# Rules of Exponents

$x^3 \cdot x^2 =$

$x \cdot x \cdot x \cdot x \cdot x = x^5$

$(x^3)^2 =$

$(x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x^6$

$$\frac{x^7}{x^4} = \frac{x\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot x\cdot x}{\cancel{x}\cdot \cancel{x}\cdot \cancel{x}\cdot \cancel{x}} = x^3$$

$$\frac{y^2x^{-3}z^5}{y^{-4}x^6z^7} = \frac{y^2\boxed{x^{-3}}z^5}{\boxed{y^{-4}}x^6z^7} = \frac{y^4y^2z^5}{x^3x^6z^7} =$$

$$\frac{y\cdot y\cdot y\cdot y\ \cdot y\cdot y\ \cdot \cancel{z}\cdot \cancel{z}\cdot \cancel{z}\cdot \cancel{z}\cdot \cancel{z}}{x\cdot x\cdot x\ \cdot x\cdot x\cdot x\cdot x\cdot x\cdot x\ \cdot \cancel{z}\cdot \cancel{z}\cdot \cancel{z}\cdot \cancel{z}\cdot \cancel{z}\cdot z\cdot z}$$

$$\frac{y^6}{x^9\cdot z^2}$$

$$\sqrt[5]{x^2} = x^{\frac{2}{5}}$$

1. Simplify

a. $(5x^2)(14x^3)$

$5\cdot x\cdot x \cdot 14\cdot x\cdot x\cdot x$

$\underbrace{5\cdot 14}\cdot x\cdot x\cdot x\cdot x\cdot x$

$70x^5$

$\boxed{70x^5}$

b. $\dfrac{12x^4y^9}{3x^5y^7}$

$= \dfrac{\cancel{3}\cdot 2\cdot 2\ xxxx\ yyyyyyyyy}{\cancel{3}\ xxxxx\ yyyyyyy}$

12 → 6 2 → 3 2 2

$= \dfrac{4y^2}{x}$

$\boxed{\dfrac{4y^2}{x}}$

c. $(3x^3y^2)^2 = (3xxxyy)^2$

$= 3xxxyy \cdot 3xxxyy$

$= 3\cdot 3\ xxx\ xxx\ yy\ yy$

$= 9x^6y^4$

$\boxed{9x^6y^4}$

d. $(5x^3y^2)(-6x^6y^4)$

$5xxxyy \cdot -6\ xxxxxx \cdot yyyy$

$-6\cdot 5\ xxx\ xxxxxx\ yy\ yyyy$

$-30x^9y^6$

$\boxed{-30x^9y^6}$

e. $(-3m^3n^4)^2(4m^4n^7)$

$-3mmm\ nnnn \cdot -3mmm\ nnnn \cdot 4mmmm\ nnnnnnn$

$-3\cdot -3\cdot 4\ mmm\ mmm\ mmmm\ nnnn\ nnnn\ nnnnnnn$

$36m^{10}n^{15}$

$\boxed{36m^{10}n^{15}}$

f. $\dfrac{24a^{-3}b^7c^2}{16a^2b^4c^7} = \dfrac{24a^{-3}b^7c^2}{16a^2b^4c^7}$

$= \dfrac{24b^7c^2}{16a^2a^3b^4c^7}$

24 → 6 4 → 3 2 2 2

16 → 4 4 → 2 2 2 2

$= \dfrac{3\cdot \cancel{2}\cdot \cancel{2}\cdot 2\ bbbbbbb\ cc}{\cancel{2}\cdot \cancel{2}\cdot 2\cdot 2\ aaaaa\ bbbb\ ccccccc}$

