Sec 4.6 – Rational & Radical Relationships
Solving Rational Equations & Inequalities Name:

Find the value of the variable that makes each of the statements true.

1. $\frac{4}{x^2} = \frac{1}{9}$

$x^2 = 36$

$\sqrt{x^2} = \pm\sqrt{36}$

$x = \pm 6$

CHECK:

$w=6$: $\frac{4}{(6)^2} = \frac{1}{9}$ $\quad \frac{4}{36} = \frac{1}{9}$ ✓

$w=-6$: $\frac{4}{(-6)^2} = \frac{1}{9}$ $\quad \frac{4}{36} = \frac{1}{9}$ ✓

1. $x = \pm 6$

2. $\frac{2t}{t-2} = \frac{t+4}{t-2}$

$\frac{(t-2)}{1} \cdot \frac{2t}{t-2} = \frac{(t-2)}{1} \cdot \frac{t+4}{t-2}$

$2t = t+4$

$-t \quad -t$

$t = 4$

CHECK: $t=4$

$\frac{2(4)}{(4)-2} = \frac{(4)+4}{(4)-2}$

$\frac{8}{2} = \frac{8}{2}$ ✓

2. $t = 4$

3. $\frac{b+2}{b-3} = \frac{3b-4}{b-3}$

$\frac{(b-3)}{1} \cdot \frac{b+2}{b-3} = \frac{(b-3)}{1} \cdot \frac{3b-4}{b-3}$

$b+2 = 3b-4$

$-b \quad -b$

$2 = 2b-4$

$+4 \quad +4$

$\frac{6}{2} = \frac{2b}{2}$

$3 = b$ EXTRANEOUS

CHECK: $b=3$

$\frac{(3)+2}{(3)-3} = \frac{3(3)-4}{(3)-3}$

$\frac{5}{0} = \frac{5}{0}$

UNDEFINED

3. $\varnothing$ NO SOLUTION

4. $\frac{w^2}{w-4} - \frac{8}{w-4} = \frac{2w}{w-4}$

$\frac{(w-4)}{1} \cdot \frac{w^2}{w-4} - \frac{(w-4)}{1} \cdot \frac{8}{w-4} = \frac{(w-4)}{1} \cdot \frac{2w}{w-4}$

$w^2 - 8 = 2w$

$-2w \quad -2w$

$w^2 - 2w - 8 = 0$

$(w+2)(w-4) = 0$

-8: $1,-8$ $\quad -1,8$ $\quad 2,-4$ $\quad -2,4$

$w+2=0$ $\quad -2 \quad -2$

$w = -2$

$w-4=0$ $\quad +4 \quad +4$

$w = 4$ EXTRANEOUS

CHECK:

$w=-2$: $\frac{(-2)^2}{(-2)-4} - \frac{8}{(-2)-4} = \frac{2(-2)}{(-2)-4}$

$\frac{4}{-6} - \frac{8}{-6} = \frac{-4}{-6}$

$-\frac{4}{6} + \frac{8}{6} = \frac{4}{6}$ ✓

$w=4$: $\frac{(4)^2}{(4)-4} - \frac{8}{(4)-4} = \frac{2(4)}{(4)-4}$

$\frac{16}{0} - \frac{8}{0} = \frac{8}{0}$

4. $w = -2$

5. $\frac{5}{x-4} = \frac{3}{x}$ LCD: $(x-4)(x)$

$\frac{(x-4)(x)}{1} \cdot \frac{5}{(x-4)} = \frac{(x-4)(x)}{1} \cdot \frac{3}{x}$

$5x = (x-4)3$

$5x = 3x-12$

$-3x \quad -3x$

$2x = -12$

$\frac{2}{2} \quad \frac{2}{2}$

$x = -6$

CHECK: $x=-6$

$\frac{5}{(-6)-4} = \frac{3}{(-6)}$

$\frac{5}{-10} = \frac{3}{-6}$

$-\frac{1}{2} = -\frac{1}{2}$ ✓

5. $x = -6$

6. $\frac{p^2}{p+2} = \frac{4p+12}{p+2}$

$\frac{(p+2)}{1} \cdot \frac{p^2}{p+2} = \frac{(p+2)}{1} \cdot \frac{4p+12}{p+2}$

