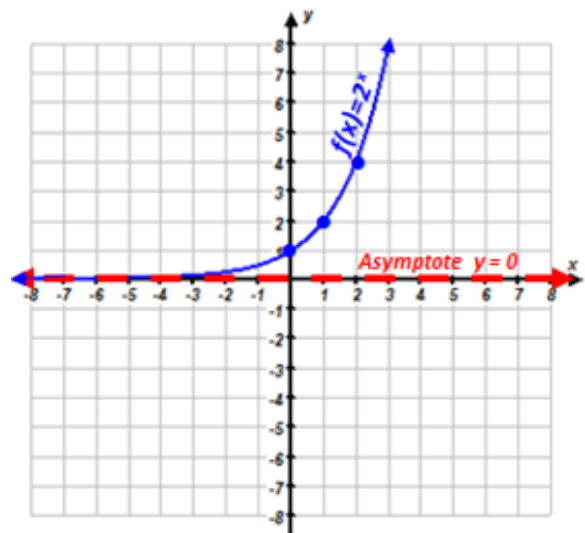
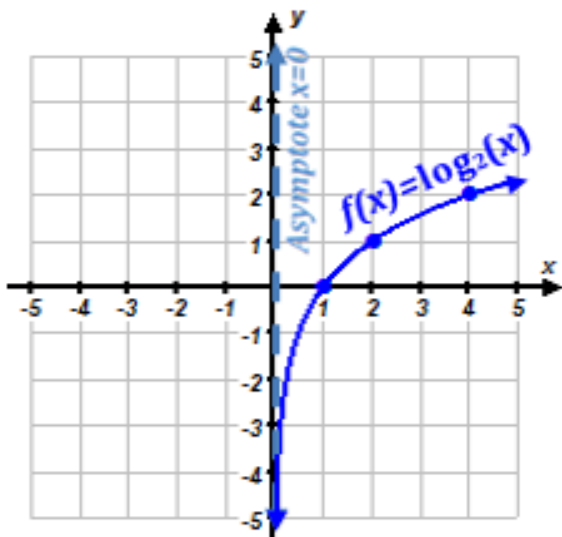


Algebra 2

Unit 5:

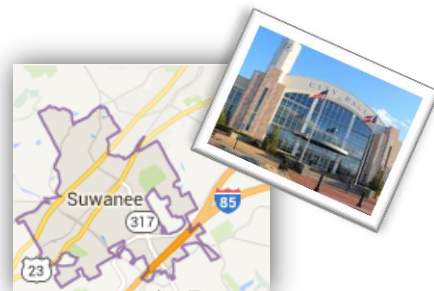
Exponential & Logarithmic Functions



1. The population of the city Suwanee, GA has consistently **grown** by 4% for the last several years. In the year 2000, the population was 9,500 people.

What would be the growth factor (multiplier)?

If the trend continues what would be the population in 2020?



2. Lisa purchases a house for \$150,000 near Lake Jackson. The value of houses in the area where the house was purchased is averaging an **increase** of 6% per year.

What would be the growth factor (multiplier)?

If the trend continues how much would the house be worth 12 years after Lisa purchased the house?



3. Esther purchased a used car, a Ford Focus, for \$8400. The car is expected to **decrease** in value by 20% per year over the next couple of years.

What would be the decay factor (multiplier)?

If the trend continues how much would the car be worth 6 years after Esther purchased the car?



4. Freddie purchased a pair of never worn Vintage 1997 Nike Air Jordan XII Playoff Black Varsity Shoe Size 12 for \$380. The shoes have shown an average **growth** rate of 14% per year.

What would be the growth factor (multiplier)?

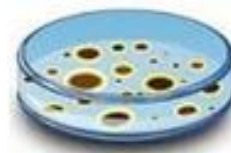
If the trend continues how much would the shoes be worth 5 years after Freddie purchased the shoes?



5. A culture of bacteria **triples** by the end of each hour. There were initially 50 bacteria present in the petri dish.

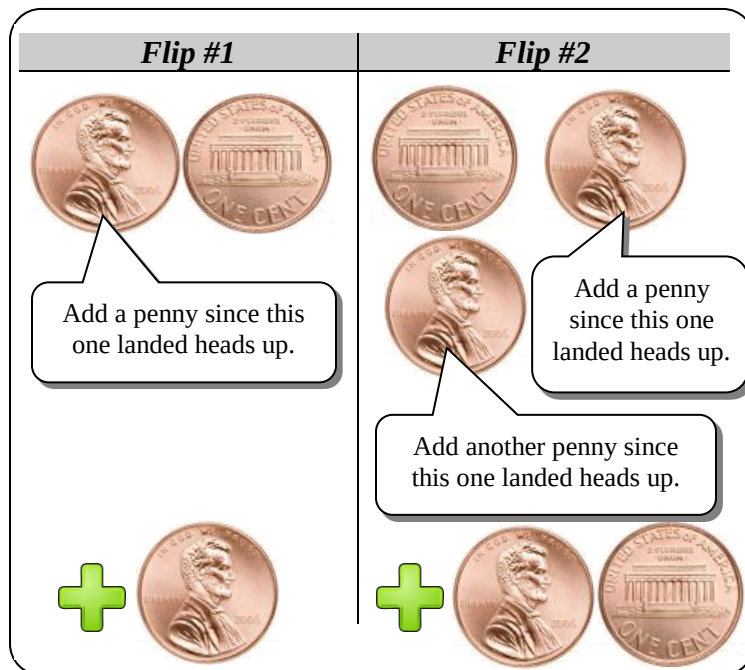
What would be the growth factor (multiplier)?

If the trend continues how many bacteria would there be 5 hours after the analysis began?

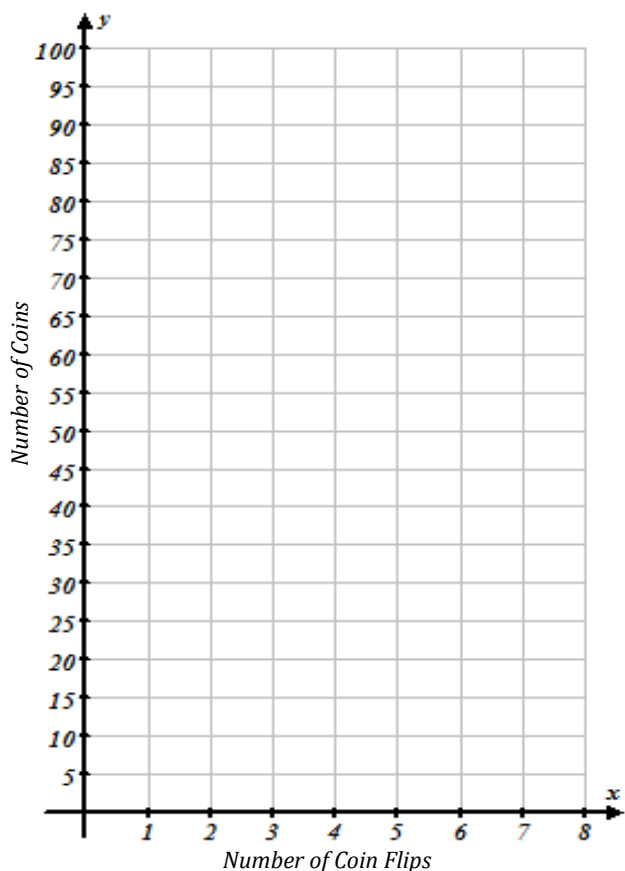


6. Consider starting with 2 pennies. Flip them both and for each one that lands heads up, add a penny to the pile. So, the pile should increase in size. Again, flip the new pile of pennies which could be a size of 2, 3, or 4. For every penny that lands heads up add another penny to the pile. Repeat this process several times and record how the penny pile grows after each flip. Your values may differ on Flips 3 and 4.

Number of Flips	Number of Pennies
0	2
1	3
2	5
3	
4	
5	
6	
7	



Create a graph of the data.



- What is an appropriate growth factor (multiplier)?
- Create an equation that describes the relationship between the number of flips and the number of pennies in the pile.
- Approximately how many pennies would there be on the 9th flip?
- Should the graph be continuous or discrete? Explain.
- What is an appropriate Domain and Range for the **situation**?

