

Find the value for the variable that makes the statement true. (SHOW WORK NEATLY)

1. $8\sqrt{2x+3}-10=30$

$$\begin{aligned} & \xrightarrow{+10 \quad +10} \\ \frac{8\sqrt{2x+3}}{8} &= \frac{40}{8} \\ (\sqrt{2x+3})^2 &= (5)^2 \\ 2x+3 &= 25 \\ \xrightarrow{-3 \quad -3} \\ 2x &= 22 \\ \frac{2x}{2} &= \frac{22}{2} \end{aligned}$$

$x = 11$

CHECK: $8\sqrt{2(11)+3}-10=30$
 $8\sqrt{25}-10=30$
 $40-10=30 \checkmark$

2. $2\sqrt{x+4}+9=1$

$$\begin{aligned} & \xrightarrow{-9 \quad -9} \\ \frac{2\sqrt{x+4}}{2} &= \frac{-8}{2} \\ (\sqrt{x+4})^2 &= (-4)^2 \\ x+4 &= 16 \\ \xrightarrow{-4 \quad -4} \\ x &= 12 \end{aligned}$$

~~$x = 12$~~
EXTRANEIOUS

EMPTY SET

CHECK: $2\sqrt{(12)+4}+9=1$
 $2\sqrt{16}+9=1$
 $2(4)+9=1$
 $8+9=1 \times$

NO SOLUTION

3. $3\sqrt{4a}-5=13$

$$\begin{aligned} & \xrightarrow{+5 \quad +5} \\ \frac{3\sqrt{4a}}{3} &= \frac{18}{3} \\ (\sqrt{4a})^2 &= (6)^2 \\ 4a &= 36 \\ \frac{4a}{4} &= \frac{36}{4} \end{aligned}$$

$a = 9$

CHECK: $3\sqrt{4(9)}-5=13$
 $3\sqrt{36}-5=13$
 $3(6)-5=13$
 $18-5=13 \checkmark$

4. $\frac{12}{2} = 2\sqrt{2(x+1)-4}$

$$\begin{aligned} 6 &= \sqrt{2(x+1)-4} \\ (6)^2 &= (\sqrt{2(x+1)-4})^2 \\ 36 &= 2(x+1)-4 \\ 36 &= 2x+2-4 \\ 36 &= 2x-2 \\ \xrightarrow{+2 \quad +2} \end{aligned}$$

$\frac{38}{2} = \frac{2x}{2}$

$19 = x$

CHECK: $12 = 2\sqrt{2(19+1)-4}$
 $12 = 2\sqrt{40-4}$
 $12 = 2\sqrt{36}$
 $12 = 2(6) \checkmark$

One Radical Basic Equation

I. Isolate the Radical if possible.

Example: $3\sqrt{x+2}+4=19$
 $\xrightarrow{-4 \quad -4}$
 $\frac{3\sqrt{x+2}}{3} = \frac{15}{3}$
 $\sqrt{x+2} = 5$

II. Square Both Sides

Example: $(\sqrt{x+2})^2 = (5)^2$
 $x+2 = 25$

III. Isolate the Variable

Example: $x+2 = 25$
 $\xrightarrow{-2 \quad -2}$
 $x = 23$

IV. Must Verify the Solution
(This is not optional some solutions are extraneous.)

Example: $3\sqrt{(23)+2}+4=19$
 $3\sqrt{25}+4=19$
 $3(5)+4=19$
 $15+4=19$
 $19=19 \checkmark$

5. $\sqrt[3]{2x+1}+2=5$

$$\begin{aligned} & \xrightarrow{-2 \quad -2} \\ \sqrt[3]{2x+1} &= 3 \\ (\sqrt[3]{2x+1})^3 &= (3)^3 \\ 2x+1 &= 27 \\ \xrightarrow{-1 \quad -1} \\ 2x &= 26 \end{aligned}$$

$\frac{2x}{2} = \frac{26}{2}$

$x = 13$

CHECK: $\sqrt[3]{2(13)+1}+2=5$
 $\sqrt[3]{27}+2=5$
 $3+2=5 \checkmark$

6. $\sqrt[3]{2x}-10=-6$

$$\begin{aligned} & \xrightarrow{+10 \quad +10} \\ \sqrt[3]{2x} &= 4 \\ (\sqrt[3]{2x})^3 &= (4)^3 \end{aligned}$$

$\frac{2x}{2} = \frac{64}{2}$

$x = 32$

CHECK: $\sqrt[3]{2(32)}-10=-6$
 $\sqrt[3]{64}-10=-6$
 $4-10=-6$
 $-6=-6 \checkmark$

Find the value for the variable that makes the statement true. (SHOW WORK NEATLY)

7. $\sqrt{5x+2} = \sqrt{3x+12}$

$$\begin{aligned} (\sqrt{5x+2})^2 &= (\sqrt{3x+12})^2 \\ 5x+2 &= 3x+12 \\ -3x & \quad -3x \\ \hline 2x+2 &= 12 \\ -2 & \quad -2 \\ \hline 2x &= 10 \\ \frac{2x}{2} &= \frac{10}{2} \end{aligned}$$