$p^2 = 4p+12$

$-4p \quad -4p \quad -12$

$p^2 - 4p - 12 = 0$

$(p+2)(p-6) = 0$

-12: $-1,12$ $\quad 1,-12$ $\quad 2,-6$ $\quad -2,6$ $\quad 3,-4$ $\quad -3,4$

$p+2=0$ $\quad -2 \quad -2$

$p = -2$ EXTRANEOUS

$p-6=0$ $\quad +6 \quad +6$

$p = 6$

CHECK:

$x=-2$: $\frac{(-2)^2}{(-2)+2} = \frac{4(-2)+12}{(-2)+2}$

$\frac{4}{0} = \frac{4}{0}$ ✗

$x=6$: $\frac{(6)^2}{(6)+2} = \frac{4(6)+12}{(6)+2}$

$\frac{36}{8} = \frac{36}{8}$ ✓

6. $p = 6$

Find the value of the variable that makes each of the statements true.

7. $\frac{4a^2-9}{2a-3} = 9$

$\frac{(2a-3)}{1}\cdot\frac{4a^2-9}{2a-3} = (2a-3)\,9$

$4a^2-9 = 18a-27$

$-18a+27 \quad -18a+27$

$4a^2-18a+18 = 0$

$2(2a^2-9a+9)=0$

$2a-3 \quad 2a-6$

$2(2a-3)(a-3)=0$

18: 1,18 −1,−18 2,9 −2,−9 3,6 −3,−6

$2(2a-3)(a-3)=0$

$2a-3=0 \quad a-3=0$

$+3\ +3 \quad +3\ +3$

$\frac{2a}{2} = \frac{3}{2} \quad a=3$

$a = 3/2$ EXTRANEOUS (DIVIDE BY 0)

CHECK $a=3$

$(4(3)^2-9)/(2(3)-3)$ ✓ 9

7. $a=3$

8. $\frac{x^2-3x+4}{x-4} = -4$

$\frac{(x-4)}{1}\cdot\frac{x^2-3x+4}{(x-4)} = (x-4)\cdot -4$

$x^2-3x+4 = -4x+16$

$+4x-16 \quad +4x-16$

$x^2+x-12=0$

$(x-3)(x+4)=0$

$x-3=0 \quad x+4=0$

$+3\ +3 \quad -4\ -4$

$x=3 \quad x=-4$

−12: 1,−12 −1,12 2,−6 −2,6 3,−4 −3,4

CHECK $x=3$

$\frac{(3)^2-3(3)+4}{(3)-4} = -4$

$\frac{9-9+4}{-1} = -4$ ✓

$x=-4$

$\frac{(-4)^2-3(-4)+4}{(-4)-4} = -4$

$\frac{16+12+4}{-8} = \frac{32}{-8} = -4$ ✓

8. $x=-4,3$

9. $\frac{2}{3x^2} = \frac{1}{x} - \frac{1}{3}$

LCD $= 3x^2$

$\frac{3x^2}{1}\cdot\frac{2}{3x^2} = \frac{3x^2}{1}\cdot\frac{1}{x} - \frac{3x^2}{1}\cdot\frac{1}{3}$

$2 = 3x - x^2$

$+x^2 \quad -3x \quad -3x \quad +x^2$

$x^2-3x+2=0$

2: 1,2 −1,−2

$(x-1)(x-2)=0$

$x-1=0 \quad x-2=0$

$+1\ +1 \quad +2\ +2$

$x=1 \quad x=2$

CHECK $x=1$

$\frac{2}{3(1)^2} = \frac{1}{(1)} - \frac{1}{3}$

$\frac{2}{3} = \frac{3}{3} - \frac{1}{3}$ ✓

$x=2$

$\frac{2}{3(2)^2} = \frac{1}{(2)} - \frac{1}{3}$

$\frac{1}{6} = \frac{3}{6} - \frac{2}{6}$ ✓

9. $x=1,2$

10. $\frac{x}{x-4} = \frac{x+10}{x-2}$

LCD: $(x-4)(x-2)$

$\frac{(x-4)(x-2)}{1}\cdot\frac{x}{(x-4)} = \frac{(x-4)(x-2)}{1}\cdot\frac{(x+10)}{(x-2)}$