7. Determine which of the following functions are exponential models of **Growth** and which are models of **Decay**.

a. $f(x) = 2 \cdot (1.05)^x$

b. $g(x) = 540 \cdot (0.92)^x + 1$

c. $h(x) = 4 \cdot \left(\frac{3}{5}\right)^x$

Circle the Answer

Growth Decay Neither

Circle the Answer

Growth Decay Neither

Circle the Answer

Growth Decay Neither

d. $y = 230 \cdot \left(\frac{7}{5}\right)^x$

e. $y = 4200 \cdot e^x - 5$

f. $y = 9 \cdot (2)^{-x}$

Circle the Answer

Growth Decay Neither

Circle the Answer

Growth Decay Neither

Circle the Answer

Growth Decay Neither

g. $y = \frac{1}{4} \cdot (3)^x$

h. $p(x) = 520 \cdot e^{-x} + 3$

i. $y = 230 \cdot \left(\frac{2}{5}\right)^{-x}$

Circle the Answer

Growth Decay Neither

Circle the Answer

Growth Decay Neither

Circle the Answer

Growth Decay Neither

8. Consider the Compound Interest Formula: $A = P \left(1 + \frac{r}{n}\right)^{nt}$

- a. Determine the value of an account in which a person invested \$6000 for 12 years at an annual rate of 9% compounded annually ($n = 1$).

A = Value of Account after Compounding
P = Original Amount Invested
r = Annual Interest Rate as a decimal
n = Compounds per Year
t = Number of Years Interest is Accrued

$n = 1$:Annually
 $n = 2$:Semi-Annually
 $n = 4$:Quarterly
 $n = 12$:Monthly
 $n = 52$:Weekly
 $n = 365$:Daily

- b. Determine the value of an account in which a person invested \$6000 for 12 years at an annual rate of 9% compounded quarterly ($n = 4$).

- c. Determine the value of an account in which a person invested \$6000 for 12 years at an annual rate of 9% compounded weekly ($n = 52$).

9. Consider the Compound Interest Formula: $A = P \cdot e^{rt}$

Determine the value of an account in which a person invested \$6000 for 12 years at an annual rate of 9% compounded continuously.

A = Value of Account after Compounding
P = Original Amount Invested
r = Annual Interest Rate as a decimal
t = Number of Years Interest is Accrued

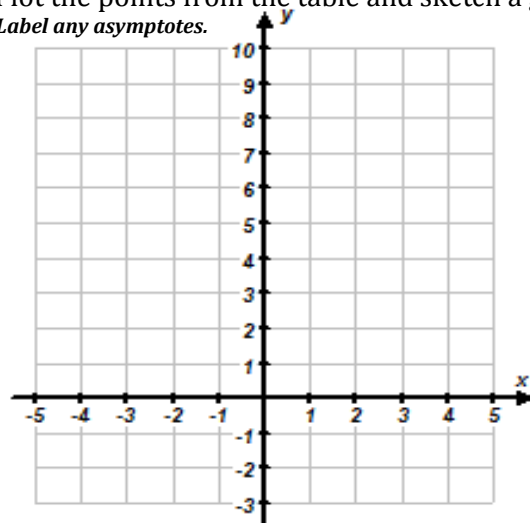
1. Consider the **exponential function**, $f(x) = 3^x$.

A. Fill in the missing values in the table below.

x	$f(x)$
2	
0	
	27
1	
-1	
-3	

C. Determine the **Domain** & **Range** of the function.

- B. Plot the points from the table and sketch a graph
Label any asymptotes.



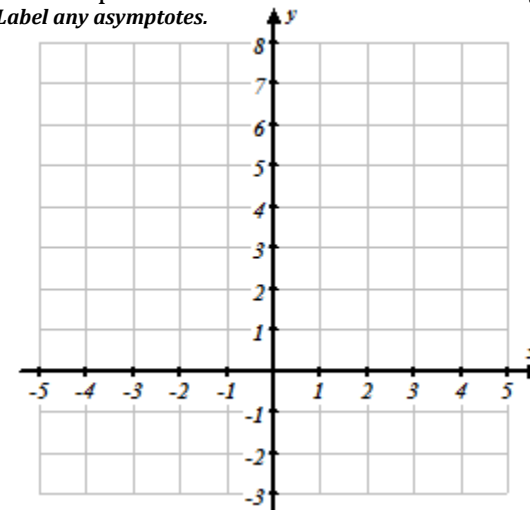
2. Consider the **exponential function**, $g(x) = 2^x - 2$.

A. Fill in the missing values in the table below.

x	$g(x)$
2	
3	
	0
0	
-1	
-3	

C. Determine the **Domain** & **Range** of the function.

- B. Plot the points from the table and sketch a graph
Label any asymptotes.



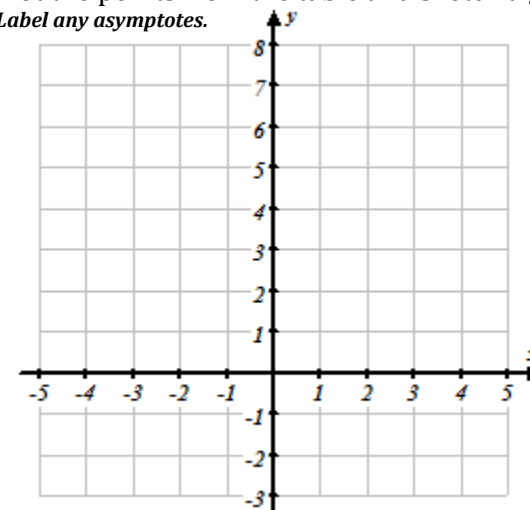
3. Consider the **exponential function**, $h(x) = \left(\frac{1}{2}\right)^x + 1$.

A. Fill in the missing values in the table below.

x	$h(x)$
-3	
-1	
0	
1	
2	
3	

C. Determine the **Domain** & **Range** of the function.

- B. Plot the points from the table and sketch a graph
Label any asymptotes.

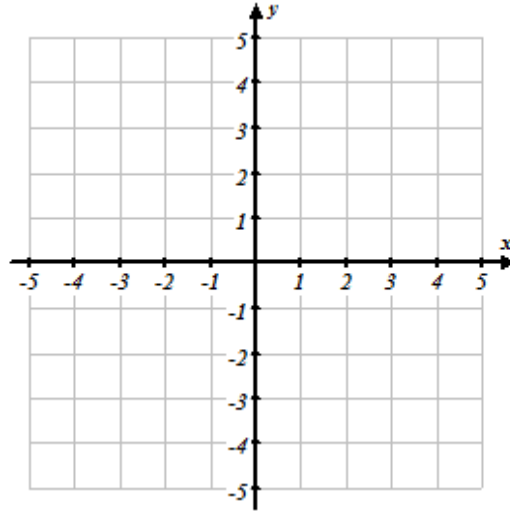
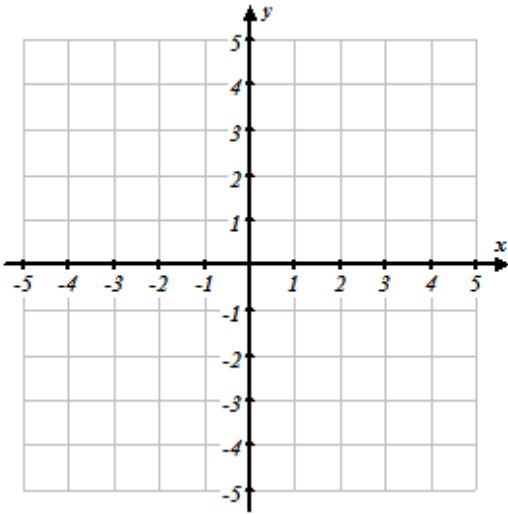
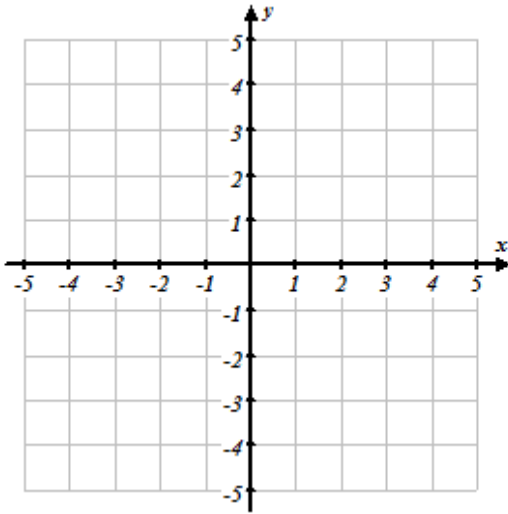


4. Determine the asymptote and sketch a graph (label the any intercepts, points when $x = 0$, and when $x = 1$.)

A. $f(x) = 3^x - 4$

B. $g(x) = \left(\frac{1}{2}\right)^x + 2$

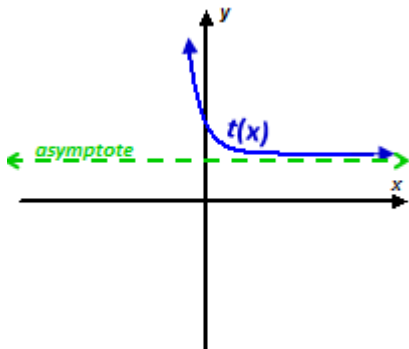
C. $h(x) = -2^x - 3$



5. Create two different exponential functions of the form $f(x) = a \cdot b^x + c$ that have a horizontal asymptote at $y = 5$.

