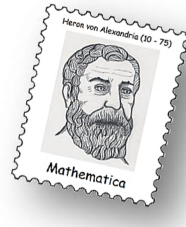
Sec 1.4 – Revisiting Quadratics
The Imaginary Number and Complex Numbers

Name:

For possibly several well over a thousand years, mathematicians disregarded the idea of attempting to find the square root of a negative number because at the time they thought it was absurd and useless. There is no "real" number that when multiplied by itself is a negative value which makes finding something like $\sqrt{-4}$ seemingly impossible.



$2 \cdot 2 = 4$ $\quad\quad -2 \cdot -2 = 4$ $\quad\quad 0 \cdot 0 = 0$ $\quad\quad -1 \cdot -1 = 1$

According to some references the first written suggestion of attempting to find the square root of a negative number may have dated all the way back to 50 A.D when Heron of Alexandria was trying to determine the volume of an impossible section of the pyramid. $0 = x^3 + 2x^2 - 3x + 1$

The first substantial noted works about finding the square root of a negative number didn't appear again until the 1500's when there was a math duel to see who could solve general cubic equations more effectively between three Italian mathematicians, Cardano and Ferrari versus Tartaglia. Ferrari was a student of Cardono and stood in for him during the duel. Cardano eventually published these findings in the Ars Magna. There was some argument as to who was first and who was better but the end result was that the solutions required taking square roots of negative numbers.


Geronimo Cardano (1501-1576) Lodovico Ferrari (1522-1565)

VS.


Niccolò Fontana Tartaglia (1499-1557)

Later in the 1600's it was Rene Descartes, considered the father of analytical geometry, that accidentally coined the term '**imaginary**' to represent the number $\sqrt{-1}$ as well as the standard form for complex numbers of $a + b\sqrt{-1}$.

$\sqrt{-1} = i$


Rene Descartes

Finally, it was Leonhard Euler that gave us the standard notation of i to represent the number $\sqrt{-1}$ and finalize the complex notation we use today of $a + bi$. Today, the understanding of imaginary numbers are commonly used in several engineering studies of such topics as force stresses, electrical engineering, and resonance.


Leonhard Euler

1. Find all square roots of each of the following and circle the principal square root.

a. 196 $\sqrt{196} = 14$ **b.** 1 $\sqrt{1} = 1$ **c.** 0 **d.** – 1 $\sqrt{-1} = i$ **e.** – 25 $\sqrt{-25} = \sqrt{25} \cdot \sqrt{-1}$ → $5 \quad i$

PRINCIPAL PRINCIPAL

a.	b.	c.	d.	e.
-14 or 14	-1 or 1	0	$-i$ or i	$-5i$ or $5i$

2. What does *i* represent? $= \sqrt{-1}$

Which mathematician was the first to call the number imaginary?
RENE DESCARTES

What is standard form?
$3 + 2i$ $\quad a + bi$

Which mathematician was the first to use the symbol *i*?
EULER

What do you think i^2 should be?
$i^2 = \left(\sqrt{-1}\right)^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

Can you explain why it MUST be defined?
B/C ALT ANSWERS COULD BE POSSIBLE

3. Simplify:

a. $i^3 = i^2 \cdot i$ → $(-1) \cdot i = -i$

b. $i^4 = i^2 \cdot i^2$ → $(-1) \cdot (-1) = 1$

c. $i^{11} = i^4 \cdot i^4 \cdot i^3$ → $(1) \cdot (1) \cdot (-i) = -i$

$i^1 = \sqrt{-1} = i$	$i^5 = i^4 \cdot i = 1 \cdot i = i$	$i^9 = i^8 \cdot i = 1 \cdot i = i$
$i^2 = (\sqrt{-1})^2 = -1$	$i^6 = i^4 \cdot i^2 = 1 \cdot -1 = -1$	$i^{10} = i^8 \cdot i^2 = 1 \cdot -1 = -1$
$i^3 = i^2 \cdot i = -1 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = 1 \cdot -i = -i$	$i^{11} = i^8 \cdot i^3 = 1 \cdot -i = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$	$i^{12} = i^8 \cdot i^4 = 1 \cdot 1 = 1$

