

Rules of E^xp^on^en^ts

$$\begin{array}{c}
 X^3 \\
 \swarrow \downarrow \searrow \\
 X \cdot X \cdot X
 \end{array}
 \cdot
 \begin{array}{c}
 X^2 \\
 \swarrow \searrow \\
 X \cdot X
 \end{array}
 =
 \begin{array}{c}
 X^5 \\
 \swarrow \searrow \\
 X \cdot X
 \end{array}$$

$$\begin{array}{c} (x^3)^2 = \\ \swarrow \quad \searrow \\ (x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x^6 \end{array}$$

$$\frac{x^7}{x^4} = \frac{\cancel{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}}{\cancel{x \cdot x \cdot x \cdot x}} = x^3$$

$$\frac{y^2 x^{-3} z^5}{y^{-4} x^6 z^7} = \frac{\overset{\uparrow}{y^2} \boxed{x^{-3}} z^5}{\boxed{y^{-4}} \downarrow x^6 z^7} = \frac{y^4 y^2 z^5}{x^3 x^6 z^7} =$$

$$\frac{y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z}}{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z} \cdot \cancel{z}}$$

$$\frac{y^6}{x^9 \cdot z^2}$$

$$\sqrt[5]{x^2} = x^{\frac{2}{5}}$$

1. Simplify

a. $(5x^2)(14x^3)$

$$5 \cdot x \cdot x \cdot 14 \cdot x \cdot x \cdot x$$

$$5.14 \cdot x \cdot x \cdot x \cdot x \cdot x$$

70×5

$70x^5$

$$\begin{aligned} \text{b. } \frac{12x^4y^9}{3x^5y^7} &= \frac{\cancel{3}^1 \cancel{20}^{\cancel{20}} \cancel{x}^{\cancel{x}} \cancel{x}^{\cancel{x}} \cancel{x}^{\cancel{x}} \cancel{x}^{\cancel{x}} \cancel{y}^{\cancel{y}} \cancel{y}^{\cancel{y}} \cancel{y}^{\cancel{y}} \cancel{y}^{\cancel{y}} \cancel{y}^{\cancel{y}}}{\cancel{3}^1 \cancel{x}^{\cancel{x}} \cancel{x}^{\cancel{x}} \cancel{x}^{\cancel{x}} \cancel{x}^{\cancel{x}} \cancel{y}^{\cancel{y}} \cancel{y}^{\cancel{y}} \cancel{y}^{\cancel{y}}} \\ &= \frac{4y^2}{x} \end{aligned}$$

$$\frac{4y^2}{x}$$

c. $(3x^3y^2)^2 = (3xxx yy)^2$

$\swarrow \quad \searrow$

$= 3xxx yy \cdot 3xxx yy$

$= 33 xxx xxx yy yy$

$= 9x^6y^4$

$9x^6y^4$

d. $(5x^3y^2)(-6x^6y^4)$

$$5xxx yy \cdot -6xxxxx \cdot yyy$$

-6.5xxx xxxxxx yy yyy

$$-30x^9y^6$$

$$-30x^9y^6$$

e. $(-3m^3n^4)^2(4m^4n^7)$

$-3\text{mm llll} \cdot -3\text{mm llll} \cdot 4\text{mm llllll}$

$-3 \cdot 3 \cdot 4$ mm mm m llll ll ll nnnnnn

$36m^{10}n^{15}$

$$36m^{10}n^{15}$$

$$f. \frac{24a^{-3}b^7c^2}{16a^2b^4c^7} = \frac{24 \cancel{a^{-3}} b^7 c^2}{16a^2 b^4 c^7}$$

$$= \frac{24b^7c^2}{16a^2a^3b^4c^7}$$

$$= \frac{3 \cdot 7 \cdot 2 \cdot 2 \cdot bbbbbb}{2 \cdot 2 \cdot 7 \cdot 2 \cdot aaaaaa \cdot bbbbbb \cdot cccccccc}$$

$$= \frac{3b^3}{2a^5c^5}$$

$$\frac{3b^3}{2a^5c^5}$$

(1 continued) Simplify

g. $\left(\frac{3a^6b^3c^2}{2a^2b^4}\right)^2$

$$= \frac{3 \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{b} \cancel{b} \cancel{b} c c}{2 \cancel{a} \cancel{a} \cancel{b} \cancel{b} \cancel{b} \cancel{b}} \cdot \frac{3 \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{b} \cancel{b} \cancel{b} c c}{2 \cancel{a} \cancel{a} \cancel{b} \cancel{b} \cancel{b} \cancel{b}}$$

$$= \frac{3 \cdot 3 \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{a} \cancel{b} \cancel{b} \cancel{b} c c c c}{2 \cdot 2 \cancel{b} \cancel{b}}$$