6. Given the function $f(x)$ is of the form $f(x) = a \cdot b^x + c$, has a horizontal asymptote at $y = -1$, and passes through the point $(0, 2)$, create a possible function for $f(x)$.

7. Consider $t(x)$ is of the form $t(x) = a^x + c$.



Which of the following must be true for the parameter ' a '?

$a > 1$

$0 < a < 1$

$a < 0$

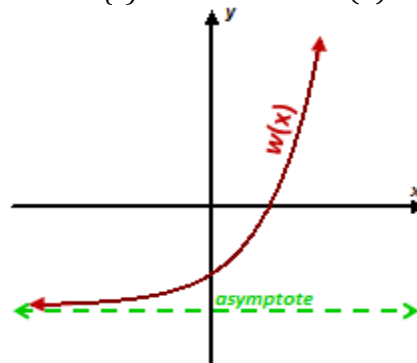
Which of the following must be true for the parameter ' c '?

$c > 0$

$c = 0$

$c < 0$

8. Consider $w(x)$ is of the form $w(x) = a^x + c$.



Which of the following must be true for the parameter ' a '?

$a > 1$

$0 < a < 1$

$a < 0$

Which of the following must be true for the parameter ' c '?

$c > 0$

$c = 0$

$c < 0$

9. Determine the x-intercept and y-intercept of the following exponential functions:

a. $r(x) = 3 \cdot 2^x - 6$

b. $r(x) = -1 \cdot 3^x + 9$

10. The parent graph is shown in light gray on the graph. Graph the transformed function on the same Cartesian coordinate grid and describe the transformations based on the function $t(x)$.

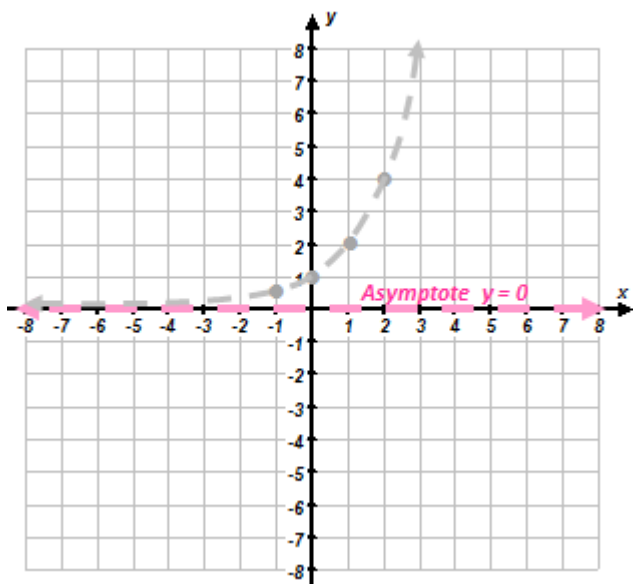
a. Parent Function: $f(x) = 2^x$

Transformed Function: $t(x) = 2^{(x-2)} - 6$

b. Parent Function: $f(x) = 2^x$

Transformed Function: $t(x) = 2 \cdot 2^{(x-4)}$

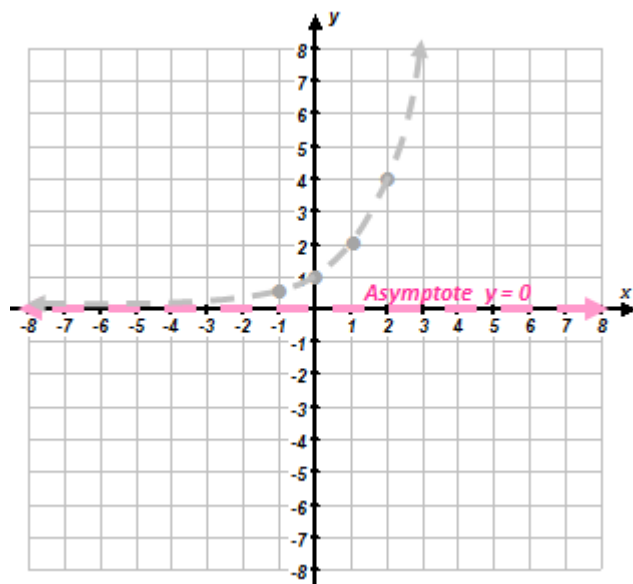
Determine the Domain & Range of the function.



c. Parent Function: $f(x) = 3^x$

Transformed Function: $t(x) = -3^{(x+3)}$

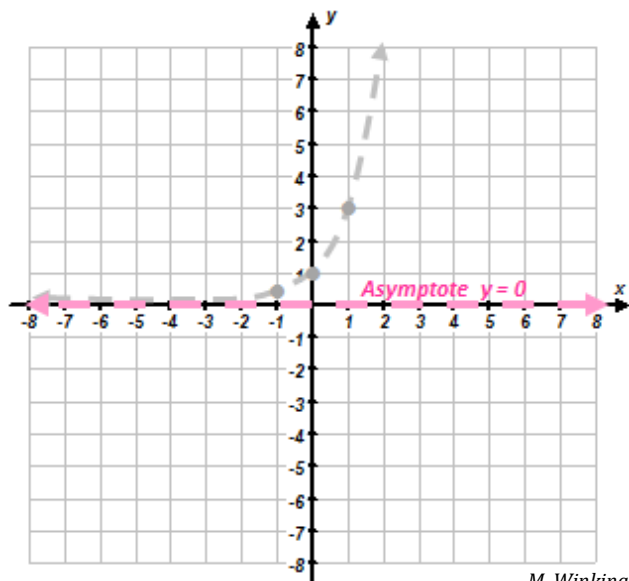
Determine the Domain & Range of the function.



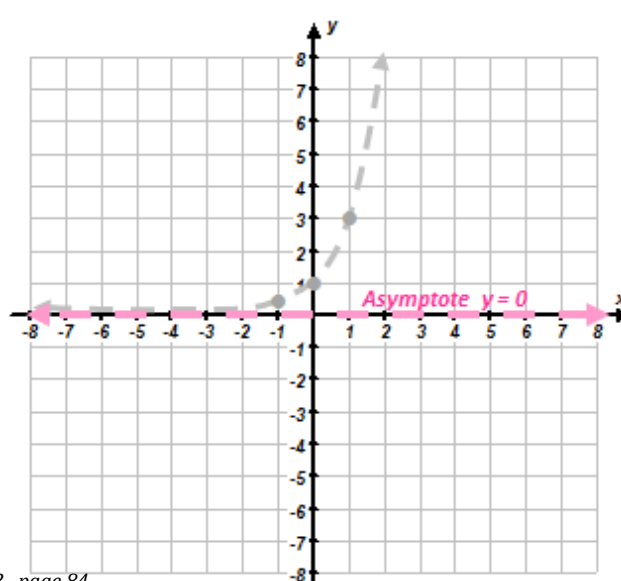
d. Parent Function: $f(x) = 3^x$

Transformed Function: $t(x) = 3^{(-x)} + 2$

Determine the Domain & Range of the function.

