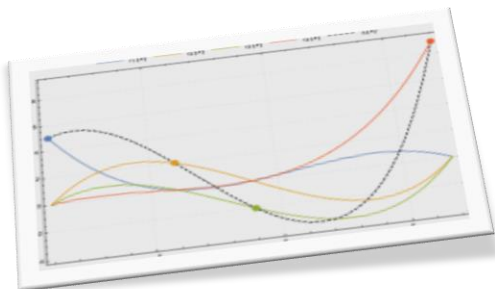


Algebra 2

Unit 2:

Operations with Polynomials



$$\begin{array}{cccc} 20x^2 & + & 5x & - & 12x & - & 3 \\ \text{F} & & \text{O} & & \text{I} & & \text{L} \\ 20x^2 & - & 7x & - & 3 & & \end{array}$$

MA_tTHew M. Winking

Sec 2.1 –Operations with Polynomials
Polynomial Classification and Operations

Name: _____

Name	Examples	Non-Examples
Monomial (one term)	1. $3x^4$ degree:4 or quartic 2. a^2 degree:2 or quadratic 3. 5 degree:0 or constant	1. $2x^{-4}$ 2. $5\sqrt{m}$ 3. $3t^{\frac{2}{3}}$
Binomial (two terms)	1. $2n^3 - n$ degree:3 or cubic 2. $p - 3$ degree:1 or linear(monic) 3. $-3a^3b^4 + a^4b^5$ degree:9 or nonic	1. $\frac{2x+1}{x}$ 2. $\sqrt{c^3 - 2}$
Trinomial (three terms)	1. $-2x^3 + 2x - 3$ degree:3 or cubic 2. $d(d^2 + 2d^4 - 2)$ degree:5 or quintic	1. $x^{-3} + 2x - 5$ 2. $2^x + 3x - 5$
Polynomial (one or more terms)	1. $3x^4 + 2x^3 - 5x + 1$ degree:4 or quartic 2. $5y^6$ degree:6 or sextic 3. $\frac{1}{2}x^2 + \sqrt{3}x^3 - 6x^4 + 1x - 3$ degree:4 or quartic	1. $3q^3 + \frac{p}{q}$ 2. $2^x + 3\sqrt{x}$

1. EXPAND and SIMPLIFY

a. $(7x + 3) - (2 - 2x)$

b. $(5x^3 - 3x^4 - 2x - 9x^2 - 2) + (3x^3 + 2x^2 - 5x - 7)$

c. $3(x + 5) + 8x$

d. $-2(3x + 2y) - (5x - 6y) + 2x - 7$

e. $(2x^2 + 5x) - (6x^2 - 2x)$

f. $(2x^3 + 5x - 8) + (5x^3 - 9x^2 - 11x + 5)$

g. $(x + 3)(x + 5)$

h. $(2x - 5)^2$

(1 Continued). EXPAND and SIMPLIFY

i. $4y^2(y^2 + 2y)$

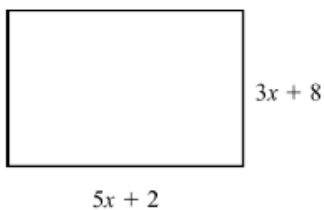
j. $-6y^2(3y^2 - 2y - 7)$

k. $(2x + 3)(3x - 5)$

l. $(2a + 3)(a^2 + 2a - 4)$

m. $(x - 2)(x^2 + 4)(2x + 3)$

n.

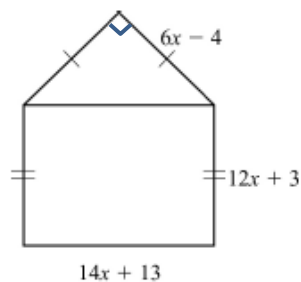


Determine an expression that represents:

Perimeter =

Area =

o.



Determine an expression that represents:

Perimeter =

Area =

Sec 2.2 –Operations with Polynomials
Pascal's Triangle & The Binomial Theorem

Name: _____

1. Expand each of the following.

a. $(a + b)^0$ _____ →

b. $(a + b)^1$ _____ →

c. $(a + b)^2$ _____ →

d. $(a + b)^3$ _____ →

e. $(a + b)^4$ _____ →

f. $(a + b)^5$ _____ →

2. Create Pascal's triangle to the 7th row.

3. Using Pascal's Triangle expand $(2a - 3b)^4$

The Binomial Theorem permits you to determine any row of Pascal's Triangle Explicitly.

