Rules of Exponents

$x^3 \cdot x^2 =$

$x \cdot x \cdot x \cdot x \cdot x = x^5$

x^5

$(x^3)^2 =$

$(x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x^6$

$$\frac{x^7}{x^4} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = x^3$$

$$\frac{y^2 x^{-3} z^5}{y^{-4} x^6 z^7} = \frac{y^2 y^4 x^{-3} z^5}{x^6 z^7} = \frac{y^4 y^2 z^5}{x^3 x^6 z^7} =$$

$$\frac{y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z \cdot z}$$

$$\frac{y^6}{x^9 \cdot z^2}$$

$$1 = \frac{x^5}{x^5} = x^{5-5} = x^0$$

1. Simplify (answers should use positive exponents)

a. $(3x^4)(2x^3)$

$3xxxx \cdot 2xxx$

$3 \cdot 2 \; xxxx \cdot xxx$

$6x^7$

I usually take the time to write out the full expansion of these types of problems until students completely visualize why the exponent rules work.

$6x^7$

b. $\dfrac{10a^4b^7}{12a^8b^5}$

$5 \cdot 2 \; aaaa \; bbbbbbb$

$3 \cdot 2 \; aaaaaaaa \; bbbbb$

$\dfrac{5b^2}{3 \cdot 2 \, a^4}$

$\dfrac{5b^2}{6a^4}$

c. $(5p^4q^2)^2 = 5^2 p^8 q^4$

$= 5pppp \cdot qq \cdot 5pppp \cdot qq$

$= 5 \cdot 5 \; pppp \cdot pppp \cdot qq \cdot qq$

$25p^8q^4$

d. $(4t^3v^2)(-8tv^5)(u^0)$

$4ttt \, vv \cdot -8t \cdot vvvvv \cdot 1$

$4 \cdot -8 \cdot 1 \; ttt \cdot t \cdot vv \cdot vvvvv$

$-32t^4v^7$

$-32t^4v^7$

e. $(-3m^3n)^2(2m^2n^4)$

$-3mmm \cdot n \cdot -3mmm \cdot 2mmnnnn$

$-3 \cdot -3 \cdot 2 \; mmm \; mmm \; mm \; n \; n \; nnnn$

$18m^8n^6$

$18m^8n^6$

f. $\dfrac{18a^{-3}b^2c^6}{24a^2b^5c^4} = \dfrac{18 \, b^2 c^6}{24 \, a^3 a^2 b^5 c^4}$

$= \dfrac{3 \cdot 3 \cdot 2 \; bb \; ccccccc}{3 \cdot 2 \cdot 2 \cdot 2 \; aaa \; aa \; bbbbb \; ccccc}$

$= \dfrac{3 \cdot c^2}{2 \cdot 2 \cdot a^5 b^3}$

$\dfrac{3c^2}{4a^5b^3}$

(1 continued) Simplify

g. $\left(\frac{2x^5y^3z^{-2}}{5x^2y^4}\right)^2$

$= \frac{2xxxxx\,yyy}{5xx\,yyyy\,zz} \cdot \frac{2xxxxx\,yyy}{5xx\,yyyy\,zz}$

$= \frac{2xxxxx\,yyy}{5xx\,yyyy\,zz} \cdot \frac{2xxxxx\,yyy}{5xx\,yyyy\,zz}$

$= \frac{2\cdot2\,xxx\,xxx}{5\cdot5\,y\,y\,zz\,zz} = \boxed{\frac{4x^6}{25y^2z^4}}$

h. $(6a^2b^5c)(4a^3b^{-2}c^8)$

$= \frac{6aabbbbbbc}{1} \cdot \frac{4aaa\,cccccccc}{bb}$

$= 6\cdot4\,aa\,aaa\,bbb\,c\,cccccccc$

$= 24a^5b^3c^9$ $\boxed{24a^5b^3c^9}$

i. $\left(4p^3q^{-2}t^4\right)^2$

$= \frac{4ppp\,tttt}{qq} \cdot \frac{4ppp\,tttt}{qq}$

$= \frac{4\cdot4\,ppppppp\,tttt\,tttt}{qq\,qq}$

$= \boxed{\frac{16p^6t^8}{q^4}}$

j. $\left(\frac{4x^2y^{-3}}{y^{-2}}\right)^{-1} = \left(\frac{y^{-2}}{4x^2y^{-3}}\right)^1$

$= \frac{y^{-2}}{4x^2y^{-3}} = \frac{y^3}{4x^2y^2}$

$= \frac{yyy}{4xx\,yy}$

$= \boxed{\frac{y}{4x^2}}$

k. $(8x^7y^3)(3x^{-4}y^8)^2$

$\frac{8xxxxxxxyyy}{1} \cdot \frac{3yyyyyyyy}{xxxx} \cdot \frac{3yyyyyyyy}{xxxx}$

$= \frac{8\cdot3\cdot3\cdot yyy\cdot yyyyyyyy\cdot yyyyyyyy}{x}$

$= \boxed{\frac{72y^{19}}{x}}$

l. $(9a^3b^5)(-4a^3b^7)^2$

$9aaabbbbb \cdot -4aaabbbbbbbb \cdot -4aaabbbbbbbb$

$= 9\cdot-4\cdot-4\,aaaaaaaaa\,bbbbb\,bbbbbbb\,bbbbbbb$

$144a^9b^{19}$ $\boxed{144a^9b^{19}}$

m. $\left(\frac{4m^4n^{-3}p^2}{6m^2n^2}\right)^{-2} = \left(\frac{6m^2n^2}{4m^4n^{-3}p^2}\right)^2$

$= \left(\frac{6m^2n^3n^2}{4m^4p^2}\right)^2 = \frac{3\cdot2\,mm\,nnnnn}{2\cdot2\,mmmm\,pp} \cdot \frac{3\cdot2\,mm\,nnnnn}{2\cdot2\,mmmm\,pp}$

$= \frac{3\cdot3nnnnnnnnn}{2\cdot2\,mm\,mm\,pp\,pp}$

$\boxed{\frac{9n^{10}}{4m^4p^4}}$

n. $\left(\frac{3x}{y^{-3}}\right)^3\left(\frac{5x^{-10}yz^2}{2x^{-1}y^3}\right)^{-2} = \left(\frac{3x}{y^{-3}}\right)^3\left(\frac{2x^{-1}y^3}{5x^{-10}yz^2}\right)^2$

$\left(\frac{3xy^3}{1}\right)^3 \cdot \left(\frac{2x^{10}y^3}{5x^1yz^2}\right)^2$

$= \frac{3xyyy}{1}\,\frac{3xyyy}{1}\,\frac{3xyyy}{1} \cdot \frac{2xxxxxxxxxyyy}{5\cdot y\,zz} \cdot \frac{2xxxxxxxxxyyy}{5\cdot y\,zz}$

$= \frac{3\cdot3\cdot3\cdot2\cdot2xyyyxyyyxyyy\;x\;xxxxxxxx\;yy\;xxxxxxxxx\;yy}{5\cdot5\cdot zz\cdot zz}$

$= \boxed{\frac{108x^{21}y^{13}}{25z^4}}$

USING RULES

$\frac{27x^3}{y^{-9}} \cdot \frac{4x^{-2}y^6}{25x^{-20}y^2z^4} = \frac{108xy^6}{25x^{-20}y^{-7}z^4} = \frac{108x^{21}y^{13}}{25z^4}$ ✓