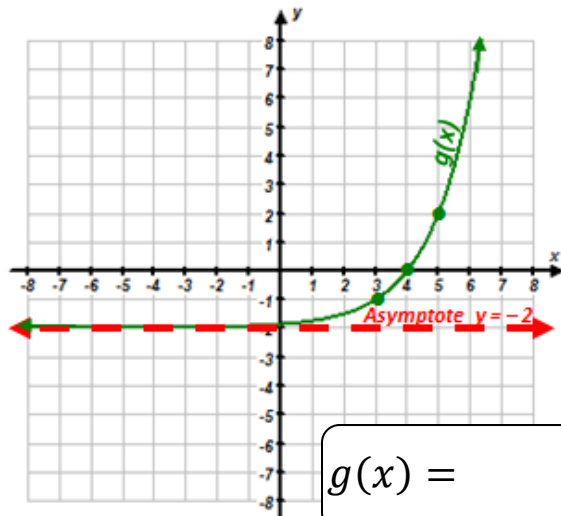
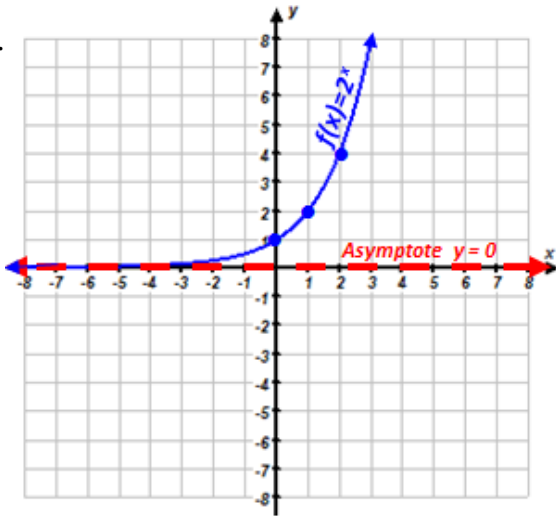


Determine the Domain & Range of the function.



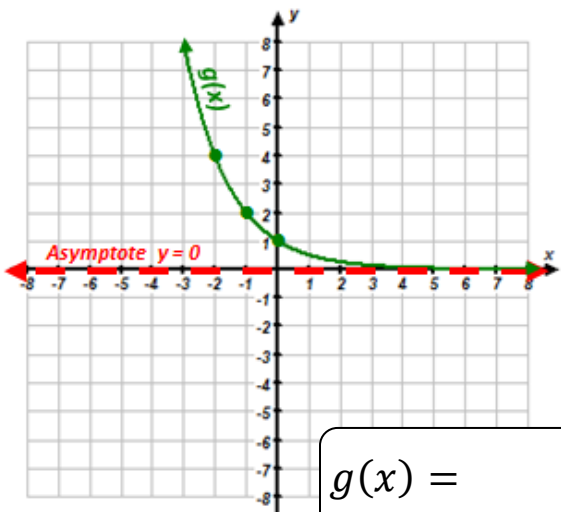
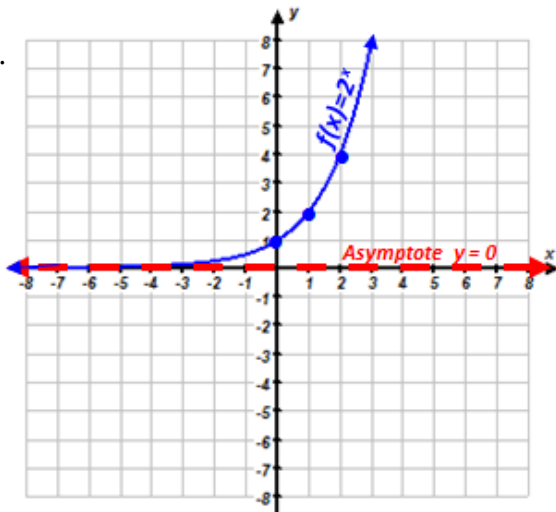
11. Given the graph of $f(x)$ on the left, determine an equation for $g(x)$ on the right in terms of $f(x)$.

a.



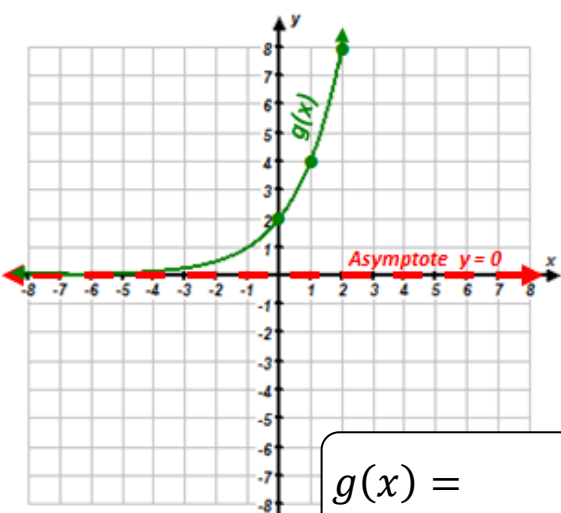
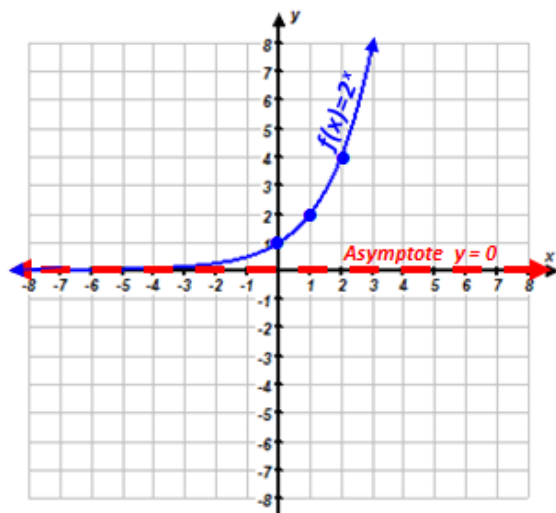
$g(x) =$

b.



$g(x) =$

c.



$g(x) =$

12. Given a table of values for the exponential function $f(x)$ and a description of the transformations for the function $g(x)$, fill out the table of values based on the original points for $g(x)$, the transformed function.

a.

x	-1	0	1	2	3
$f(x)$	$\frac{1}{3}$	1	3	9	27

Translated Down 4

x	-1	0	1	2	3
$g(x)$					

b.

x	-1	0	1	2	3
$f(x)$	$\frac{1}{2}$	1	2	4	8

Translated Left 1 & Up 2

x					
$g(x)$					

c.

x	-1	0	1	2	3
$f(x)$	$\frac{1}{4}$	1	4	16	64

Reflect over x -axis

x					
$g(x)$					

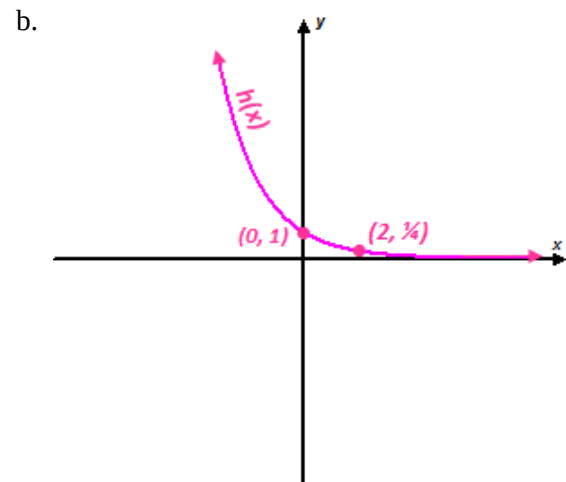
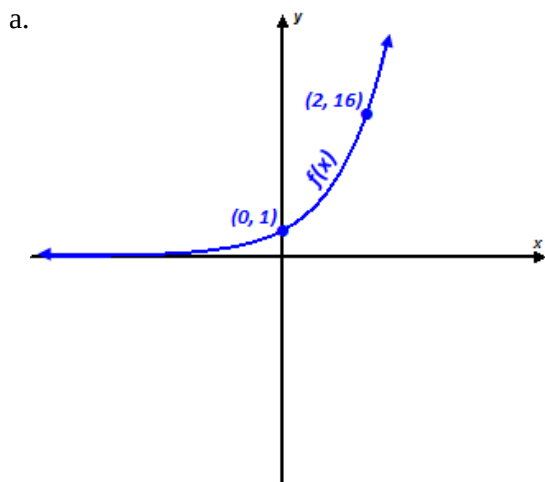
d.

x	-1	0	1	2	3
$f(x)$	$\frac{1}{2}$	1	2	4	8

Vertical Stretch of Factor 3

x					
$g(x)$					

13. Given each of the graphs below are exponential functions of the form $f(x) = a^x$, determine the parameter 'a' in each graph.