The **Binomial Theorem** is shown below:

$$(a + b)^n = \binom{n}{0}(a)^n(b)^0 + \binom{n}{1}(a)^{n-1}(b)^1 + \binom{n}{2}(a)^{n-2}(b)^2 + \dots + \binom{n}{n-1}(a)^1(b)^{n-1} + \binom{n}{n}(a)^0(b)^n$$

EXAMPLE: Expand $(3x - 2y)^5$

$$\binom{5}{0}(3x)^5(-2y)^0 + \binom{5}{1}(3x)^4(-2y)^1 + \binom{5}{2}(3x)^3(-2y)^2 + \binom{5}{3}(3x)^2(-2y)^3 + \binom{5}{4}(3x)^1(-2y)^4 + \binom{5}{5}(3x)^0(-2y)^5$$

$\binom{5}{0} = 1$ $(3)^5 = 243$ $(-2)^0 = 1$ $1 \cdot 243 \cdot 1 = 243$	$\binom{5}{1} = 5$ $(3)^4 = 81$ $(-2)^1 = -2$ $5 \cdot 81 \cdot -2 = -810$	$\binom{5}{2} = 10$ $(3)^3 = 27$ $(-2)^2 = 4$ $10 \cdot 27 \cdot 4 = 1080$	$\binom{5}{3} = 10$ $(3)^2 = 9$ $(-2)^3 = -8$ $10 \cdot 9 \cdot -8 = -720$	$\binom{5}{4} = 5$ $(3)^1 = 3$ $(-2)^4 = 16$ $5 \cdot 3 \cdot 16 = 240$	$\binom{5}{5} = 1$ $(3)^0 = 1$ $(-2)^5 = -32$ $1 \cdot 1 \cdot -32 = -32$
--	---	---	---	--	--

$$243x^5 - 810x^4y^1 + 1080x^3y^2 - 720x^2y^3 + 240x^1y^4 - 32y^5$$

1st term

2nd term

3rd term

4th term

5th term

6th term

4. Using the Binomial Theorem expand $(3x - 2y)^6$

Use the **Binomial Theorem** to answer the following:

5. What is just the 4th term of $(3c + 4d)^7$

6. What is just the 7th term of $(3q - 2p)^6$

7. What is just the coefficient of the 3rd term of $(3t + 5m)^8$

8. Which term of $(5a - 3b)^9$ could be represented by $({}_9C_7)(5a)^2(-3b)^7$

9. The Binomial Theorem also has some applications in counting. For example if you wanted to know the probability of 6 coins being flipped and the probability that 5 of the flipped coins will land on heads by expanding.

First, expand $(h + t)^6$ using which ever method you would prefer.

Each coefficient represents the number of different ways you can flip a the 6 coins that way.

(e.g. $15h^4t^2$ suggests there are 15 different ways the 6 coins could land with 4 heads up and 2 tails up)

a. Determine the probability of having 5 coins land heads up.

b. Determine the probability of having 2 or more tails landing heads up.