$$= \frac{9a^8c^4}{4b^2}$$

$$\boxed{\frac{9a^8c^4}{4b^2}}$$

h. $(3x^2y^4z)(4x^3y^{-2}z^8)$

$$= \frac{3 \cancel{x} \cancel{x} y y \cancel{z}}{1} \cdot \frac{4 \cancel{x} \cancel{x} \cancel{x} z z z z z z z z}{\cancel{z}}$$

$$= 3 \cdot 4 \cancel{x} \cancel{x} \cancel{x} \cancel{x} y y z z z z z z z z$$

$$= 12x^5y^2z^9$$

$$\boxed{12x^5y^2z^9}$$

i. $(5a^3b^{-2}c^5)^2$

$$\frac{5 \cancel{a} \cancel{a} \cancel{a} c c c c c}{b b} \cdot \frac{5 \cancel{a} \cancel{a} \cancel{a} c c c c c}{b b}$$

$$\boxed{\frac{25a^6c^{10}}{b^4}}$$

j. $\left(\frac{4x^2y^{-3}}{y^{-2}}\right)^{-1}$

$$= \frac{y^{-2}}{4x^2y^{-3}} = \frac{y^{-2}}{4x^2y^{-3}}$$

$$= \frac{y^3}{4x^2y^2} = \frac{\cancel{y} y}{4x \cancel{y}}$$

$$\boxed{= \frac{y}{4x^2}}$$

k. $(8x^7y^3)(3x^{-4}y^8)$

$$= \frac{8 \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{y} y y}{1} \cdot \frac{3 \cancel{x} \cancel{x} \cancel{x} \cancel{x} y y y y y y y y}{\cancel{y}}$$

$$= \frac{72y^{11}}{x}$$

$$\boxed{\frac{72y^{11}}{x}}$$

l. $(9a^3b^5)(-4a^3b^7)^2$

$$9 \cancel{a} \cancel{a} \cancel{a} b b b b b \cdot -4 \cancel{a} \cancel{a} \cancel{a} b b b b b b b b b \cdot -4 \cancel{a} \cancel{a} \cancel{a} b b b b b b b b b$$

$$144a^9b^{19}$$

$$\boxed{144a^9b^{19}}$$

m. $\left(\frac{4m^4n^{-3}p^2}{6m^2n^2}\right)^{-2} = \left(\frac{6m^2n^3p^2}{4m^4n^{-3}p^2}\right)^2$

$$\left(\frac{6m^2n^3p^2}{4m^4n^{-3}p^2}\right)^2 = \frac{3 \cdot 2 \cancel{m} \cancel{m} \cancel{n} \cancel{n} \cancel{n} \cancel{n} \cancel{p} \cancel{p}}{2 \cdot 2 \cancel{m} \cancel{m} \cancel{m} \cancel{m} \cancel{p} \cancel{p}} \cdot \frac{3 \cdot 2 \cancel{m} \cancel{m} \cancel{n} \cancel{n} \cancel{n} \cancel{n} \cancel{p} \cancel{p}}{2 \cdot 2 \cancel{m} \cancel{m} \cancel{m} \cancel{m} \cancel{p} \cancel{p}}$$

$$= \frac{9n^{10}}{4m^4p^4}$$

$$\boxed{\frac{9n^{10}}{4m^4p^4}}$$

n. $\left(\frac{3x}{y^{-3}}\right)^3 \left(\frac{5x^{-10}yz^2}{2x^{-1}y^3}\right)^{-2} = \left(\frac{3xy^3}{1}\right)^3 \cdot \left(\frac{2x^{-1}y^3}{5x^{-10}yz^2}\right)^2$

$$\left(\frac{3xy^3}{1}\right)^3 \cdot \left(\frac{2x^{-1}y^3}{5x^{-10}yz^2}\right)^2 =$$

$$\frac{3 \cdot 3 \cdot 3 \cancel{x} \cancel{x} \cancel{x} y y y}{1} \cdot \frac{2 \cdot 2 \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} y y y}{5 \cdot 5 \cancel{z} \cancel{z} \cancel{z} \cancel{z}}$$

$$\frac{108x^{21}y^{13}}{25z^4}$$

$$\boxed{\frac{108x^{21}y^{13}}{25z^4}}$$

Rules of Logarithms

a. $\log_2 8 + \log_2 4 = \log_2 (32) = 5$

$$\underbrace{\log_2(2 \cdot 2 \cdot 2)}_3 + \underbrace{\log_2(2 \cdot 2)}_2 = 5$$

$$\log_2(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) = 5$$

$$\text{d. } \log_c a + \log_c b = \log_c(ab)$$

$$\text{b. } \log_3 81 - \log_3 3 = \log_3 \left(\frac{81}{3} \right) = \log_3 (27) = 3$$

$$\underbrace{\log_3(3 \cdot 3 \cdot 3 \cdot 3)}_4 - \underbrace{\log_3(3)}_1 = 3$$

$$\log_3\left(\frac{3 \cdot 3 \cdot 3}{3}\right) = \log_3(3 \cdot 3 \cdot 3) = 3$$

$$\text{e. } \log_d x - \log_d y = \log_d \left(\frac{x}{y} \right)$$

c. $\log_2(8^3)$

$$= \log_2(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$$
$$= 9$$

$$3 \cdot \log_2(8) = 3 \cdot \underbrace{\log_2(2 \cdot 2 \cdot 2)}_{3 \cdot 1} = 9$$

f. $b \cdot \log_t(a)$

$$\log_t(a^b) = b \cdot \log_t(a)$$

2. Rewrite the following as a single logarithm expression and simplify.

a. $\log_2(40) - \log_2(10)$

$$\begin{aligned} &= \log_2 \left(\frac{40}{10} \right) = \log_2 (4) \\ &= \log_2 (2^2) \\ &= 2 \end{aligned}$$