$= \dfrac{3b^3}{2a^5c^5}$

$\boxed{\dfrac{3b^3}{2a^5c^5}}$

(1 continued) Simplify

g. $\left(\frac{3a^6b^3c^2}{2a^2b^4}\right)^2$

$= \frac{3aaaaaabbbcc}{2aabbbb} \cdot \frac{3aaaaaabbbcc}{2aabbbb}$

$= \frac{3 \cdot 3\, aaaa\, aaaa\, cccc}{2 \cdot 2\, b\, b}$

$= \frac{9a^8c^4}{4b^2}$

$\boxed{\frac{9a^8c^4}{4b^2}}$

h. $(3x^2y^4z)(4x^3y^{-2}z^8)$

$= \frac{3xxyyyyz}{1} \cdot \frac{4xxx\,zzzzzzzz}{yy}$

$= 3 \cdot 4\, xxxxx\, yy\, zzzzzzzzz$

$= 12x^5y^2z^9$

$\boxed{12x^5y^2z^9}$

i. $(5a^3b^{-2}c^5)^2$

$\frac{5aaa\,ccccc}{bb} \cdot \frac{5aaa\,ccccc}{bb}$

$\boxed{\frac{25a^6c^{10}}{b^4}}$

j. $\left(\frac{4x^2y^{-3}}{y^{-2}}\right)^{-1}$

$= \frac{y^{-2}}{4x^2y^{-3}} = \frac{y^{-2}}{4x^2y^{-3}}$

$= \frac{y^3}{4x^2y^2} = \frac{yyy}{4xx\,yy}$

$\boxed{= \frac{y}{4x^2}}$

k. $(8x^7y^3)(3x^{-4}y^8)^2$

$= \frac{8xxxxxxxyyy}{1} \cdot \frac{3yyyyyyyy}{xxxx} \cdot \frac{3yyyyyyyy}{xxxx}$

$= \frac{72y^{19}}{x}$

$\boxed{\frac{72y^{19}}{x}}$

l. $(9a^3b^5)(-4a^3b^7)^2$

$9aaa\,bbbbb \cdot -4aaabbbbbbb \cdot -4aaabbbbbbb$

$144a^9b^{19}$

$\boxed{144a^9b^{19}}$

m. $\left(\frac{4m^4n^{-3}p^2}{6m^2n^2}\right)^{-2} = \left(\frac{6m^2n^2}{4m^4n^{-3}p^2}\right)^2$

$\left(\frac{6m^2n^3n^2}{4m^4p^2}\right)^2 = \frac{3 \cdot 2\, mm\, nnnnn}{2 \cdot 2\, mmmm\, pp} \cdot \frac{3 \cdot 2\, mm\, nnnnn}{2 \cdot 2\, mmmm\, pp}$

$= \frac{9n^{10}}{4m^4p^4}$

$\boxed{\frac{9n^{10}}{4m^4p^4}}$

n. $\left(\frac{3x}{y^{-3}}\right)^3\left(\frac{5x^{-10}yz^2}{2x^{-1}y^3}\right)^{-2} = \left(\frac{3xy^3}{1}\right)^3 \cdot \left(\frac{2x^{-1}y^3}{5x^{-10}yz^2}\right)^2$

$\left(\frac{3xy^3}{1}\right)^3 \cdot \left(\frac{2x^{10}y^3}{5x^1yz^2}\right)^2 =$

$\frac{3xyyy}{1} \cdot \frac{3xyyy}{1} \cdot \frac{3xyyy}{1} \cdot \frac{2xxxxxxxxxxyyy}{5xyzz} \cdot \frac{2xxxxxxxxxxyyy}{5xyzz}$

$\frac{108x^{21}y^{13}}{25z^4}$

$\boxed{\frac{108x^{21}y^{13}}{25z^4}}$

## Rules of Logarithms

a. $\log_2 8 + \log_2 4 = \log_2(32) = 5$

$\underbrace{\log_2(2\cdot2\cdot2)}_{3} + \underbrace{\log_2(2\cdot2)}_{2} = 5$

$\log_2(2\cdot2\cdot2\cdot2\cdot2) = 5$

b. $\log_3 81 - \log_3 3 = \log_3\left(\frac{81}{3}\right) = \log_3(27) = 3$

$\underbrace{\log_3(3\cdot3\cdot3\cdot3)}_{4} - \underbrace{\log_3(3)}_{1} = 3$

$\log_3\left(\frac{3\cdot3\cdot3\cdot\cancel{3}}{\cancel{3}}\right) = \log_3(3\cdot3\cdot3) = 3$

c. $\log_2(8^3)$

$= \log_2(2\cdot2\cdot2 \cdot 2\cdot2\cdot2 \cdot 2\cdot2\cdot2)$

$= 9$

$3\cdot\log_2(8) = 3\cdot\underbrace{\log_2(2\cdot2\cdot2)}_{3} = 3 \cdot 3 = 9$

d. $\log_c a + \log_c b = \log_c(ab)$

e. $\log_d x - \log_d y = \log_d\left(\frac{x}{y}\right)$

f. $b \cdot \log_t(a)$

$\log_t(a^b) = b\cdot\log_t(a)$

2. Rewrite the following as a single logarithm expression and simplify.

a. $\log_2(40) - \log_2(10)$

$= \log_2\left(\frac{40}{10}\right) = \log_2(4)$

$= \cancel{\log_2}(\cancel{2}^2)$

$= 2$

$\boxed{2}$

b. $\underbrace{\log_5(30) + \log_5(2)} - \log_5(4)$

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log(15)/log(5)
        1.682606194
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$\log_5(30\cdot2) - \log_5(4)$