Sec 2.3 –Operations with Polynomials
Dividing Polynomials

Name: _____

1. Divide each of the following polynomials by the suggested monomial.

a. $\frac{32a^5 + 24a^3}{8a^3}$

b. $\frac{36x^5 + 72x^3 + 48x^2}{6x^2}$

c. $\frac{12m^5 + 20m^4 + 32m^2}{4m^3}$

2. (REVIEW) Complete the following long division problem: $385274 \div 12$

$$12 \overline{)385274}$$

3. Use long division to divide the following polynomials. $(x^4 + 5x^3 + 3x^2 - 8x + 3) \div (x + 3)$

$$x + 3 \overline{)x^4 + 5x^3 + 3x^2 - 8x + 3}$$

4. Use long division to divide $(2x^4 - 7x^3 + 10x^2 - 6x - 3)$ by $(x - 2)$

5. Use long division to divide: $\frac{2x^4 - 3x^3 - x^2 + 8x - 6}{2x + 3}$

6. Use long division to divide $(4x^4 + 3x^2 + 2x - 2)$
by $(2x - 1)$

7. Use long division to divide:
$$\frac{x^5 + 5x^4 - x^3 - 22x^2 + x + 6}{x^2 + 3x - 2}$$

8. Use long division to find the quotient of $3x^3 - 12$
and $x^2 + 2x + 3$

9. Use long division to determine.
 $(6x^4 + 8x^3 - 11x^2 - 7x + 6) \div (2x^2 + 2x - 3)$

10. Rewrite $2x^3 + 3x^2 + 4x - 5$ as a nested polynomial.

11. Use the nested polynomial to easily evaluate $2x^3 + 3x^2 + 4x - 5$ when $x = 2$.

12. Use the following format to quickly evaluate $2x^3 + 3x^2 + 4x - 5$ when $x = 2$.

$$\begin{array}{r} \underline{2} \mid \quad 2 \quad 3 \quad 4 \quad -5 \\ \quad \downarrow \\ \hline \quad \quad 2 \end{array}$$

13. Use long division to find the following quotient: $(2x^3 + 3x^2 + 4x - 5) \div (x - 2)$

(Compare the answer from problem #12 with problem #11.)

14. Use synthetic division to divide

$$(x^4 + 7x^3 + 17x^2 + 13x - 6) \div (x + 3)$$

15. Use synthetic division to divide

$$(x^4 + 3x^3 + 2x^2 + x - 6) \div (x + 2)$$

16. Use synthetic division to divide

$$\frac{(2x^4 - 5x^3 - 7x - 6)}{(x - 3)}$$

17. Use synthetic division to evaluate

$$2x^3 + 3x^2 - 4x - 2 \text{ at } x = 3$$

18. Use synthetic division find the remainder of

$$(3x^4 + 10x^3 - 6x^2 + 5x - 7) \div (x + 4)$$

19. Which of the following is a factor of

$$2x^5 - 3x^3 + 6x^2 - 5x + 6$$

- | | |
|------------|------------|
| a. $(x+1)$ | b. $(x-2)$ |
| c. $(x+2)$ | d. $(x-1)$ |

Sec 2.4 –Operations with Polynomials
Composition of Functions

Name: _____

1. Consider the following functions.

$$f(x) = 6x - 2$$

$$g(x) = 3x$$

$$h(x) = 2^x + 3$$

$$p(x) = 2$$

a. Determine $(f + g)(x)$

b. Determine $(f + h)(x)$

c. Determine $(g - f)(x)$

d. Determine $(p \cdot g)(x)$

e. Determine $(f \cdot g)(2)$

f. Determine $\left(\frac{f}{g}\right)(x)$

2. Given the following partial set of values of function evaluate the following.

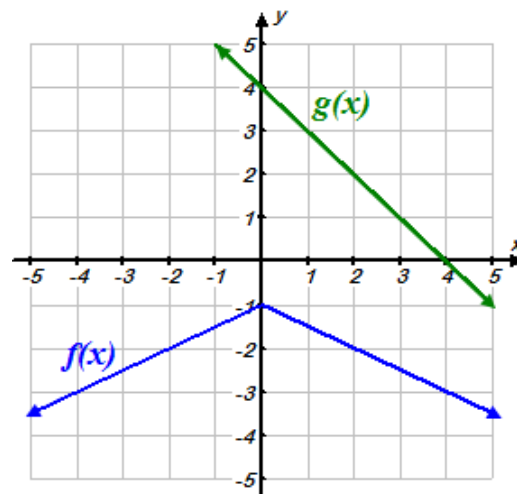
x	-1	0	1	2	3
$f(x)$	1	2	4	8	16
$g(x)$	-5	-3	-1	1	3