$f(x) =$

$h(x) =$

1. Rewrite the following **exponential statements** as **logarithmic statements**. (**EXP**→**LOG**)

a. $125 = 5^3$

b. $2^6 = 64$

c. $4^x = 16$

d. $243 = x^5$

e. $e^x = 9$

f. $x = e^5$

2. Rewrite the following **logarithmic statements** as **exponential statements**. (**LOG** → **EXP**)

a. $3 = \log_2(8)$

b. $5 = \log_x(243)$

c. $\log_6(x) = 3$

d. $\ln(x) = 5$

e. $\log_4(256) = 2x$

f. $x = \ln(3)$

3. Evaluate the following basic logarithm statements.

a. $\log_2(32)$

b. $\log_7(49)$

c. $\log_6(6)$

d. $\log_4(256)$

e. $\log(1000)$

f. $\ln(e^7)$

4. Evaluate the following logarithm statements.

a. $\log_5(5^{12})$

b. $(\log_3(3^x))^2$

c. $\log_3(9^3)$

d. $\log_2(16^5)$

e. $4^{\log_4(16)}$

f. $3^{\log_3(81)}$

d. $5^{\log_5(12)}$

e. $4^{\log_2(32)}$

f. $e^{\ln(5x)}$

Evaluate the following using the prime factorization of 9^4 .

$\log_3(9^4)$

Evaluate the following using a recognized property.

$\log_3(9^4)$

5. Rewrite each of the following using the property above.

- a. $\log_5(25^3)$
- b. $\log_3(14^5)$
- c. $\ln(9^3)$

Evaluate the following with your calculator by changing the base to 3 decimal places
(show the work to provide reasoning)

$\log_2 9 = x$

6. Evaluate the following with your calculator by changing the base to 3 decimal places

- a. $\log_5(50)$

b. $\log_8(12)$

c. $\log_4(4194304)$
- d. $\log_3(212)$

e. $\log(532)$

f. $\ln(28)$

1. Consider the **logarithmic function** , $f(x) = \log_2(x)$.

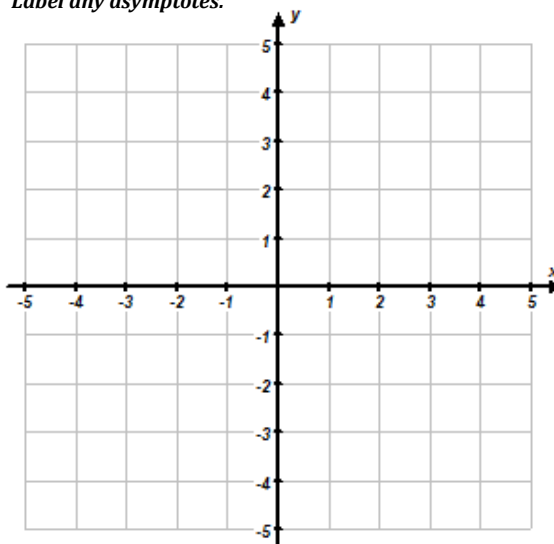
A. Fill in the missing values in the table below.

x	$f(x)$
0	
$\frac{1}{2}$	
1	
2	
4	
$\frac{1}{4}$	

C. Determine the **Domain** & **Range** of the function.

D. Determine the **End Behavior**.

- B. Plot the points from the table and sketch a graph
Label any asymptotes.



2. Consider the **logarithmic function** , $g(x) = \log_2(x + 3) + 2$

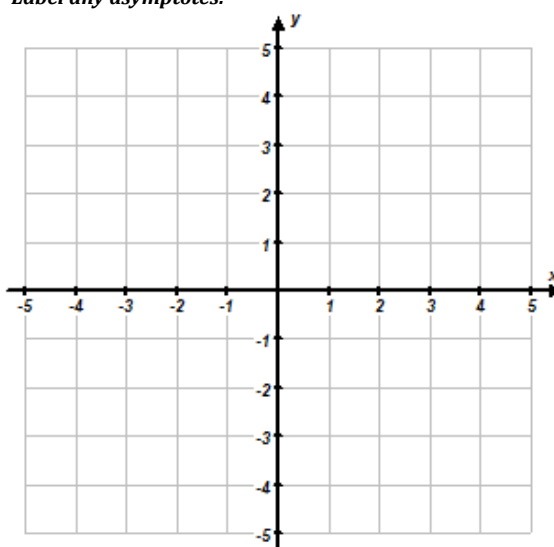
A. Fill in the missing values in the table below.

x	$g(x)$
-3	
-2.5	
-1	
0	
1	
5	

C. Determine the **Domain** & **Range** of the function.

D. Determine the **End Behavior**.

- B. Plot the points from the table and sketch a graph
Label any asymptotes.



3. Consider the **logarithmic function** , $h(x) = \ln(x - 1) + 3$.

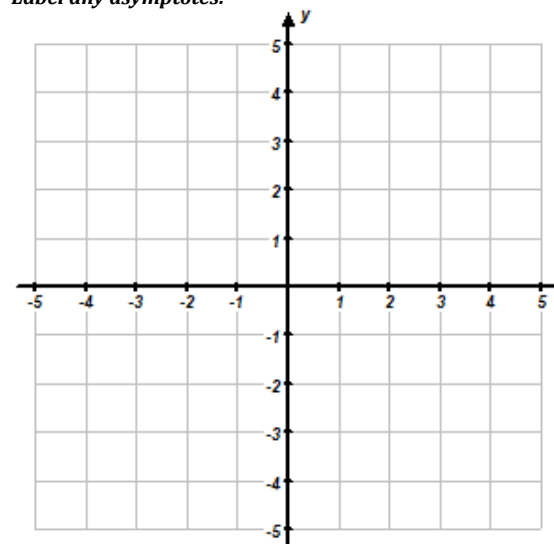
A. Fill in the missing values in the table below.

x	$h(x)$
2	
1	
1.2	
1.5	
3	
5	

C. Determine the **Domain** & **Range** of the function.

D. Determine the **End Behavior**.

- B. Plot the points from the table and sketch a graph
Label any asymptotes.

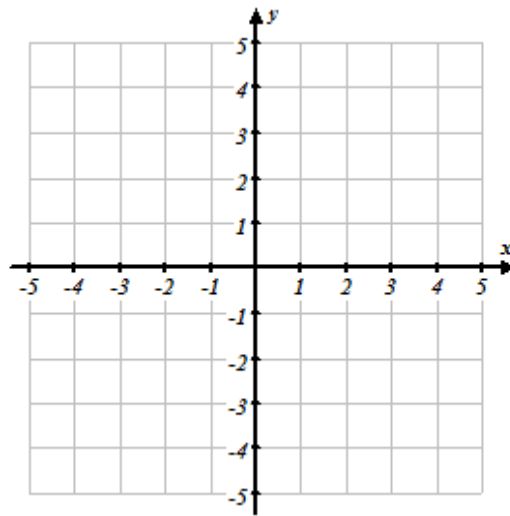
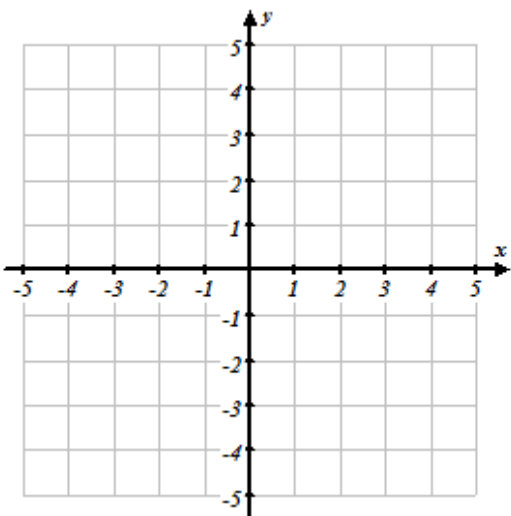
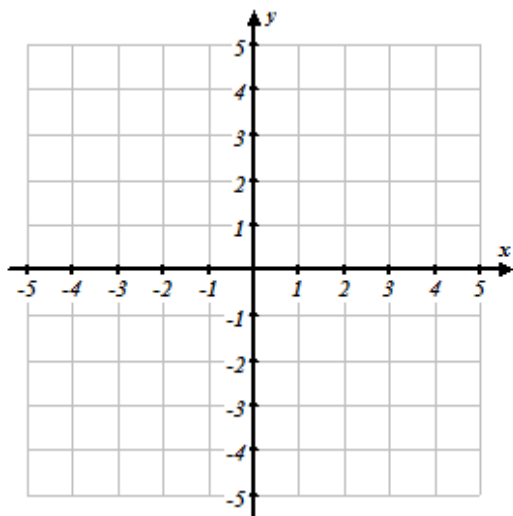


4. Determine the asymptote and sketch a graph (label the any intercepts, points when you locate $\log(1)$).

A. $f(x) = \log_2(x + 3)$

B. $g(x) = \log_5(x - 2) - 1$

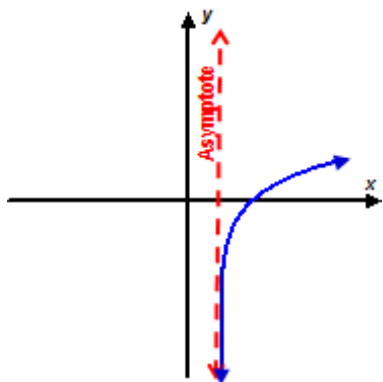
C. $h(x) = -\ln(x + 1)$



5. Create two different logarithmic functions of the form $f(x) = a \cdot \log_2(x + b) + c$ that have a vertical asymptote at $x = 4$.

