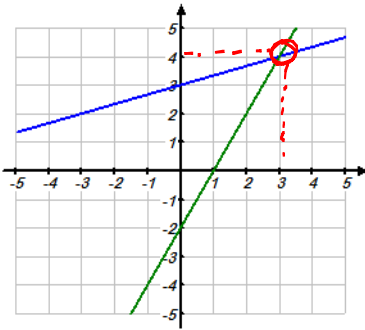


Each system of equation is shown in graph. Using the graph find the solutions to each of the systems.

1. $y = \frac{1}{3}x + 3$

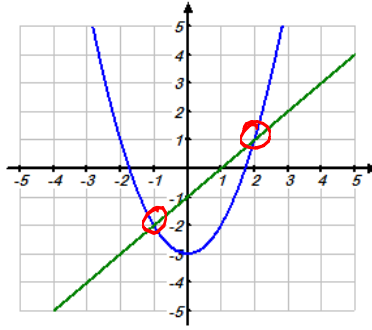
$y = 2x - 2$



$(3, 4)$

2. $y = x^2 - 3$

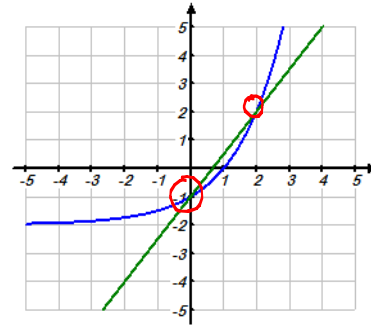
$y = x - 1$



$(-1, -2)$ or $(2, 1)$

3. $y = 2^x - 2$

$y = \frac{3}{2}x - 1$



$(0, -1)$ or $(2, 2)$

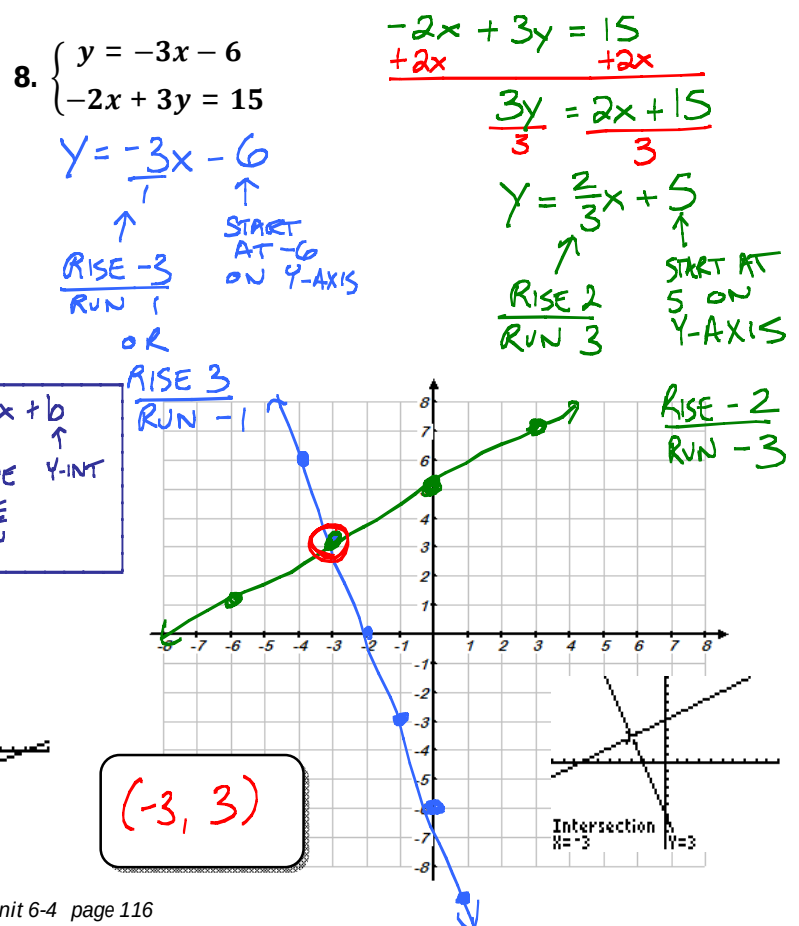
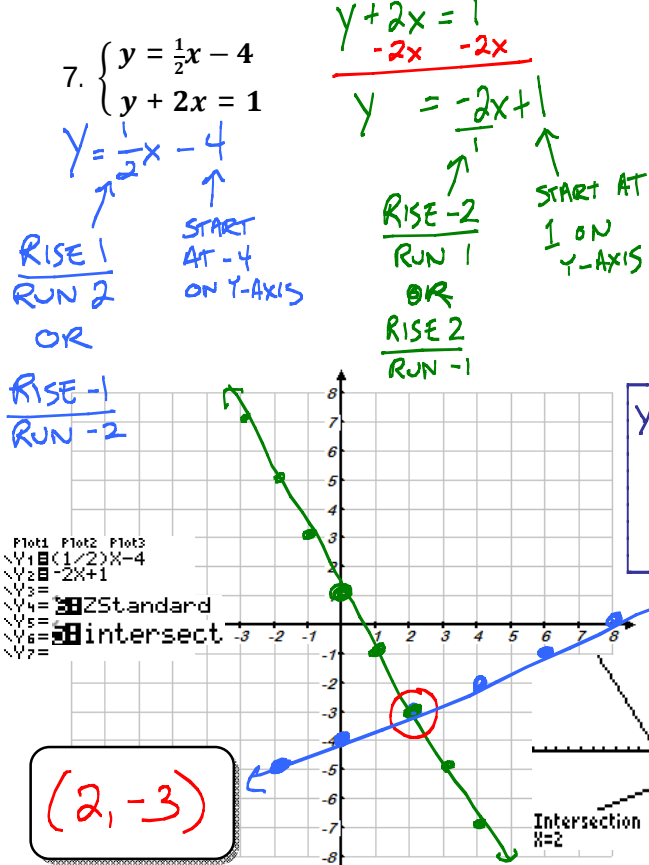
Which of the system of equations below have a solution of $(-3, 2)$?