a. Determine $f(1) - 2 \cdot g(2)$

b. Determine $(f + g)(2)$

3. Given the following partial set of values of function evaluate the following.

a. Determine $f(4) + 2 \cdot g(1)$



4. Consider the following functions.

$$f(x) = 6x - 2$$

$$g(x) = 3x$$

$$h(x) = 2^x + 3$$

$$p(x) = 2$$

a. Determine $(f \circ h)(1)$

b. Determine $(g \circ f)(2)$

c. Determine $(f \circ g)(x)$

d. Determine $(g \circ h)(x)$

5. Given the following partial set of values of function evaluate the following.

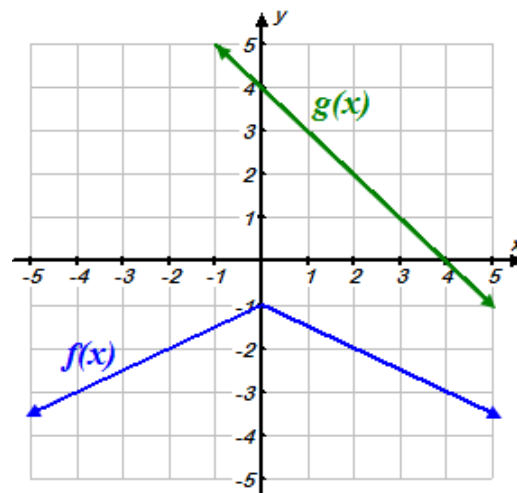
x	-1	0	1	2	3
$f(x)$	1	2	4	8	16
$g(x)$	-5	-3	-1	1	3

a. Determine $(f \circ g)(2)$

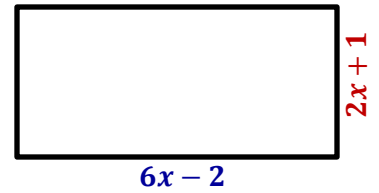
b. Determine $(g \circ f)(0)$

6. Given the following partial set of values of function evaluate the following.

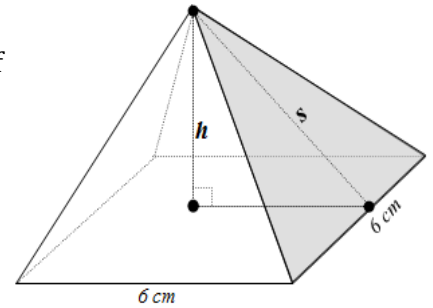
a. Determine $(f \circ g)(0)$



7. Given the **length** of a rectangle can be described by the function $f(x) = 6x - 2$ and the **width** of the same rectangle can be described by $g(x) = 2x + 1$



- Determine $(f \cdot g)(x)$ and tell what it would represent as far as the rectangle is concerned.
 - Determine an expression that represents the perimeter of the rectangle.
8. A pyramid is created from a square base where each side is 6 cm. The volume of the pyramid can be given by the function, $V(h) = 12h$.



The function to calculate the height of the same square based pyramid given the slant height could be described by $H(s) = \sqrt{s^2 - 9}$ where 's' is the slant height in cm.

- Evaluate $V(H(5)) =$
- Explain what $V(H(5))$ represents.

9. After being filled a party balloon's volume is dependent of the temperature in the room. The volume of the balloon can be modeled by $V(c) = 4100 + 15c$ where the volume, V, is measured in cubic centimeters and the temperature, c, is measured in degrees Celsius.

A function that enables you to change from degrees Celsius ($^{\circ}\text{C}$) to degrees Fahrenheit ($^{\circ}\text{F}$) is $C(f) = \frac{5}{9}(f - 32)$.



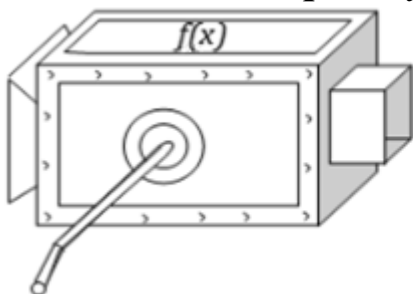
Show the composition of function required to determine the volume of the balloon when the temperature is 98°F .

10. A square is increased by doubling one side and decreasing the other sided by 3 units. Let x be the length of a side of the square.

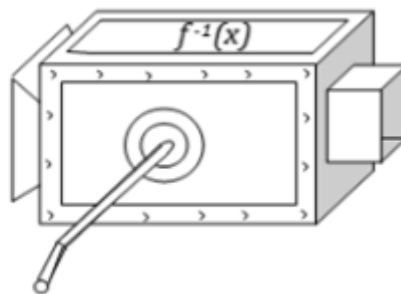
Create a function that would represent a change in the area of the rectangle after the transformation.