$x=5$

CHECK: $\sqrt{5(5)+2} = \sqrt{3(5)+12}$
 $\sqrt{25+2} = \sqrt{15+12}$
 $\sqrt{27} = \sqrt{27} \checkmark$

8. $2\sqrt{x-5} = \sqrt{3x+2}$

$$\begin{aligned} (2\sqrt{x-5})^2 &= (\sqrt{3x+2})^2 \\ 4(x-5) &= 3x+2 \\ 4x-20 &= 3x+2 \\ -3x & \quad -3x \\ \hline x-20 &= 2 \\ +20 & \quad +20 \\ \hline x &= 22 \end{aligned}$$

CHECK: $2\sqrt{(22)-5} = \sqrt{3(22)+2}$
 $2\sqrt{17} = \sqrt{68}$
 $2\sqrt{17} = \sqrt{4 \cdot 17}$
 $2\sqrt{17} = 2\sqrt{17} \checkmark$

9. $(2\sqrt{5x-4})^2 = (3\sqrt{x+8})^2$

$$\begin{aligned} 4(5x-4) &= 9(x+8) \\ 20x-16 &= 9x+72 \\ -9x & \quad -9x \\ \hline 11x-16 &= 72 \\ +16 & \quad +16 \\ \hline 11x &= 88 \\ \frac{11x}{11} &= \frac{88}{11} \end{aligned}$$

$x=8$

CHECK: $2\sqrt{5(8)-4} = 3\sqrt{(8)+8}$
 $2\sqrt{36} = 3\sqrt{16}$
 $2(6) = 3(4)$
 $12 = 12 \checkmark$

10. $(2\sqrt[3]{5x-3})^3 = (\sqrt[3]{35x+6})^3$

$$\begin{aligned} 8(5x-3) &= 35x+6 \\ 40x-24 &= 35x+6 \\ -35x & \quad -35x \\ \hline 5x-24 &= 6 \\ +24 & \quad +24 \\ \hline 5x &= 30 \\ \frac{5x}{5} &= \frac{30}{5} \end{aligned}$$

$x=6$

CHECK: $2\sqrt[3]{5(6)-3} = \sqrt[3]{35(6)+6}$
 $2\sqrt[3]{27} = \sqrt[3]{216}$
 $2(3) = 6$
 $6 = 6 \checkmark$

11. $(x+3)^2 = \sqrt{15+x}^2$

$$\begin{aligned} (x+3)(x+3) &= 15+x \\ x^2+3x+3x+9 &= 15+x \\ x^2+6x+9 &= 15+x \\ -x & \quad -15 \quad -15 \quad -x \\ \hline x^2+5x-6 &= 0 \\ (x-1)(x+6) &= 0 \\ \downarrow & \quad \downarrow \\ x-1=0 & \quad x+6=0 \\ +1 & \quad -6 \quad -6 \end{aligned}$$

$x=1$

~~$x=6$~~

EXTRANEOUS

CHECK: $(1)+3 = \sqrt{15+(1)}$
 $4 = \sqrt{16}$
 $4 = 4 \checkmark$

CHECK: $(-6)+3 = \sqrt{15+(-6)}$
 $-3 = \sqrt{9}$
 $-3 = 3 \times$

$x=1$

Two Radical Basic Equation

I. Square Both Sides

Examp 1 e $\sqrt{5x+2} = 3\sqrt{x-2}$
 $(\sqrt{5x+2})^2 = (3\sqrt{x-2})^2$
 $5x+2 = 9(x-2)$

II. Eliminate Parenthesis

Examp 1 e $5x+2 = 9(x-2)$
 $5x+2 = 9x-18$

III. Move variables to one side and constants to the other

Examp 1 e $5x+2 = 9x-18$
 $-5x \quad -5x$
 $2 = 4x-18$
 $+18 \quad +18$
 $20 = 4x$

IV. Divide both sides by the coefficient

Examp 1 e $\frac{20}{4} = \frac{4x}{4}$
 $5 = x$

V. Must Verify the Solution (This is not optional some solutions are extraneous.)

Examp 1 e $\sqrt{5(5)+2} = 3\sqrt{(5)-2}$
 $\sqrt{25+2} = 3\sqrt{3}$
 $\sqrt{27} = 3\sqrt{3}$
 $3\sqrt{3} = 3\sqrt{3} \checkmark$

Find the value for the variable that makes the statement true. (SHOW WORK NEATLY)

$$12. (x-1)^2 = (\sqrt{5x-9})^2$$

$$\begin{aligned} (x-1)(x-1) &= 5x-9 \\ x^2 - 1x - 1x + 1 &= 5x-9 \\ x^2 - 2x + 1 &= 5x-9 \\ -5x + 9 & \quad -5x + 9 \end{aligned}$$

$$\begin{aligned} x^2 - 7x + 10 &= 0 & \frac{10}{-1, -10} \\ (x-2)(x-5) &= 0 & \frac{2, 5}{-2, -5} \\ x-2=0 & \quad x-5=0 \\ +2 +2 & \quad +5 +5 \end{aligned}$$

$$\boxed{X=2} \text{ or } \boxed{X=5}$$

CHECK: $(2)-1 = \sqrt{5(2)-9}$
 $(x=2)$
 $1 = \sqrt{10-9}$
 $1 = \sqrt{1} \checkmark$