4. $\begin{cases} y = 2x + 8 \\ 3x + 2y = -5 \end{cases}$
 $(2) = 2(-3) + 8$
 $2 = -6 + 8$
 $2 = 2$ ✓
 $3(-3) + 2(2) = -5$
 $-9 + 4 = -5$
 $-5 = -5$ ✓
YES
 $(-3, 2)$

5. $\begin{cases} y = \frac{2}{3}x + 4 \\ x = \frac{1}{2}y - 2 \end{cases}$
 $(2) = \frac{2}{3}(-3) + 4$
 $2 = -2 + 4$
 $2 = 2$ ✓
 $(-3) = \frac{1}{2}(2) - 2$
 $-3 = 1 - 2$
 $-3 = -1$ ✗
NO, A NOT A SOLUTION

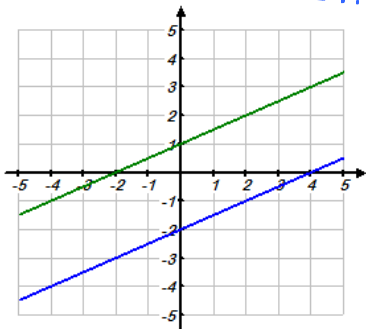
6. $\begin{cases} y + 2x = -4 \\ 3y + x = 6 \end{cases}$
 $(2) + 2(-3) = -4$
 $2 - 6 = -4$
 $-4 = -4$ ✓
 $3^2 + (-3) = 6$
 $9 - 3 = 6$
 $6 = 6$ ✓
YES
 $(-3, 2)$

Graph each system and use the graph to determine a solution.



Each system of equation is shown in graph. How many solutions does each system have?

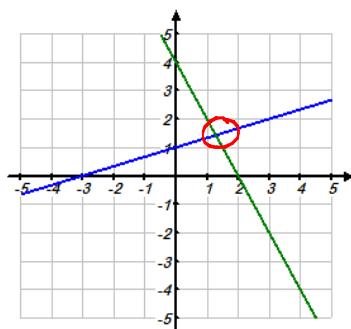
9. $y = \frac{1}{2}x - 2$ SAME SLOPE
 $y = \frac{1}{2}x + 1$ DIFFERENT Y-INTERCEPT



No SOLUTIONS
 (INCONSISTENT)