$(x-2)x = (x-4)(x+10)$

$x^2-2x = x^2+10x-4x-40$

$x^2-2x = x^2+6x-40$

$-x^2 \quad -x^2$

$-2x = 6x-40$

$-6x \quad -6x$

$-8x = -40$

$-8 \quad -8$

$x=5$

CHECK $x=5$

$\frac{(5)}{(5)-4} = \frac{(5)+10}{(5)-2}$

$\frac{5}{1} = \frac{15}{3}$ ✓

10. $x=5$

11. $\frac{p-1}{p+3} - \frac{2}{p-3} = \frac{7-3p}{p^2-9}$

$(p+3) \quad (p-3) \quad (p+3)(p-3)$

LCD: $(p+3)(p-3)$

$\frac{(p-3)(p+3)}{1}\cdot\frac{(p-1)}{(p+3)} - \frac{(p-3)(p+3)}{1}\cdot\frac{2}{(p-3)} = \frac{(p-3)(p+3)}{1}\cdot\frac{7-3p}{(p-3)(p+3)}$

$(p-3)(p-1) - 2(p+3) = 7-3p$

$p^2-4p+3-2p-6 = 7-3p$

$p^2-6p-3 = 7-3p$

$+3p \quad -7 \quad -7+3p$

$p^2-3p-10=0$

$(p-5)(p+2)=0$

$p-5=0 \quad p+2=0$

$+5\ +5 \quad -2\ -2$

$p=5 \quad p=-2$

CHECK $p=5$

$\frac{(5)-1}{(5)+3} - \frac{2}{(5)-3} = \frac{7-3(5)}{(5)^2-9}$

$\frac{4}{8} - \frac{2}{2} = \frac{-8}{16}$

$\frac{-4}{8} = -\frac{1}{2}$ ✓

11. $p=-2,5$

12. $\frac{x}{x-2} + \frac{2}{x+3} = \frac{3x+4}{x^2+x-6}$

$(x-2) \quad (x+3) \quad (x+3)(x-2)$

LCD: $(x+3)(x-2)$

$\frac{(x+3)(x-2)}{1}\cdot\frac{x}{(x-2)} + \frac{(x+3)(x-2)}{1}\cdot\frac{2}{(x+3)} = \frac{(x+3)(x-2)}{1}\cdot\frac{3x+4}{(x+3)(x-2)}$

$(x+3)x + (x-2)\cdot 2 = 3x+4$

$x^2+3x+2x-4 = 3x+4$

$x^2+5x-4 = 3x+4$

$-3x \quad -4 \quad -3x \quad -4$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x+4=0 \quad x-2=0$

$-4\ -4 \quad +2\ +2$

$x=-4 \quad x=2$ EXTRANEOUS

CHECK $x=-4$

$\frac{(-4)}{(-4)-2} + \frac{2}{(-4)+3} = \frac{3(-4)+4}{(-4)^2+(-4)-6}$

$\frac{-4}{-6} + \frac{2}{-1} = \frac{-8}{6}$

$\frac{2}{3} - \frac{6}{3} = -\frac{4}{3} = -\frac{4}{3}$ ✓

12. $x=-4$

Find the set of values of the variable that makes each of the inequality statements true.

