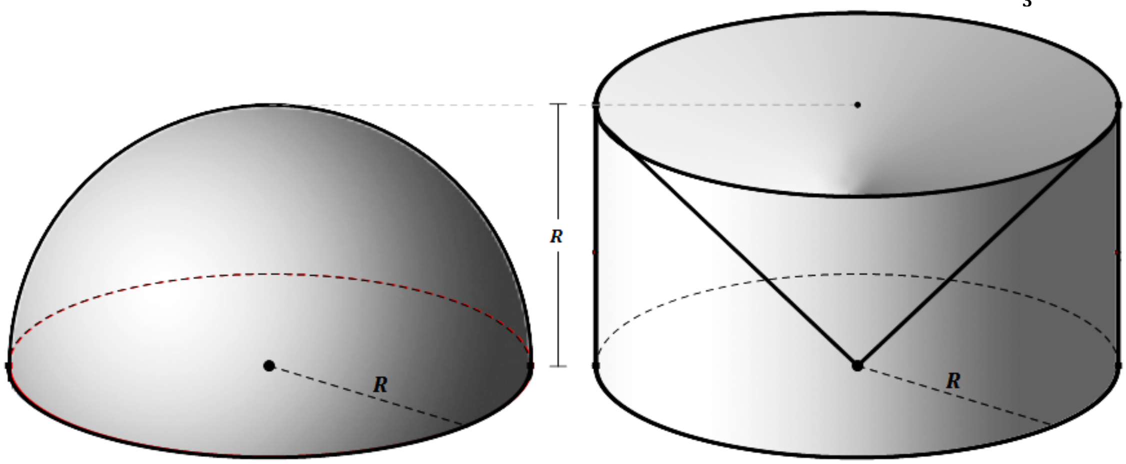
Volume:

Consider triangle ABC with vertices at A (0,0), B(4, 6), and C(0,6) plotted and a coordinate grid. Determine the volume of the solid created by rotating the triangle around the y-axis.

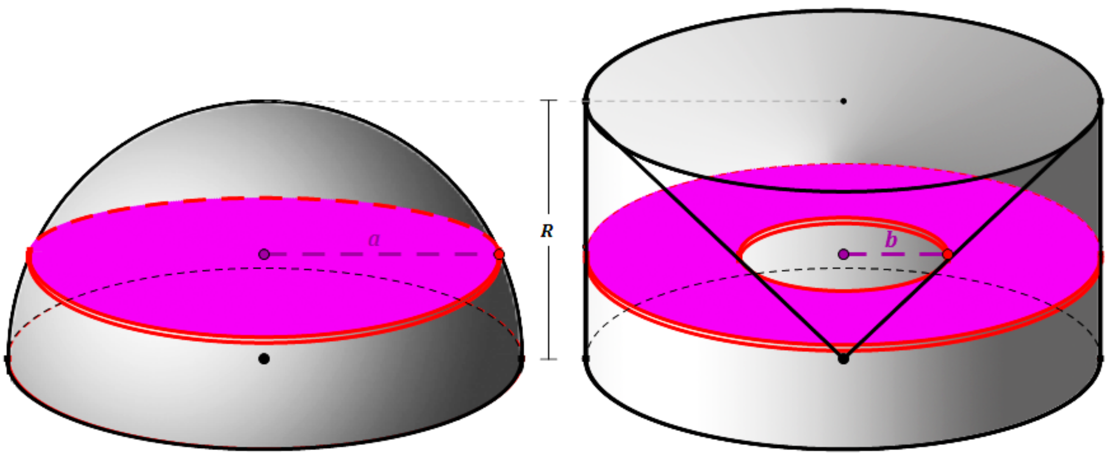


Volume:

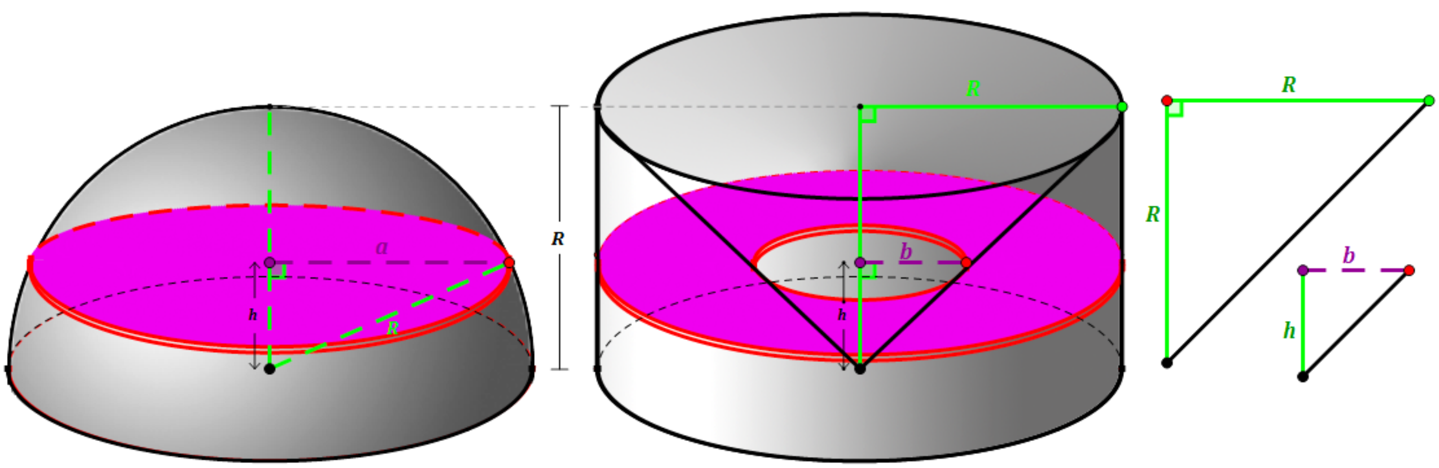
Using Cavalieri's Principle we can show that the volume of a sphere can be found by $\frac{4}{3}\pi \cdot r^3$



First, consider a hemisphere with a radius of R. Create a cylinder that has a base with the same radius R and a height equal to the radius R. Then, remove a cone from the cylinder that has the same base and height.



Next, consider a cross section that is parallel to the base and cuts through both solids using the same plane.



Cavalieri's Principle suggests if the 2 cross sections have the same area then the 2 solids must have the same volume.

The area of the cross section of the sphere is:	**The area of the cross section of the second solid is:**
$CA = \pi \cdot a^2$ **Using the Pythagorean theorem we know:** $a^2 + h^2 = R^2$ **or** $a^2 = R^2 - h^2$ **So, with simple substitution:** $CA = \pi \cdot (R^2 - h^2) = \pi R^2 - \pi h^2$	$CA = \pi \cdot R^2 - \pi \cdot b^2$ **Using similar triangles we know that h = b and then, using simple substitution** $CA = \pi \cdot R^2 - \pi \cdot h^2$

Volume of Hemisphere = Volume of Cylinder – Volume of Cone = $\pi \cdot R^2 h - \frac{1}{3}\pi \cdot b^2 h$

We also know that R = b = h. So, Volume of Hemisphere = $\pi \cdot R^2(R) - \frac{1}{3}\pi \cdot R^2(R) = \pi \cdot R^3 - \frac{1}{3}\pi \cdot R^3 = \frac{2}{3}\pi \cdot R^3$

To find the volume of a complete sphere, we can just double the hemisphere: Volume of Sphere = = $\frac{4}{3}\pi \cdot R^3$