EMPTY SET

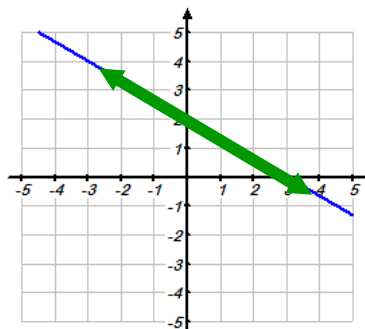
10. $y = \frac{1}{3}x + 1$ DIFFERENT SLOPE
 $y = -2x + 4$



ONE SOLUTION
 (CONSISTENT INDEPENDENT)

(1.5, 1.4)

11. $y = -\frac{2}{3}x + 2$ SAME SLOPE
 $y = -\frac{2}{3}x + 2$ SAME Y-INTERCEPT



INFINITE SOLUTIONS
 (CONSISTENT DEPENDENT)

$\{y = -\frac{2}{3}x + 2\}$

Graph each system and use the graph to determine a solution.

9. $3y = 2x - 6$

$4x - 6y = 12$

$\frac{3y}{3} = \frac{2x-6}{3}$

$y = \frac{2}{3}x - 2$

RISE 2
 RUN 3

START AT
 -2 ON
 Y-AXIS

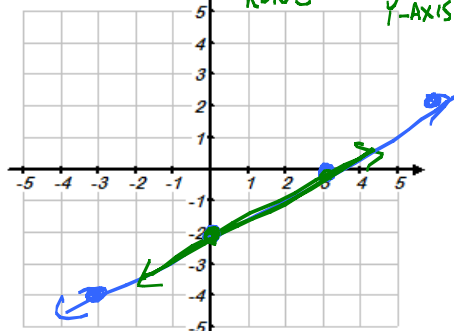
$4x - 6y = 12$
 $-4x$

$-6y = -4x + 12$
 -6

$y = \frac{2}{3}x - 2$

RISE 2
 RUN 3

START AT
 -2 ON
 Y-AXIS



INFINITE SOLUTIONS

$\{y = \frac{2}{3}x - 2\}$

$3y = \frac{2}{3}x - 6$

$3y = 2x - 6$

$-2x$

$-2x + 3y = -6$

10. $4y - 7 = 2x + 1$

$2y - x = -6$

$\frac{4y-7}{4} = \frac{2x+1}{4}$

$y = \frac{1}{2}x + 2$

RISE 1
 RUN 2

START AT
 2 ON
 Y-AXIS

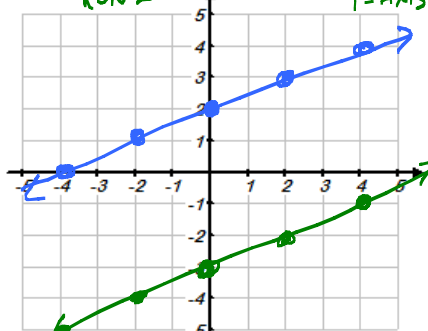
$2y - x = -6$
 $+x$

$2y = 1x - 6$
 2

$y = \frac{1}{2}x - 3$

RISE 1
 RUN 2

START AT
 -3 ON
 Y-AXIS



NO SOLUTIONS

EMPTY SETS

EMPTY SETS

11. $2x = y + 2$

$3y = -2x + 9$

$\frac{2x}{-2} = \frac{y+2}{-2}$

$2x - 2 = y$

$y = 2x - 2$

RISE 2
 RUN 1

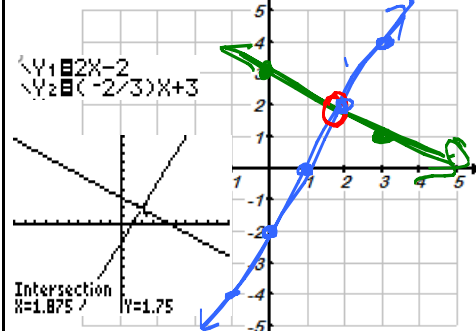
START AT
 -2 ON
 Y-AXIS

$\frac{3y}{3} = \frac{-2x+9}{3}$

$y = -\frac{2}{3}x + 3$

RISE -2
 RUN 3

START AT
 3 ON
 Y-AXIS



ONE SOLUTION

$(1.875, 1.75)$

STANDARD FORM $Ax + By = C$

Solve each of the following systems using the substitution method.