Inverse of a Function *conceptually*



$$f(x) = \frac{3x-4}{2}$$



$$f^{-1}(x) =$$



1. Find the inverse functions of the following.

a. $f(x) = 5x + 3$



b. $g(x) = \frac{3x+1}{5}$



c. $h(x) = \frac{x}{2} - 6$

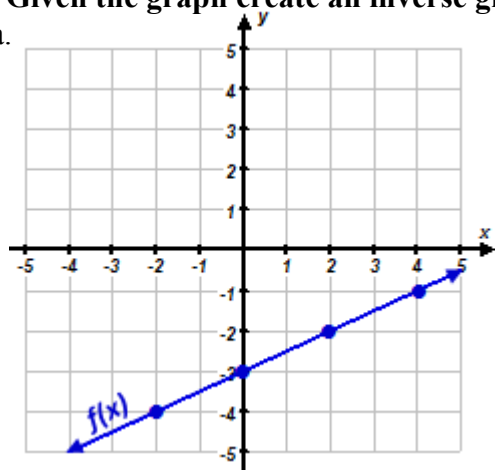


d. $g(x) = 2x^3 + 6$



2. Given the graph create an inverse graph and determine if the inverse is a function.

a.

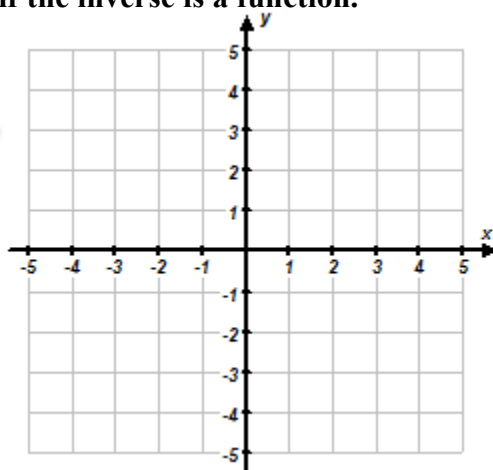


Create an
inverse of the
graph shown

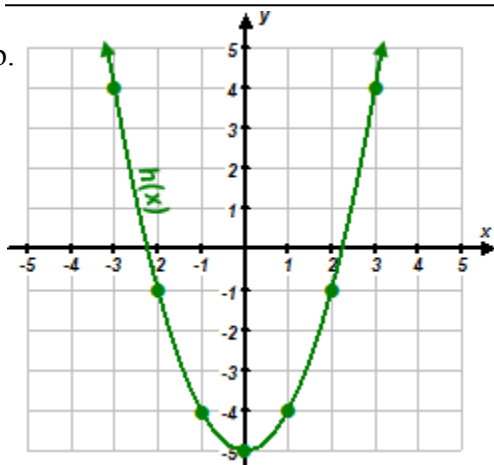
Is the inverse a
function?

CIRCLE ONE:

YES NO



b.

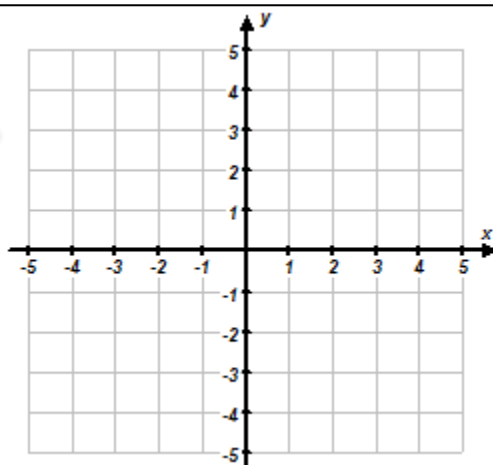


Create an
inverse of the
graph shown

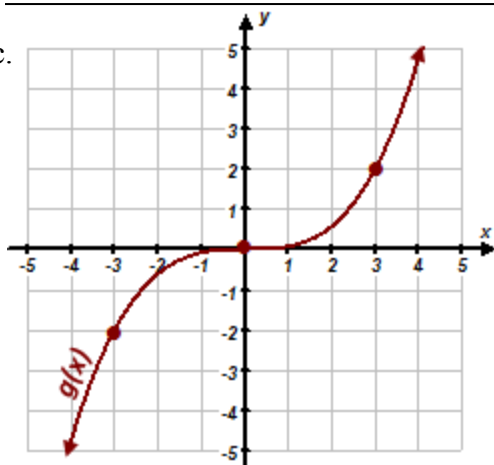
Is the inverse a
function?