CHECK: $(5)-1 = \sqrt{5(5)-9}$
 $(x=5)$
 $4 = \sqrt{25-9}$
 $4 = \sqrt{16} \checkmark$

$$13. (2x+1)^2 = (\sqrt{11-2x})^2$$

$$\begin{aligned} (2x+1)(2x+1) &= 11-2x \\ 4x^2 + 2x + 2x + 1 &= 11-2x \\ 4x^2 + 4x + 1 &= 11-2x \\ +2x - 11 & \quad -11 + 2x \end{aligned}$$

$$\begin{aligned} 4x^2 + 6x - 10 &= 0 \\ 2(2x^2 + 3x - 5) & \\ \downarrow & \quad \downarrow \\ \frac{2x-2}{2} & \quad \frac{2x+5}{1} \end{aligned}$$

$$2(x-1)(2x+5) = 0$$

$x-1=0$ $2x+5=0$
 $+1 +1$ $\frac{2x}{2} = \frac{-5}{2}$
 $\boxed{X=1}$ ~~$\boxed{X=-\frac{5}{2}}$~~ **EXTRANEOUS**

CHECK: $2(1)+1 = \sqrt{11-2(1)}$
 $(x=1)$
 $2+1 = \sqrt{11-2}$
 $3 = \sqrt{9} \checkmark$

CHECK: $2(-2.5)+1 = \sqrt{11-2(-2.5)}$
 $(x=-2.5)$
 $-5+1 = \sqrt{11+5}$
 $-4 = \sqrt{16}$
 $-4 = 4 \times$

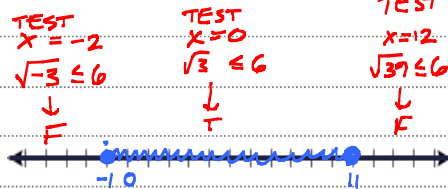
$$\boxed{X=1}$$

Find the values for the variable that make the inequality statement true. (SHOW WORK NEATLY)

$$14. (\sqrt{3x+3})^2 \leq (6)^2$$

$$\begin{aligned} 3x+3 &\geq 0 & 3x+3 &\leq 36 \\ -3 -3 & & -3 -3 & \\ \frac{3x}{3} &\geq \frac{-3}{3} & \frac{3x}{3} &\leq \frac{33}{3} \\ X &\geq -1 & X &\leq 11 \end{aligned}$$

SET: $-1 \leq X \leq 11$ INT: $[-1, 11]$



$$15. -2\sqrt{x+1} < -8$$

$$\begin{aligned} \frac{-2\sqrt{x+1}}{-2} &< \frac{-8}{-2} \\ \sqrt{x+1} &> (4)^2 \\ X+1 &> 16 \\ -1 -1 & \quad -1 -1 \\ X &> 15 \end{aligned}$$

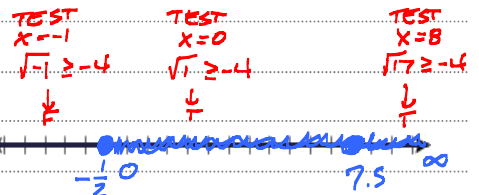
SET: $X > 15$ INT: $(15, \infty)$



$$16. \sqrt{2x+1} \geq -4$$

$$\begin{aligned} 2x+1 &\geq 0 & 2x+1 &\geq 16 \\ -1 -1 & & -1 -1 & \\ \frac{2x}{2} &\geq \frac{-1}{2} & \frac{2x}{2} &\geq \frac{15}{2} \\ X &\geq -\frac{1}{2} & X &\geq 7.5 \end{aligned}$$

SET: $X \geq -\frac{1}{2}$ INT: $[-\frac{1}{2}, \infty)$



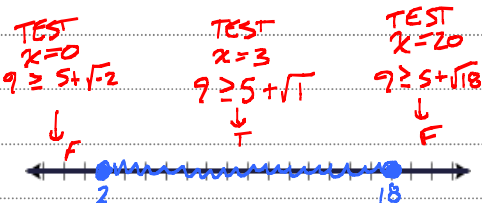
$$17. 9 \geq 5 + \sqrt{x-2}$$

$$(4)^2 \geq (\sqrt{x-2})^2$$

$$\begin{aligned} X-2 &\geq 0 & 16 &\geq X-2 \\ +2 +2 & & +2 +2 & \\ X &\geq 2 & 18 &\geq X \\ & & X &\leq 18 \end{aligned}$$

SET: $2 \leq X \leq 18$

INT: $[2, 18]$



$$18. (\sqrt[3]{4x-1})^3 \leq (3)^3$$

$$\begin{aligned} 4x-1 &\leq 27 \\ +1 +1 & & +1 +1 & \\ 4x &\leq 28 \\ \frac{4x}{4} &\leq \frac{28}{4} \\ X &\leq 7 \end{aligned}$$

SET: $X \leq 7$

INT: $(-\infty, 7]$

