

Sec 2.2 –Operations with Polynomials
Pascal's Triangle & The Binomial Theorem

Name: _____

1. Expand each of the following.

a. $(a + b)^0$

ANY TERM OR EXPRESSION RAISED TO THE POWER OF ZERO IS JUST 1.

1

b. $(a + b)^1$

$a + b$

$1a + 1b$

c. $(a + b)^2$

$(a+b)(a+b)$

$a^2 + ab + ab + b^2$

$1a^2 + 2ab + 1b^2$

d. $(a + b)^3$

$(a+b)(a+b)(a+b)$

$(a+b)(a^2 + 2ab + b^2)$

$a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$

$1a^3 + 3a^2b + 3ab^2 + 1b^3$

e. $(a + b)^4$

$(a+b)(a+b)(a+b)(a+b)$

$(a+b)(a^3 + 3a^2b + 3ab^2 + b^3)$

$a^4 + 3a^3b + 3a^2b^2 + ab^3 + a^3b + 3a^2b^2 + 3ab^3 + b^4$

$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

f. $(a + b)^5$

$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

2. Create Pascal's triangle to the 7th row.

1										Row 0
	1									Row 1
		1								Row 2
			1	2	1					Row 3
				3	3	1				Row 4
					4	6	4	1		Row 5
						5	10	10	5	Row 6
							6	15	20	Row 7
								7	21	
									7	

3. Using Pascal's Triangle expand $(2a - 3b)^4 = (2a - 3b)(2a - 3b)(2a - 3b)(2a - 3b)$

$$\begin{aligned}
 & \underbrace{1(2a)^4(-3b)^0}_{1 \cdot 16a^4 \cdot 1} + \underbrace{4(2a)^3(-3b)^1}_{4 \cdot 8a^3 \cdot -3b} + \underbrace{6(2a)^2(-3b)^2}_{6 \cdot 4a^2 \cdot 9b^2} + \underbrace{4(2a)^1(-3b)^3}_{4 \cdot 2a \cdot -27b^3} + \underbrace{1(2a)^0(-3b)^4}_{1 \cdot 1 \cdot 81b^4} \\
 & \boxed{16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4}
 \end{aligned}$$

The Binomial Theorem permits you to determine any row of Pascal's Triangle Explicitly.

The **Binomial Theorem** is shown below:

$$(a + b)^n = \binom{n}{0}(a)^n(b)^0 + \binom{n}{1}(a)^{n-1}(b)^1 + \binom{n}{2}(a)^{n-2}(b)^2 + \dots + \binom{n}{n-1}(a)^1(b)^{n-1} + \binom{n}{n}(a)^0(b)^n$$

EXAMPLE: Expand $(3x - 2y)^5$

$$\begin{aligned}
 & \binom{5}{0}(3x)^5(-2y)^0 + \binom{5}{1}(3x)^4(-2y)^1 + \binom{5}{2}(3x)^3(-2y)^2 + \binom{5}{3}(3x)^2(-2y)^3 + \binom{5}{4}(3x)^1(-2y)^4 + \binom{5}{5}(3x)^0(-2y)^5 \\
 & \begin{array}{l} \text{Cloud 1: } \binom{5}{0}=1, (3)^5=243, (-2)^0=1, 1 \cdot 243 \cdot 1 = 243 \\ \text{Cloud 2: } \binom{5}{1}=5, (3)^4=81, (-2)^1=-2, 5 \cdot 81 \cdot -2 = -810 \\ \text{Cloud 3: } \binom{5}{2}=10, (3)^3=27, (-2)^2=4, 10 \cdot 27 \cdot 4 = 1080 \\ \text{Cloud 4: } \binom{5}{3}=10, (3)^2=9, (-2)^3=-8, 10 \cdot 9 \cdot -8 = -720 \\ \text{Cloud 5: } \binom{5}{4}=5, (3)^1=3, (-2)^4=16, 5 \cdot 3 \cdot 16 = 240 \\ \text{Cloud 6: } \binom{5}{5}=1, (3)^0=1, (-2)^5=-32, 1 \cdot 1 \cdot -32 = -32 \end{array} \\
 & 243x^5 - 810x^4y^1 + 1080x^3y^2 - 720x^2y^3 + 240x^1y^4 - 32y^5 \\
 & \text{1st term} \quad \quad \quad \text{2nd term} \quad \quad \quad \text{3rd term} \quad \quad \quad \text{4th term} \quad \quad \quad \text{5th term} \quad \quad \quad \text{6th term}
 \end{aligned}$$