6. Given the function $f(x)$ is of the form $f(x) = \log_2(x + b) + c$, has a vertical asymptote at $x = -1$, and passes through the point $(0, 2)$, create a possible function for $f(x)$.

7. Consider $t(x)$ is of the form $t(x) = a \cdot \log_2(x + b)$.



Which of the following must be true for the parameter ' b '?

$b < 1$

$b = 0$

$b > 0$

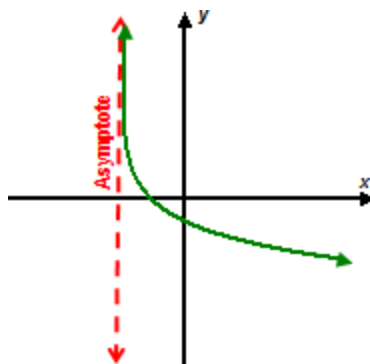
Which of the following must be true for the parameter ' a '?

$a < 0$

$a = 0$

$a > 0$

8. Consider $w(x)$ is of the form $w(x) = a \cdot \log_2(x + b)$



Which of the following must be true for the parameter ' b '?

$b < 0$

$b = 0$

$b > 0$

Which of the following must be true for the parameter ' a '?

$a < 0$

$a = 0$

$a > 0$

9. Determine the y-intercept of the following logarithmic functions:

a. $r(x) = 2 \log_3(x + 9)$

b. $p(x) = \log_3(x - 2)$

c. $m(x) = \log_5(x + 1) + 9$

Consider the parent function of $f(x) = \log_m(x)$. The following would be a transformed function

$$t(x) = a \cdot \log_m(b(x - c)) + d$$

$a > 1$: Vertical Stretch (eg. $a = 3$)
(factor ' a ')

$0 < a < 1$: Vertical Compress (e.g. $a = 0.2$)
(factor ' a ')

$-1 < a < 0$: Reflect over x-axis & Vertical Compress (e.g. $a = -0.2$)
(factor ' a ')

$a = -1$: Reflect over x-axis

$a < -1$: Reflect over x-axis & Vertical Stretch (e.g. $a = -4$)
(factor ' a ')

$b > 1$: Horizontal Compress (eg. $b = 3$)
(factor ' $\frac{1}{b}$ ')

$0 < b < 1$: Horizontal Stretch (e.g. $b = 0.2$)
(factor ' $\frac{1}{b}$ ')

$-1 < b < 0$: Reflect over y-axis & Horizontal Stretch (e.g. $b = -0.2$)
(factor ' $\frac{1}{b}$ ')

$b = -1$: Reflect over y-axis

$b < -1$: Reflect over y-axis & Horizontal Compress (e.g. $b = -4$)
(factor ' $\frac{1}{b}$ ')

d = Vertical Translation

c = Horizontal Translation
(opposite direction)

10. Describe the transformations based on the function $t(x)$.

a. Parent Function: $f(x) = \log_3(x)$

Transformed Function: $t(x) = 3 \cdot \log_3(x + 2) - 1$

b. Parent Function: $f(x) = \ln(x)$

Transformed Function: $t(x) = -\ln(2(x + 4))$

11. Given a table of values for the exponential function $f(x)$ and a description of the transformations for the function $g(x)$, fill out the table of values based on the original points for $g(x)$, the transformed function.

a.

x	0	1	2	4	8
$f(x)$	Undefined	0	1	2	3

Translated Down 4

x	0	1	2	4	8
$g(x)$					

b.

x	0	1	3	9	27
$f(x)$	Undefined	0	1	2	3

Translated Left 1 & Up 2

x					
$g(x)$					

12. The parent graph is shown in light gray on the graph. Graph the transformed function on the same Cartesian coordinate grid and describe the transformations based on the function $t(x)$.

a. Parent Function: $f(x) = \log_2(x)$

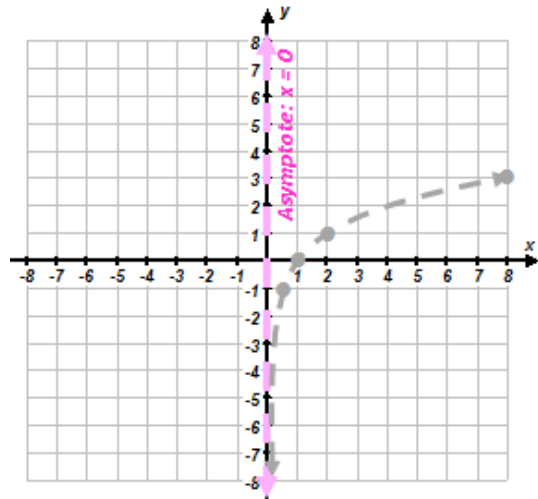
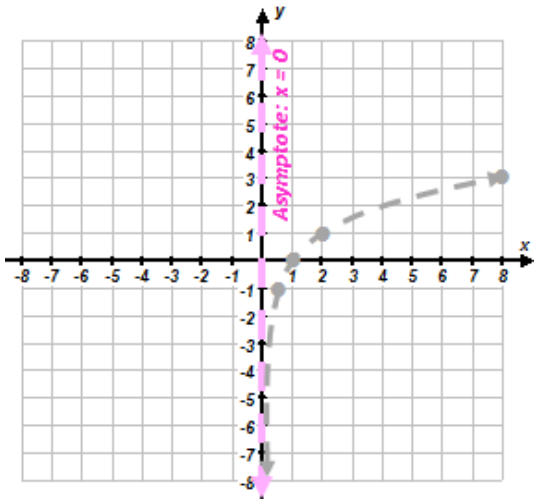
Transformed Function: $t(x) = \log_2(-x) + 3$

b. Parent Function: $f(x) = \log_2(x)$

Transformed Function: $t(x) = 3 \cdot \log_2(x + 4)$

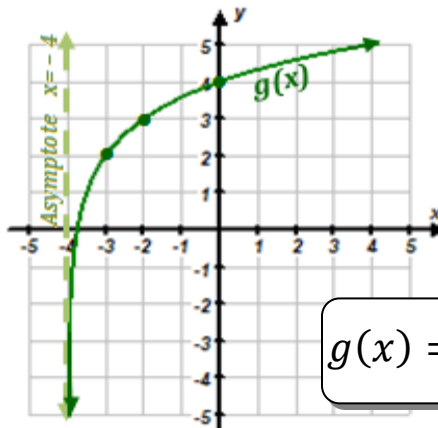
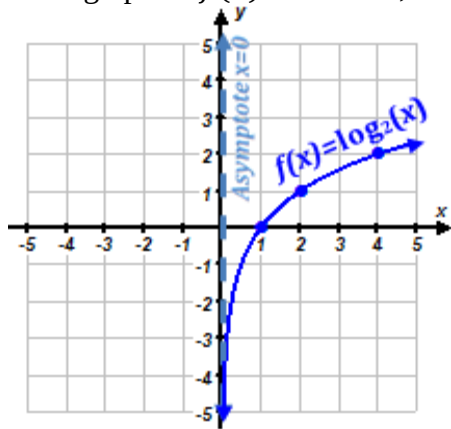
Determine the Domain & Range of the transformed function.

Determine the Domain & Range of the transformed function.