$i, i^2, i^3, i^4, i^5, i^6, i^7, i^8, i^9, i^{10}, i^{11}, i^{12}, i^{13}, i^{14}, i^{15}, i^{16}, i^{17}, \ldots$

$i, -1, -i, 1, i, -1, -i, 1, i, -1, -i, 1, i, -1, -i, 1, i$

EVERY 4TH TERM IS EQUAL TO ONE.

ex. $i^{100} = 1$ $\quad$ ex. $i^{103} = i^{100} \cdot i^3 = 1 \cdot -i = -i$

4. Simplify the following:

a. $\sqrt{-45}$

$= \sqrt{-1}\cdot\sqrt{45}$

$= i\sqrt{45}$

9 5

3 3 5

$= \sqrt{9}\cdot\sqrt{-1}\cdot\sqrt{5}$

3 i $\sqrt{5}$

$\boxed{= 3i\sqrt{5}}$

b. $2\sqrt{-72}$

$= 2i\sqrt{72}$

9 8

3 3 4 2

3 3 2 2 2

$= 2i\cdot 3\cdot 2\cdot\sqrt{2}$

$= 12i\sqrt{2}$

$\boxed{12i\sqrt{2}}$

5. Add/Sub and simplifying the following and write the answer in standard form (a + b*i*):

a. $2i+5i$ $\boxed{7i}$

b. $(3+5i)-(2-6i)$

$3+5i-2+6i$

$1+11i$

$\boxed{1+11i}$

(3+5i)-(2-6i)
1+11i

c. $(2-3i)+(8-2i)$

$2-3i+8-2i$

$10-5i$

$\boxed{10-5i}$

(2-3i)+(8-2i)
10-5i

6. Add/Sub and simplifying the following and write the answer in standard form (a + b*i*):

a. $(3i+5)+(6+\sqrt{-25})$

$i\cdot\sqrt{25}$

$= (3i+5)+(6+5i)$

$= 3i+5+6+5i$

$= 11+8i$

$\boxed{11+8i}$

(3i+5)+(6+√(-25))
11+8i

b. $(\sqrt{18}+\sqrt{-12})+(4\sqrt{2}-\sqrt{-3})$

9 2   3 4

3 3 2   3 2 2

$(3\sqrt{2}+2i\sqrt{3})+(4\sqrt{2}-i\sqrt{3})$

$7\sqrt{2}+1i\sqrt{3}$

$\boxed{7\sqrt{2}+i\sqrt{3}}$

(√(18)+√(-12))+(4√(2)-√(-3))
9.899494937+1.7…
7√(2)+i√(3)
9.899494937+1.7…

7. Mult/Div and simplify the following:

a. $2i\times 5i$

$= 10i^2$

$= 10(-1)$

$= -10$

$\boxed{-10}$

c. $\sqrt{-3}\cdot\sqrt{-12}$

$= i\sqrt{3}\cdot i\sqrt{12}$

$= i^2\cdot\sqrt{36}$

$= (-1)\ (6)$

$= -6$

$\boxed{-6}$

√(-3)*√(-12)
-6

d. $i\sqrt{15}\times i\sqrt{20}$

$= i^2\sqrt{300}$

3 100

3 10 10

3 5 2 2 5

$= (-1)(5)(2)\sqrt{3} = -10\sqrt{3}$

$\boxed{-10\sqrt{3}}$

i√(15)*i√(20)
-17.32050808
-10√(3)
-17.32050808

e. $(2+4i)(7-8i)$

$14-16i+28i-32i^2$

$14+12i-32(-1)$

$14+12i+32$

$46+12i$

$\boxed{46+12i}$

(2+4i)(7-8i)
46+12i

f. $(3+4i)^2$

$(3+4i)(3+4i)$

$9+12i+12i+16i^2$

$9+24i-16$

$-7+24i$

$\boxed{-7+24i}$

(3+4i)²
-7+24i

g. $(3+5i)(3-5i)$

$= 9-\cancel{15i}+\cancel{15i}-25i^2$

$= 9-25(-1)$

$= 9+25 = 34$

$\boxed{34}$

(3+5i)(3-5i)
34

8. **What are the complex conjugates of each of the following?**

a. $(4+6i)$ $\boxed{4-6i}$

b. $8i$ $0+8i$ $\boxed{0-8i \quad -8i}$

c. 6 $6+0i$ $\boxed{6-0i \quad 6}$

9. **Mult/Div and simplify the following:**