15. $y = 2x + 5$
 $3x + 2y = -4$

$$3x + 2(2x + 5) = -4$$

$$3x + 4x + 10 = -4$$

$$7x + 10 = -4$$

$$\begin{array}{r} 7x + 10 = -4 \\ -10 \quad -10 \\ \hline 7x = -14 \\ \frac{7x}{7} = \frac{-14}{7} \\ x = -2 \end{array}$$

TRANSLATION EQUATION: $y = 2(-2) + 5$
 $y = -4 + 5$
 $y = 1$

$(-2, 1)$

16. $4x + 2y = 4$
 $x - 3y = 15$

$$\begin{array}{r} x - 3y = 15 \\ +3y \quad +3y \\ \hline x = 3y + 15 \end{array}$$

$$4(3y + 15) + 2y = 4$$

$$12y + 60 + 2y = 4$$

$$14y + 60 = 4$$

$$\begin{array}{r} 14y + 60 = 4 \\ -60 \quad -60 \\ \hline 14y = -56 \\ \frac{14y}{14} = \frac{-56}{14} \\ y = -4 \end{array}$$

TRANSLATION EQUATION

$x = 3y + 15$

$x = 3(-4) + 15$

$x = -12 + 15$

$x = 3$

$(3, -4)$

Solve each of the following systems using the elimination method.

17. $2x - 3y = 9$
 $3x + 4y = 5$

$$\begin{array}{r} 2x - 3y = 9 \\ -2 \cdot (3x + 4y = 5) \\ \hline 2x - 3y = 9 \\ -6x - 8y = -10 \\ \hline -17y = 17 \\ \frac{-17y}{-17} = \frac{17}{-17} \\ y = -1 \end{array}$$

$$\begin{array}{r} 2x - 3y = 9 \\ + \cdot 4 \cdot (3x + 4y = 5) \\ \hline 2x - 3y = 9 \\ 9x + 12y = 15 \\ \hline 17x = 51 \\ \frac{17x}{17} = \frac{51}{17} \\ x = 3 \end{array}$$

$(3, -1)$

18. $4x - 2y = 6$
 $-6x + 3y = -9$

$$\begin{array}{r} 4x - 2y = 6 \\ 2 \cdot (-6x + 3y = -9) \\ \hline 4x - 2y = 6 \\ -12x + 6y = -18 \\ \hline 0 = 0 \end{array}$$

$\{(x, y) \mid 4x - 2y = 6\}$
 $\{(x, y) \mid 2x - y = 3\}$

WHEN SOLVING A SYSTEM ALGEBRAICALLY AND ALL THE VARIABLES ARE ELIMINATED AND WE ARE LEFT WITH A TRUE STATEMENT, THIS SUGGESTS A CONSISTENT DEPENDENT SYSTEM WITH INFINITE SOLUTIONS.

IF INSTEAD WE ARE LEFT WITH A FALSE STATEMENT THEN THE SYSTEM IS INCONSISTENT AND HAS NO SOLUTIONS OR EMPTY SET.

19. The Turbo Taxi Service charges a flat rate of \$5 and then \$0.40 per mile. The Express Taxi Company charges a flat rate of \$2 and then \$0.75 per mile.

- a. Write an equation that describes the cost, c , of each taxi cab in terms of miles, m , driven.

Turbo Taxi Service:

$$c = 5 + 0.40m$$

Express Taxi Co:

$$c = 2 + 0.75m$$



$$3 \div 0.35 = 8.571428571$$

- b. When do the two taxi cabs charge the same amount?

$$\begin{array}{r} 2 + 0.75m = 5 + 0.40m \\ -0.40m \quad -0.40m \\ \hline 2 + 0.35m = 5 \\ -2 \quad -2 \\ \hline 0.35m = 3 \end{array}$$

$$\frac{0.35m}{0.35} = \frac{3}{0.35}$$

$$m \approx 8.57 \text{ MILES}$$

- c. Describe when the Express Taxi Company charges more than Turbo Taxi Services.

IF THE TAXI DRIVES YOU MORE THAN 8.57 MILES THEN THE EXPRESS TAXI COMPANY CHARGES MORE.

20. A local school sold 230 tickets for their performance of Hamlet. They sold a combination of regular tickets and student tickets. The regular tickets sold for \$8 each and the student tickets sold for \$5. That night they collected \$1522 in ticket sales.

