

Sec 5.8 - Exponential & Logarithmic Functions
(Solving Logarithmic Equations)

Name: _____

1. Solve the following **basic** logarithmic equations.

a. $\log_2(3x + 3) = \log_2(5x - 15)$

$$\begin{array}{r} \cancel{3x} + 3 = \cancel{x-5} \\ -3x \quad -3x \\ \hline 3 = 2x - 15 \\ +15 \quad +15 \\ \hline 18 = 2x \\ \frac{18}{2} = \frac{2x}{2} \\ 9 = x \end{array}$$

CHECKING
 $\log_2(3(9) + 3) = \log_2(5(9) - 15)$
 $\log_2(30) = \log_2(30) \checkmark$

b. $\log_5(x^2 - 12) = \log_5(4x)$

$$\begin{array}{r} x^2 - 12 = 4x \\ -4x \quad -4x \\ \hline x^2 - 4x - 12 = 0 \\ (x+2)(x-6) = 0 \\ \downarrow \quad \downarrow \\ x+2=0 \quad x-6=0 \\ \frac{x}{-2} = \frac{0}{-2} \quad \frac{x}{+6} = \frac{0}{+6} \\ x = -2 \quad \text{or} \quad x = 6 \\ \text{EXTRANEUS} \end{array}$$

CHECKING ($x = -2$)
 $\log_5((-2)^2 - 12) = \log_5(4(-2))$
 $\log_5(-8) = \log_5(-8)$
UNDEFINED

CHECK ($x = 6$)
 $\log_5((6)^2 - 12) = \log_5(4(6))$
 $\log_5(24) = \log_5(24) \checkmark$

$x = 6$

2. Solve the following **basic** exponential equation by rewriting each as logarithmic equation and approximating the value of x.

a. $\log_2(x) = 7$

$$\begin{array}{r} \cancel{2}^{\log_2(x)} = 2^7 \\ x = 2^7 \\ x = 128 \end{array}$$

CHECKING
 $\log_2(128) = 7 \checkmark$

b. $\log_3(4x + 1) = 4$

$$\begin{array}{r} \cancel{3}^{\log_3(4x+1)} = 3^4 \\ 4x + 1 = 81 \\ -1 \quad -1 \\ \hline 4x = 80 \\ \frac{4x}{4} = \frac{80}{4} \\ x = 20 \end{array}$$

CHECK
 $\log_3(4(20) + 1) = 4$
 $\log_3(81) = 4 \checkmark$

c. $\ln(x - 3) = 5$

$$\begin{array}{r} \log_e(x-3) = 5 \\ \cancel{e}^{\log_e(x-3)} = e^5 \\ x - 3 = e^5 \\ +3 \quad +3 \\ \hline x = e^5 + 3 \\ x \approx 151.41 \end{array}$$

CHECKING
 $\ln(e^5 + 3 - 3) = 5$
 $\ln(e^5) = 5$
 $5 = 5 \checkmark$

3. Solve the following exponential equation by rewriting each as logarithmic equation and approximating the value of x.

a. $\log_2(3x + 2) + 5 = 4$

$$\begin{array}{r} \log_2(3x+2) = -1 \\ \cancel{2}^{\log_2(3x+2)} = 2^{-1} \\ 3x + 2 = 0.5 \\ -2 \quad -2 \\ \hline 3x = -1.5 \\ \frac{3x}{3} = \frac{-1.5}{3} \\ x = -0.5 \end{array}$$

CHECKING
 $\log_2(3(-0.5) + 2) + 5 = 4$
 $\log_2(-1.5 + 2) + 5 = 4$
 $\log_2(0.5) + 5 = 4$
 $-1 + 5 = 4 \checkmark$

b. $\log_6(9x) + \log_6(4x) = 6$

$$\begin{array}{r} \log_6(9x \cdot 4x) = 6 \\ \log_6(36x^2) = 6 \\ \cancel{6}^{\log_6(36x^2)} = 6^6 \\ 36x^2 = 46656 \\ \frac{36x^2}{36} = \frac{46656}{36} \end{array}$$

$x^2 = 1296$
 $\sqrt{x^2} = \pm \sqrt{1296}$
 $x = 36$ or $x = -36$
CHECK $x = 36$
 $\log_6(9(36)) + \log_6(4(36)) = 6$
 $\log_6(324) + \log_6(144) = 6$
CHECK $x = -36$
 $\log_6(-324) + \log_6(-144) = 6$
UNDEFINED

c. $3 \cdot \ln(2x + 1) - 2 = 22$

$$\begin{array}{r} 3 \cdot \ln(2x+1) = 24 \\ \frac{3 \cdot \ln(2x+1)}{3} = \frac{24}{3} \\ \ln(2x+1) = 8 \\ \cancel{e}^{\ln(2x+1)} = e^8 \\ 2x + 1 = e^8 \\ -1 \quad -1 \\ \hline 2x = e^8 - 1 \\ \frac{2x}{2} = \frac{e^8 - 1}{2} \\ x = \frac{e^8 - 1}{2} \end{array}$$