CIRCLE ONE:

YES NO



c.

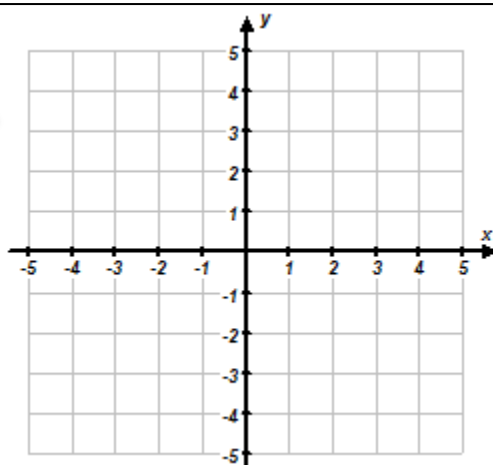


Create an
inverse of the
graph shown

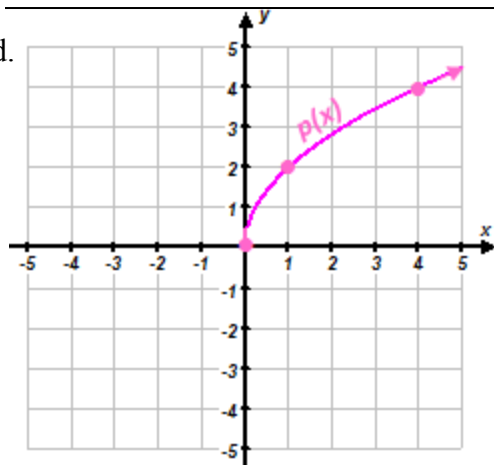
Is the inverse a
function?

CIRCLE ONE:

YES NO



d.

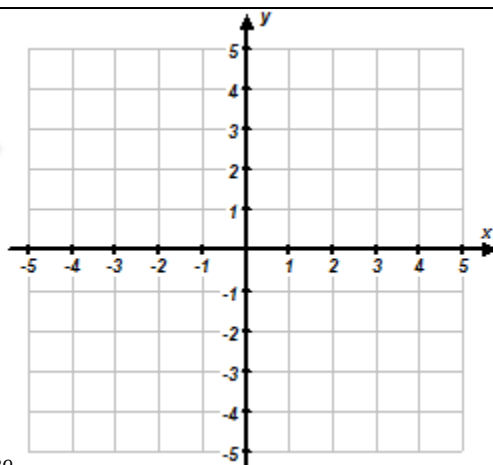


Create an
inverse of the
graph shown

Is the inverse a
function?

CIRCLE ONE:

YES NO



3. Which two functions could be inverses of one another based on the partial set of values in the table?

x	$f(x)$
-2	0
-1	2
0	-2
2	-1

x	$g(x)$
0	-2
2	-1
-2	0
-1	2

x	$h(x)$
-2	-1
-1	-2
0	2
2	0

4. Find the inverse functions of the following using the  flip technique.

a. $g(x) = \frac{3x+1}{5}$

b. $h(x) = \frac{3}{2x+1} ; x \neq -\frac{1}{2}$

c. $f(x) = \frac{x+3}{x-2} ; x \neq 2$

d. $m(x) = \frac{2x-3}{3x+1} ; x \neq -\frac{1}{3}$

5. Find the inverse functions of the following using any method:

a. $f(x) = 2x^2 - 3x$

b. $g(x) = x^2 - 4$; $x \geq 0$

6. Verify which of the following are inverses of one another by considering $f(g(x))$ and $g(f(x))$

a. $f(x) = 4x$
 $g(x) = \frac{x}{4}$

b. $f(x) = 2x + 1$
 $g(x) = \frac{x-1}{2}$

c. $f(x) = \frac{x}{2} - 3$
 $g(x) = 2x + 3$

d. $f(x) = 2x^3 - 1$
 $g(x) = \sqrt[3]{\frac{x+1}{2}}$