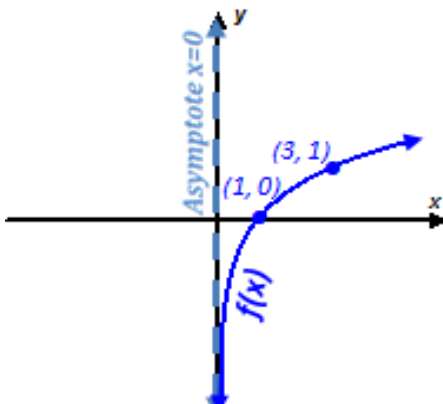
13. Given the graph of $f(x)$ on the left, determine an equation for $g(x)$ on the right in terms of $f(x)$.

a.



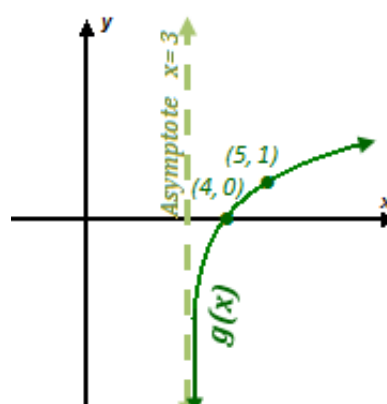
$g(x) =$

14. The graph below is a functions of the form $f(x) = \log_a x$, determine the parameter 'a'.



$f(x) =$

15. The graph below is a functions of the form $g(x) = \log_a(x + b)$, determine the parameter 'b'.

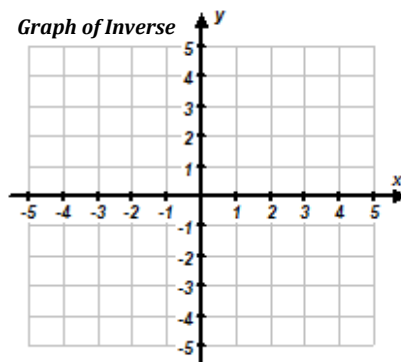
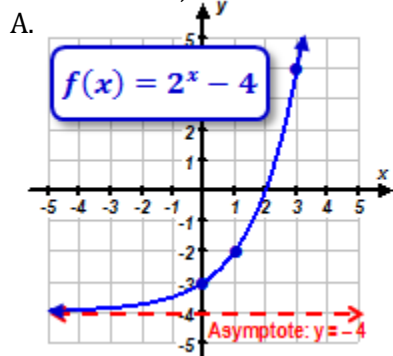


$g(x) =$

Sec 5.5 – Exponential & Logarithmic Functions
(Inverses of Exponential and Log Functions)

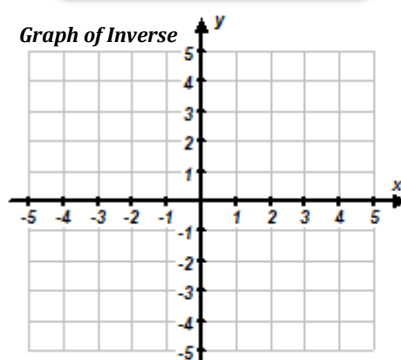
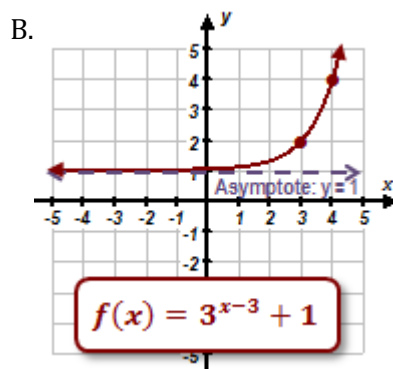
Name: _____

1. Consider the exponential function $f(x)$ shown below. Find the inverse of the function, sketch a graph of the inverse, and determine whether or not the inverse is a function.



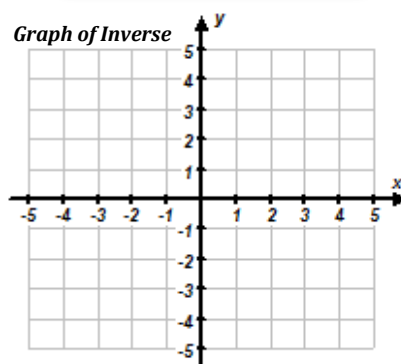
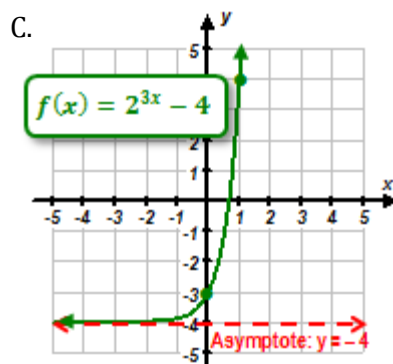
Is the Inverse a Function?

YES NO



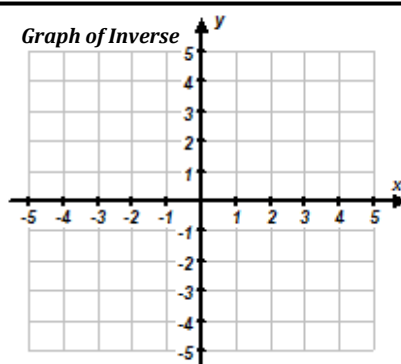
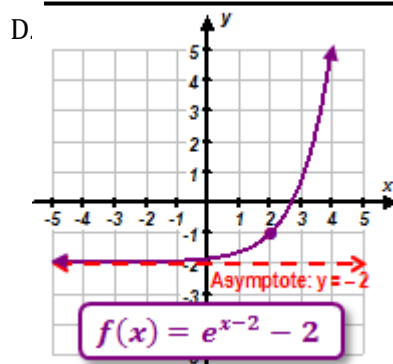
Is the Inverse a Function?

YES NO



Is the Inverse a Function?

YES NO

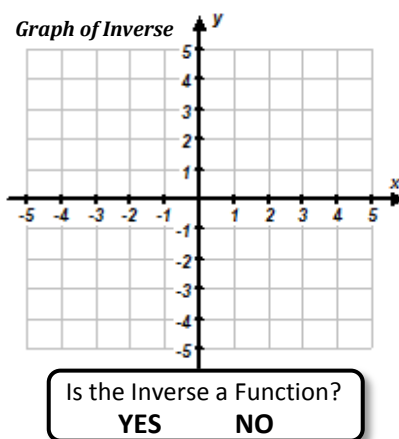
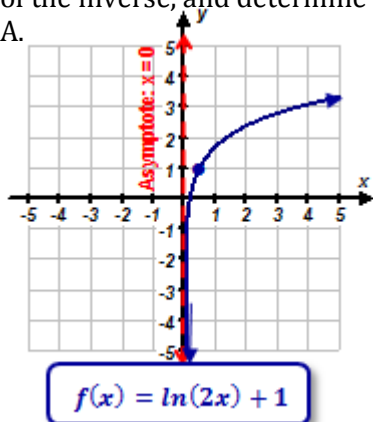


Is the Inverse a Function?

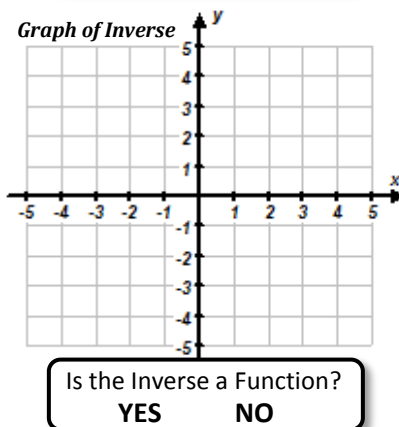
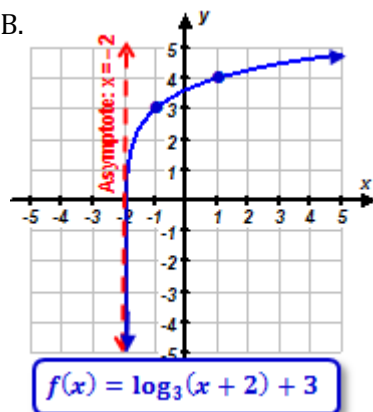
YES NO

2. Consider the logarithmic function $f(x)$ shown below. Find the inverse of the function, sketch a graph of the inverse, and determine whether or not the inverse is a function.