13. $\frac{3a}{a-1} \leq 4$

BOUNDARY POINTS

① SET INEQUALITY TO EQUAL

$\frac{3a}{a-1} = 4$ · $(a-1)$

$3a = 4a - 4$

$-4a \quad -4a$

$-1a = -4$

$\frac{-1a}{-1} = \frac{-4}{-1}$

$a = 4$ ← CLOSED DOT

② SET DENOMINATOR TO ZERO

$a - 1 \neq 0$

$+1 \quad +1$

$a \neq 1$

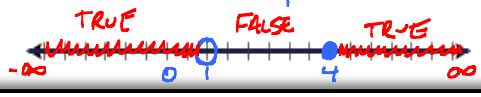
↑ OPEN DOT

TEST $a=0$: $\frac{3(0)}{0-1} \leq 4$ → $\frac{0}{-1} \leq 4$ ↓

TEST $x=2$: $\frac{3(2)}{2-1} \leq 4$ → $6 \leq 4$ ↓

TEST $x=5$: $\frac{3(5)}{5-1} \leq 4$ → $3.75 \leq 4$ ↓

13. SET: $a < 1$ OR $a \geq 4$ | INT: $(-\infty, 1) \cup [4, \infty)$



14. $\frac{m-3}{m+2} \geq 0$

BOUNDARY POINTS

① SET INEQUALITY TO EQUAL

$\frac{(m+2)}{1} \frac{m-3}{m+2} = 0 \frac{(m+2)}{1}$

$m - 3 = 0$

$+3 \quad +3$

$m = 3$

CLOSED

② SET DENOMINATOR EQUAL TO ZERO

$m + 2 = 0$

$-2 \quad -2$

$m = -2$

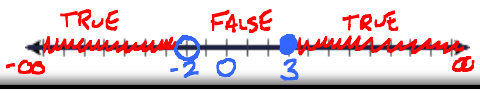
OPEN

TEST $x=-3$: $\frac{-3-3}{-3+2} \geq 0$ → $\frac{-6}{-1} \geq 0$ TRUE ↓

TEST $x=0$: $\frac{0-3}{0+2} \geq 0$ → $-1.5 \geq 0$ FALSE ↓

TEST $x=4$: $\frac{4-3}{4+2} \geq 0$ → $\frac{1}{6} \geq 0$ TRUE ↓

14. SET: $m < -2$ OR $m \geq 3$ | INT: $(-\infty, -2) \cup [3, \infty)$



15. $\frac{y}{y+2} < 3$

BOUNDARY POINT

① SET INEQUALITY TO EQUAL

$\frac{(y+2)}{1} \frac{y}{y+2} = 3(y+2)$

$y = 3y + 6$

$-3y \quad -3y$

$-2y = 6$

$\frac{-2y}{-2} = \frac{6}{-2}$

$y = -3$ ← OPEN DOT

② SET DENOMINATOR EQUAL TO ZERO

$y + 2 = 0$

$-2 \quad -2$

$y = -2$

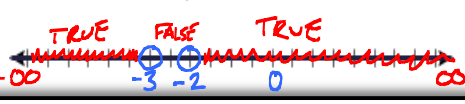
↑ OPEN DOT

TEST $x=-4$: $\frac{-4}{-4+2} < 3$ → $\frac{-4}{-2} < 3$ → $2 < 3$ ↓

TEST $x=-2.5$: $\frac{-2.5}{-2.5+2} < 3$ → $5 < 3$ ↓

TEST $x=0$: $\frac{0}{0+2} < 3$ → $0 < 3$ ↓

15. SET: $y < -3$ OR $y > -2$ | INT: $(-\infty, -3) \cup (-2, \infty)$



16. $\frac{3}{x-2} \geq \frac{1}{x+2}$

BOUNDARY POINT

① SET INEQUALITY TO EQUAL

$\frac{(x+2)(x-2)}{1} \frac{3}{x-2} = \frac{1}{x+2} \frac{(x+2)(x-2)}{1}$

$(x+2)3 = 1(x-2)$

$3x + 6 = x - 2$

$-x \quad -x$

$2x + 6 = -2$

$-6 \quad -6$

$\frac{2x}{2} = \frac{-8}{2}$

$x = -4$ ← CLOSED DOT

② SET DENOMINATOR EQUAL TO ZERO

$x - 2 = 0$ → $x = 2$

$x + 2 = 0$ → $x = -2$

OPEN DOTS

TEST $x=-5$: $\frac{3}{-5-2} \geq \frac{1}{-5+2}$ → $\frac{3}{-7} \geq \frac{1}{-3}$ FALSE

TEST $x=-3$: $\frac{3}{-3-2} \geq \frac{1}{-3+2}$ → $\frac{3}{-5} \geq -1$ TRUE

TEST $x=0$: $\frac{3}{0-2} \geq \frac{1}{0+2}$ → $-1.5 \geq 0.5$ FALSE

TEST $x=3$: $\frac{3}{3-2} \geq \frac{1}{3+2}$ → $\frac{3}{1} \geq \frac{1}{5}$ TRUE

16. SET: $-4 \leq x < -2$ OR $x > 2$ | INT: $[-4, -2) \cup (2, \infty)$