4. Using the Binomial Theorem expand $(3x - 2y)^6$

$$\begin{aligned}
 & \binom{6}{0}(3x)^6(-2y)^0 + \binom{6}{1}(3x)^5(-2y)^1 + \binom{6}{2}(3x)^4(-2y)^2 + \binom{6}{3}(3x)^3(-2y)^3 + \binom{6}{4}(3x)^2(-2y)^4 + \binom{6}{5}(3x)^1(-2y)^5 + \binom{6}{6}(3x)^0(-2y)^6 \\
 & 1 \cdot 729x^6 \cdot 1 + 6 \cdot 243x^5 \cdot -2y + 15 \cdot 81x^4 \cdot 4y^2 + 20 \cdot 27x^3 \cdot -8y^3 + 15 \cdot 9x^2 \cdot 16y^4 + 6 \cdot 3x \cdot -32y^5 + 1 \cdot 1 \cdot 64y^6 \\
 & 729x^6 - 2916x^5y + 4860x^4y^2 - 4320x^3y^3 + 2160x^2y^4 - 576xy^5 + 64y^6 \\
 & \begin{array}{l} \uparrow \text{1st TERM} \quad \uparrow \text{2nd TERM} \quad \uparrow \text{3rd TERM} \quad \uparrow \text{4th TERM} \quad \uparrow \text{5th TERM} \quad \uparrow \text{6th TERM} \quad \uparrow \text{7th TERM} \end{array}
 \end{aligned}$$

6 nCr 6

6 nCr 0
6 nCr 1
6 nCr 2

1
6
15

6 nCr 3
6 nCr 4
6 nCr 5

20
15
6

Use the **Binomial Theorem** to answer the following:

5. What is just the 4th term of $(3c + 4d)^7$

$$35 \cdot 81 \cdot 64$$

■

$$n = 3$$

$$181440$$

$$\begin{aligned} &({}_7C_3)(3c)^4(4d)^3 \\ &\quad \downarrow \quad \downarrow \quad \downarrow \\ &(35)(81c^4)(64d^3) \end{aligned}$$

$$181440 c^4 d^3$$

6. What is just the 7th term of $(3q - 2p)^6$

$$n = 6$$

$$\begin{aligned} &({}_6C_6)(3q)^0(-2p)^6 \\ &\quad (1) \cdot (1) \cdot (64p^6) \end{aligned}$$

$$64p^6$$

7. What is just the coefficient of the 3rd term of $(3t + 5m)^8$

$$28 \cdot 729 \cdot 25$$

■

$$510300$$

$$n = 2$$

$$\begin{aligned} &({}_8C_2)(3t)^6(5m)^2 \\ &\quad \downarrow \quad \downarrow \quad \downarrow \\ &(28)(729t^6)(25m^2) \end{aligned}$$

$$510300$$

8. Which term of $(5a - 3b)^9$ could be represented by $({}_9C_7)(5a)^2(-3b)^7$

$$\begin{aligned} &\underline{8^{th} \text{ TERM}} \\ &n = 7 \end{aligned}$$

MUST BE
THE 8TH
TERM

$$8^{th} \text{ TERM}$$

9. The Binomial Theorem also has some applications in counting. For example if you wanted to know the probability of 6 coins being flipped and the probability that 5 of the flipped coins will land on heads by expanding.

First, expand $(h + t)^6$ using which ever method you would prefer.

$$1h^6t^0 + 6h^5t^1 + 15h^4t^2 + 20h^3t^3 + 15h^2t^4 + 6h^1t^5 + 1h^0t^6$$

$$\text{TOTAL UNIQUE FLIPS OF 6 COINS} = 64$$

Each coefficient represents the number of different ways you can flip a the 6 coins that way.

(e.g. $15h^4t^2$ suggests there are 15 different ways the 6 coins could land with 4 heads up and 2 tails up)

a. Determine the probability of having 5 coins land heads up.

$$\frac{6}{64} = \frac{3}{32} \approx 9.4\%$$

b. Determine the probability of having 2 or more tails landing heads up.

$$\begin{aligned} &15 + 20 + 15 + 6 + 1 \\ &57/64 \quad \quad \quad 57 \\ &\quad \quad \quad .890625 \end{aligned}$$

$$\frac{57}{64} \approx 89\%$$