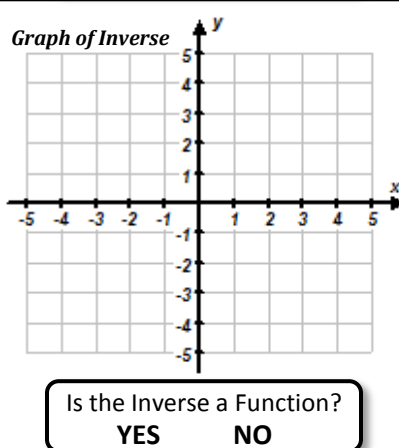
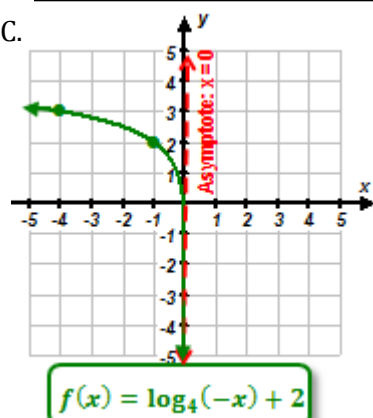
A.



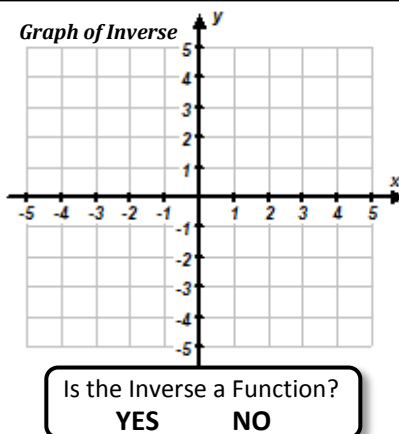
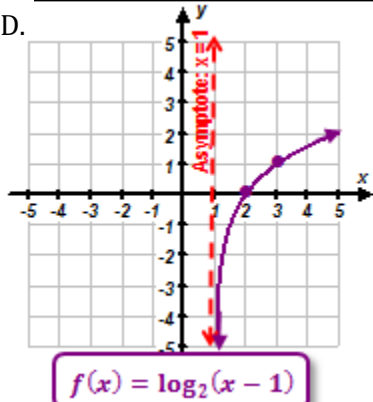
B.



C.



D.



Rules of Exponents

$$x^3 \cdot x^2 = x^5$$

$$(x^3)^2 = x^6$$

$$\frac{x^7}{x^4} = x^3$$

$$\frac{y^2 x^{-3} z^5}{y^{-4} x^6 z^7} = \frac{y^2 \boxed{x^{-3}} z^5}{\boxed{y^{-4}} x^6 z^7} = \frac{y^4 y^2 z^5}{x^3 x^6 z^7} = \frac{y^6}{x^9 z^2}$$

$$\sqrt[5]{x^2} = x^{\frac{2}{5}}$$

1. Simplify

a. $(5x^2)(14x^3)$

b. $\frac{12x^4 y^9}{3x^5 y^7}$

c. $(3x^3 y^2)^2$

d. $(5x^3 y^2)(-6x^6 y^4)$

e. $(-3m^3 n^4)^2 (4m^4 n^7)$

f. $\frac{24a^{-3} b^7 c^2}{16a^2 b^4 c^7}$

(1 continued) Simplify

g. $\left(\frac{3a^6b^3c^2}{2a^2b^4}\right)^2$

h. $(3x^2y^4z)(4x^3y^{-2}z^8)$

i. $(5a^3b^{-2}c^5)^2$

j. $\left(\frac{4x^2y^{-3}}{y^{-2}}\right)^{-1}$

k. $(8x^7y^3)(3x^{-4}y^8)^2$

l. $(9a^3b^5)(-4a^3b^7)^2$

m. $\left(\frac{4m^4n^{-3}p^2}{6m^2n^2}\right)^{-2}$

n. $\left(\frac{3x}{y^{-3}}\right)^3\left(\frac{5x^{-10}yz^2}{2x^{-1}y^3}\right)^{-2}$

Rules of Logarithms

a. $\log_2 8 + \log_2 4$

b. $\log_3 81 - \log_3 3$

c. $\log_2(8^3)$

d. $\log_c a + \log_c b$

e. $\log_d x - \log_d y$

f. $b \cdot \log_t(a)$

2. Rewrite the following as a single logarithm expression and simplify.

a. $\log_2(40) - \log_2(10)$

b. $\log_5(30) + \log_5(2) - \log_5(4)$

c. $\ln(8x) - 2 \cdot \ln(x)$

d. $\log_3(4x) + \log_3(2y^2) - \log_3(z)$

e. $2 \cdot \log_b(3x) + 3 \cdot \log_b(2x) - \log_b(x^2)$

f. $2 \cdot \ln(2x) + 3 \cdot \ln(x^3) - \ln(6)$

3. Expand each of the single logarithm expressions in to multiple logarithms.

a. $\log_5(12x^3)$

b. $\ln\left(\frac{4x^3}{3y^2}\right)$

c. $\log_2\left(\frac{8a^3b}{c}\right)$

d. $\ln\left(\frac{3a^x}{b^2}\right)$

Sec 5.7 – Exponential & Logarithmic Functions
(Solving Exponential Equations)

Name: _____

1. Solve the following **basic** exponential equations by rewriting each side using the same base.

a. $3^{x-1} = 81$

b. $2^{2x-3} = 128$

c. $2^{2x-1} = 4^3$

2. Solve the following **basic** exponential equation by rewriting each as logarithmic equation and approximating the value of x.

a. $4^x = 102$

b. $5^{2x-3} = 1953125$

c. $e^{2x} = 78$

3. Solve the following exponential equation by rewriting each as logarithmic equation and approximating the value of x.

a. $6^{2x} - 8 = 112$

b. $2 \cdot 3^{x-3} + 1 = 39367$

c. $3e^{2x} + 2 = 92$

d. $2 \cdot 4^{2x+1} + 40 = 8$

4. Solve the following exponential inequalities.

a. $3^x - 7 > 20$



c. $2e^{2x} + 1 < 33$



b. $5^{x-3} \geq -40$



d. $5 \cdot 4^{2x+1} - 2 \geq 243$



5. Solve the following applications

- a. Create an equation that represents the value P of an investment t years after the initial investment. The initial investment was \$3200 and increases by 12% each year (compounded annually). This would suggest that the account value could be modeled by $P = 3200(1.12)^t$. Determine how many years it should take for the investment to double in value.

- b. There are 15 virus particles known as a virion in a host and the number of virions doubles every hour and continues this model for the first 28 hours. The number of N virion after t hours can be found using the formula $N = 15 \cdot (2^t)$. How long will it take approximately for there to be 4,500,000 virions living in the host?



- c. There are initially 8 frogs living in a pond in the back of a farm. The number frogs can be described by the formula $P = 8 \cdot e^{(0.2t)}$. How many years will it take for the population of frogs to grow to 100 if the model continues?



Sec 5.8 – Exponential & Logarithmic Functions
(Solving Logarithmic Equations)

Name: _____

1. Solve the following **basic** logarithmic equations.

a. $\log_2(3x + 3) = \log_2(5x - 15)$

b. $\log_5(x^2 - 12) = \log_5(4x)$

2. Solve the following **basic** exponential equation by rewriting each as logarithmic equation and approximating the value of x.

a. $\log_2(x) = 7$

b. $\log_3(4x + 1) = 4$

c. $\ln(x - 3) = 5$

3. Solve the following exponential equation by rewriting each as logarithmic equation and approximating the value of x.

a. $\log_2(3x + 2) + 5 = 4$

b. $\log_6(9x) + \log_6(4x) = 6$

c. $3 \cdot \ln(2x + 1) - 2 = 22$

d. $\ln(12x) - 2 \cdot \ln(2) = 8$

4. Solve the following exponential inequalities.

a. $\log_2(3x - 2) > 4$



b. $\log_3(9x + 9) \leq 4$



c. $2 \cdot \ln(x + 2) < 12$

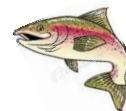


d. $\log_4(4x + 20) \geq 3$



5. Solve the following applications

- a. The population of trout in a lake could be modeled by the equation $P = 250 \cdot \log_2(2t + 2)$ where P is the number of fish and t is the number of years after 2016. If the trend continues, how many years after 2016 will it take for the population to reach 600 trout?



- b. The Richter scale measures the magnitude of an earthquake based on the amount energy determined by the ground motion from a set distance from the epicenter of the quake. The Magnitude is given by $= \frac{2}{3} \log \left(\frac{E}{10^{11.8}} \right)$. If the Magnitude of an earthquake was 7.2, how much energy was released by the earthquake?