CHECKING
 $3 \cdot \ln(2(\frac{e^8 - 1}{2}) + 1) - 2 = 22$
 $3 \ln(e^8 - 1 + 1) - 2 = 22$
 $3 \ln(e^8) - 2 = 22$
 $3 \cdot 8 - 2 = 22$
 $24 - 2 = 22 \checkmark$

d. $\ln(12x) - 2 \cdot \ln(2) = 8$

$$\begin{array}{r} \ln(12x) - \ln(2^2) = 8 \\ \ln(\frac{12x}{4}) = 8 \\ \ln(3x) = 8 \\ \cancel{e}^{\ln(3x)} = e^8 \\ 3x = e^8 \\ \frac{3x}{3} = \frac{e^8}{3} \\ x = \frac{e^8}{3} \end{array}$$

CHECKING
 $\ln(12(\frac{e^8}{3})) - 2 \ln(2) = 8$
 $\ln(4e^8) - 2 \ln(2) = 8$
 $8 = 8 \checkmark$
 $\ln(4e^8) - 2 \ln(2)$

$x \approx 1489.98$

4. Solve the following exponential inequalities.

a. $\log_2(3x - 2) > 4$

AND $3x - 2 > 0$

$$\log_2(3x - 2) > 4$$

$$3x - 2 > 16$$

$$\frac{3x}{3} > \frac{18}{3}$$

$$x > 6$$

$$\frac{3x}{3} > \frac{2}{3}$$

$$x > \frac{2}{3}$$

SET: $x > 6$ INT: $(6, \infty)$



b. $\log_3(9x + 9) \leq 4$

AND $9x + 9 > 0$

$$\log_3(9x + 9) \leq 4$$

$$9x + 9 \leq 81$$

$$\frac{9x}{9} \leq \frac{72}{9}$$

$$x \leq 8$$

$$\frac{9x + 9}{9} > \frac{-9}{9}$$

$$x > -1$$

SET: $-1 < x \leq 8$ INT: $(-1, 8]$



c. $2 \cdot \ln(x + 2) < \frac{12}{2}$

AND $x + 2 > 0$

$$2 \cdot \ln(x + 2) < 6$$

$$\ln(x + 2) < 3$$

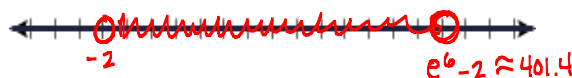
$$x + 2 < e^3$$

$$x < e^3 - 2$$

$$x < 401.4$$

$$x > -2$$

SET: $-2 < x < e^3 - 2$ INT: $(-2, e^3 - 2)$



d. $\log_4(4x + 20) \geq 3$

AND $4x + 20 > 0$

$$\log_4(4x + 20) \geq 3$$

$$4x + 20 \geq 64$$

$$\frac{4x}{4} \geq \frac{44}{4}$$

$$x \geq 11$$

$$x > -5$$

SET: $x \geq 11$ INT: $[11, \infty)$



5. Solve the following applications

- a. The population of trout in a lake could be modeled by the equation $P = 250 \cdot \log_2(2t + 2)$ where P is the number of fish and t is the number of years after 2016. If the trend continues, how many years after 2016 will it take for the population to reach 600 trout?

$$\frac{600}{250} = \frac{250 \cdot \log_2(2t + 2)}{250}$$

$$2.4 = \log_2(2t + 2)$$

$$2^{2.4} = 2^{\log_2(2t + 2)}$$

$$5.278 = 2t + 2$$

$$3.278 = \frac{2t}{2}$$

$$1.639 = t$$

t = 1.639 years
OR
2017 JUL 4



- b. The Richter scale measures the magnitude of an earthquake based on the amount energy determined by the ground motion from a set distance from the epicenter of the quake. The Magnitude is given by $M = \frac{2}{3} \log\left(\frac{E}{10^{11.8}}\right)$. If the Magnitude of an earthquake was 7.2, how much energy was released by the earthquake?

$$\frac{2}{3} \cdot 7.2 = \frac{2}{3} \log\left(\frac{E}{10^{11.8}}\right)$$

$$10.8 = \log\left(\frac{E}{10^{11.8}}\right)$$

$$10^{10.8} = 10^{\log\left(\frac{E}{10^{11.8}}\right)}$$

$$6.31 \times 10^{10} = \frac{E}{10^{11.8}}$$

$$3.981 \times 10^{22} = E$$