2

b. $\log_5(30) + \log_5(2) - \log_5(4)$

$$\log(15)/\log(5)$$

1.682606194

$$\log_5(30 \cdot 2) - \log_5(4)$$

$$\log_5(\infty) - \log_5(4)$$

$$\log_5\left(\frac{60}{4}\right) = \log_5(15) \approx 1.683$$

$$\log_5(15) \approx 1.683$$

c. $\ln(8x) - 2 \cdot \ln(x)$

$$\ln(8x) - \ln(x^2)$$

$$= \ln \left(\frac{e^x}{x^2} \right)$$

$$= \ln\left(\frac{\infty}{x}\right)$$

$$h\left(\frac{8}{x}\right)$$

d. $\log_3(4x) + \log_3(2y^2) - \log_3(z)$

$$\log_3(4x \cdot 2y^2) - \log_3(z)$$

$$\log_3 (8x y^2) - \log_3 (z)$$

$$\log_3 \left(\frac{8xy^2}{z} \right)$$

$$\log_3 \left(\frac{8xy^2}{z} \right)$$

e. $2 \cdot \log_b(3x) + 3 \log_b(2x) - \log_b(x^2)$

$$\log_b(3x^2) + \log_b(2x^3) - \log_b(x^2)$$

$$\log_b(9x^2) + \log_b(8x^3) - \log_b(x^2)$$

$$\log_b(72x^5) - \log_b(x^2)$$

$$\log_b \left(\frac{72x^5}{x^2} \right) = \log_b (72x^3)$$

$$\log_6 (72 \times 3)$$

f. $2 \cdot \ln(2x) + 3 \cdot \ln(x^3) - \ln(6)$

$$\ln(2x^2) + \ln(x^3) - \ln(6)$$

$$\ln(4x^2) + \ln(x^9) - \ln(6)$$

$$\ln(4x'') - \ln(6)$$

$$\ln\left(\frac{4x''}{6}\right) = \ln\left(\frac{2x''}{3}\right)$$

$$\ln\left(\frac{2x''}{3}\right)$$

3. Expand each of the single logarithm expressions in to multiple logarithms.

a. $\log_5(12x^3)$

$\log_5(12 \cdot x^3) \longrightarrow \text{USE ADD PROP}$

$\log_5(12) + \log_5(x^3) \longrightarrow \text{USE MULT PROP}$

$\log_5(12) + 3 \cdot \log_5(x)$

$$\log_5(12) + 3 \log_5(x)$$

b. $\ln\left(\frac{4x^3}{3y^2}\right)$

$\ln\left(\frac{4x^3}{3y^2}\right) \longrightarrow \text{USE SUBTRACT PROP}$

$\ln(4x^3) - \ln(3y^2) \longrightarrow \text{USE ADD PROP}$

$[\ln(4) + \ln(x^3)] - [\ln(3) + \ln(y^2)] \longrightarrow \text{USE MULT PROP}$

$[\ln(4) + 3 \cdot \ln(x)] - [\ln(3) + 2 \ln(y)]$

$$\ln(4) + 3 \ln(x) - \ln(3) - 2 \ln(y)$$

c. $\log_2\left(\frac{8a^3b}{c}\right)$

$\log_2\left(\frac{8a^3b}{c}\right) \longrightarrow \text{USE SUB PROP}$

$\log_2(8a^3b) - \log_2(c) \longrightarrow \text{USE ADD PROP}$

$\log_2(8) + \log_2(a^3) + \log_2(b) - \log_2(c) \longrightarrow \text{USE MULT PROP}$

$\log_2(2^3) + 3 \log_2(a) + \log_2(b) - \log_2(c)$

$$3 + 3 \log_2(a) + \log_2(b) - \log_2(c)$$

d. $\ln\left(\frac{3a^x}{b^2}\right)$

$\ln\left(\frac{3a^x}{b^2}\right) \longrightarrow \text{USE SUB. PROP}$

$\ln(3a^x) - \ln(b^2) \longrightarrow \text{USE ADD PROP}$

$\ln(3) + \ln(a^x) - \ln(b^2) \longrightarrow \text{USE MULT. PROP}$

$\ln(3) + x \ln(a) - 2 \ln(b)$

$$\ln(3) + x \ln(a) - 2 \ln(b)$$