- a. Write an equation that describes each piece of information. Let ' R ' represent the number of regular tickets and ' S ' represent the number of Student tickets.

$$\begin{aligned} R + S &= 230 \\ 8R + 5S &= 1522 \end{aligned}$$



- b. Using the system of equations determine the number of each type of ticket that was sold.

$$\begin{array}{r} 8 \cdot \\ -1 \cdot \\ \hline R + S = 230 \\ 8R + 5S = 1522 \end{array}$$

$$\begin{array}{r} R + S = 230 \\ R + 106 = 230 \\ -106 \quad -106 \\ \hline R = 124 \end{array}$$

$$\begin{array}{r} 8R + 8S = 1840 \\ -8R - 5S = -1522 \\ \hline 3S = 318 \\ \hline S = 106 \end{array}$$

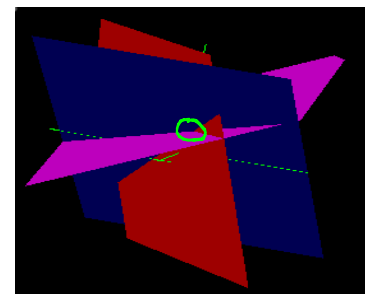
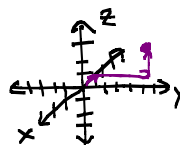
STUDENT TICKETS = 106
REGULAR TICKETS = 124

Solve each of the following systems using the substitution method.

21. $y = 2z$

$$3x + 2y - z = 3$$

$$2x + y + 4z = 10$$



EACH 3 VARIABLE LINEAR EQUATION DESCRIBES A "PLANE" IN A 3-DIMENSIONAL COORDINATE SYSTEM. THE 3 PLANES ALL INTERSECT AT ONE COMMON POINT.

$$3x + 3z = 3$$

$$2x + 6z = 10$$

$$\begin{array}{r} -6z \quad -6z \\ \hline \end{array}$$

$$\frac{2x}{2} = \frac{10-6z}{2}$$

$$x = 5 - 3z$$

$$3(5-3z) + 8z = 3$$

$$15 - 9z + 8z = 3$$

$$15 - z = 3$$

$$\begin{array}{r} -15 \quad -15 \\ \hline \end{array}$$

$$\frac{-z}{-1} = \frac{-12}{-1}$$

$$z = 2$$

$$x = 5 - 3z$$

$$x = 5 - 3(2)$$

$$x = 5 - 6 = -1$$

$$x = -1$$

$$y = 2z$$

$$y = 2(2)$$

$$y = 4$$

$$(-1, 4, 2)$$

Solve each of the following systems using the elimination method.

CHOOSE A VARIABLE TO BE ELIMINATED. FOR THIS EXAMPLE LET'S ELIMINATE "Y"

22.

$$4 \cdot [3x + 2y + z = 9]$$

$$-2 \cdot [5x + 4y - 3z = -5]$$

$$-2x + 3y + 5z = 8$$

$$\begin{array}{r} 12x + 8y + 4z = 36 \\ (-) -10x - 8y + 6z = 10 \\ \hline \end{array}$$

$$2x + 10z = 46$$

$$\begin{array}{r} 15x + 12y - 9z = -15 \\ (+) 8x - 12y - 20z = -32 \\ \hline \end{array}$$

$$23x - 29z = -47$$

$$23. \quad 2x + 10z = 46$$

$$-2 \cdot 23x - 29z = -47$$

$$\begin{array}{r} 46x + 230z = 1058 \\ (+) -46x + 58z = 94 \\ \hline \end{array}$$

$$\frac{288z}{288} = \frac{1152}{288}$$

$$z = 4$$

$$\begin{array}{r} 58x + 290z = 1334 \\ (+) 230x - 290z = -470 \\ \hline \end{array}$$

$$\frac{288x}{288} = \frac{864}{288}$$

$$x = 3$$

$$3x + 2y + z = 9$$

$$3(3) + 2y + (4) = 9$$

$$9 + 2y + 4 = 9$$

$$2y + 13 = 9$$

$$\begin{array}{r} 2y + 13 = 9 \\ -13 \quad -13 \\ \hline \end{array}$$

$$\frac{2y}{2} = \frac{-4}{2}$$

$$y = -2$$

$$(3, -2, 4)$$