b. $\frac{10\cancel{i}}{6\cancel{i}} = \frac{10}{6}$ $\boxed{\frac{5}{3}}$

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(10i)/(6i)
          1.666666667
Ans▸Frac
                  5/3
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b. $\frac{(2+5i)\cdot 3i}{3i\cdot 3i}$ $\boxed{=\frac{5}{3}-\frac{2}{3}i}$

$= \frac{6i+15i^2}{9i^2} = \frac{6i-15}{-9}$

$= \frac{2i-5}{-3}$

$= -\frac{2}{3}i+\frac{5}{3}$

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(2+5i)/(3i)▸Frac
            5/3-2/3i
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c. $\frac{(6-5i)\cdot(4+i)}{(4-i)\cdot(4+i)}$ $\boxed{\frac{29}{17}-\frac{14}{17}i}$

$= \frac{24+6i-20i-5i^2}{16+\cancel{4i}-\cancel{4i}-i^2}$

$= \frac{24-14i+5}{16+1}$

$= \frac{29-14i}{17} = \frac{29}{17}-\frac{14}{17}i$

```
(6-5i)/(4-i)▸Fra
c
      29/17-14/17i
█
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d. $\frac{(3+2i)\cdot(5-3i)}{(5+3i)\cdot(5-3i)}$ $\boxed{\frac{21}{34}+\frac{1}{34}i}$

$= \frac{15-9i+10i-6i^2}{25-\cancel{15i}+\cancel{15i}-9i^2}$

$= \frac{15+i+6}{25+9}$

$= \frac{21+i}{34} = \frac{21}{34}+\frac{1}{34}i$

```
(3+2i)/(5+3i)▸Fr
ac
        21/34+1/34i
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10. Mult/Div and simplify the following:

a. $\frac{(-4+2i)}{(2i-3)} \cdot \frac{(-2i-3)}{(-2i-3)}$ $\boxed{\frac{16}{13}+\frac{2}{13}i}$

$= \frac{8i+12-4i^2-6i}{-4i^2 \cancel{-6i} \cancel{+6i} + 9}$

(-4+2i)/(2i-3)▸F
rac
16/13+2/13i

$= \frac{2i+12+4}{4+9}$

$= \frac{16+2i}{13} = \frac{16}{13}+\frac{2}{13}i$

b. $\frac{6i(2+3i)+3}{3i}$ $\boxed{4+5i}$

$= \frac{12i+18i^2+3}{3i}$

(6i(2+3i)+3)/(3i
)
4+5i

$= \frac{12i-18+3}{3i}$

$= \frac{-15+12i}{3i}$

$= \frac{(-5+4i)}{i} \cdot \frac{i}{i} = \frac{-5i+4i^2}{i^2} = \frac{-4-5i}{-1}$

11. Solve the following complex equations

a. $8i+12=(4x)i-2y$

REAL PART	IMAGINARY PART
$12=-2y$	$8i=(4x)i$
$\frac{12}{-2}=\frac{-2y}{-2}$	$\frac{8}{4}=\frac{4x}{4}$
$-6=y$	$2=x$

$\boxed{x=2,\ y=-6}$

b. $4(3i+2)-2i=(2x)i+y$

$\underbrace{12i+8-2i}=(2x)i+y$

$8+10i=(2x)i+y$

REAL PART	IMAGINARY PART
$8=y$	$10i=(2x)i$
	$\frac{10}{2}=\frac{2x}{2}$
	$5=x$

$\boxed{x=5,\ y=8}$

c. $18i+3x=(9y)i$

$\quad -3x \quad -3x$

$0+18i=-3x+(9y)i$

REAL PART	IMAGINARY PART
$\frac{0}{-3}=\frac{-3x}{-3}$	$18i=(9y)i$
$0=x$	$\frac{18}{9}=\frac{9y}{9}$ $\quad 2=y$

$\boxed{x=0,\ y=2}$