$\underbrace{\log_5(60) - \log_5(4)}$

$\log_5\left(\frac{60}{4}\right) = \log_5(15) \approx 1.683$

$\boxed{\log_5(15) \approx 1.683}$

c. $\ln(8x) - 2\cdot \ln(x)$

$\underbrace{\ln(8x) - \ln(x^2)}$

$\frac{8\cancel{x}}{x\cancel{x}} = \frac{8}{x}$

$= \ln\left(\frac{8x}{x^2}\right)$

$= \ln\left(\frac{8}{x}\right)$

$\boxed{\ln\left(\frac{8}{x}\right)}$

d. $\underbrace{\log_3(4x) + \log_3(2y^2)} - \log_3(z)$

$\log_3(4x\cdot 2y^2) - \log_3(z)$

$\underbrace{\log_3(8xy^2) - \log_3(z)}$

$\log_3\left(\frac{8xy^2}{z}\right)$

$\boxed{\log_3\left(\frac{8xy^2}{z}\right)}$

e. $2\cdot\log_b(3x) + 3\cdot\log_b(2x) - \log_b(x^2)$

$\log_b((3x)^2) + \log_b((2x)^3) - \log_b(x^2)$

$\underbrace{\log_b(9x^2) + \log_b(8x^3)} - \log_b(x^2)$

$\underbrace{\log_b(72x^5) - \log_b(x^2)}$

$\log_b\left(\frac{72x^5}{x^2}\right) = \log_b(72x^3)$

$\frac{72\cancel{x}\cancel{x}xxx}{\cancel{x}\cancel{x}}$

$\boxed{\log_b(72x^3)}$

f. $2\cdot\ln(2x) + 3\cdot\ln(x^3) - \ln(6)$

$\ln((2x)^2) + \ln((x^3)^3) - \ln(6)$

$\underbrace{\ln(4x^2) + \ln(x^9)} - \ln(6)$

$\underbrace{\ln(4x^{11}) - \ln(6)}$

$\ln\left(\frac{4x^{11}}{6}\right) = \ln\left(\frac{2x^{11}}{3}\right)$

$\boxed{\ln\left(\frac{2x^{11}}{3}\right)}$

**3. Expand each of the single logarithm expressions in to multiple logarithms.**

**a.** $\log_5(12x^3)$

$\log_5(12 \cdot x^3) \longrightarrow$ USE ADD PROP

$\log_5(12) + \log_5(x^3) \longrightarrow$ USE MULT PROP

$\log_5(12) + 3 \cdot \log_5(x)$

$$\boxed{\log_5(12) + 3\log_5(x)}$$

**b.** $\ln\left(\frac{4x^3}{3y^2}\right)$

$\ln\left(\frac{4x^3}{3y^2}\right) \longrightarrow$ USE SUBTRACT PROP

$\ln(4x^3) - \ln(3y^2) \longrightarrow$ USE ADD PROP

$[\ln(4) + \ln(x^3)] - [\ln(3) + \ln(y^2)] \longrightarrow$ USE MULT PROP

$[\ln(4) + 3 \cdot \ln(x)] - [\ln(3) + 2\ln(y)]$

$$\boxed{\ln(4) + 3\ln(x) - \ln(3) - 2\ln(y)}$$

**c.** $\log_2\left(\frac{8a^3b}{c}\right)$

$\log_2\left(\frac{8a^3b}{c}\right) \longrightarrow$ USE SUB PROP

$\log_2(8a^3 b) - \log_2(c) \longrightarrow$ USE ADD PROP.

$\log_2(8) + \log_2(a^3) + \log_2(b) - \log_2(c) \rightarrow$ USE MULT. PROP

$\log_2(2^3) + 3\log_2(a) + \log_2(b) - \log_2(c)$

$$\boxed{3 + 3\log_2(a) + \log_2(b) - \log_2(c)}$$

**d.** $\ln\left(\frac{3a^x}{b^2}\right)$

$\ln\left(\frac{3a^x}{b^2}\right) \longrightarrow$ USE SUB. PROP

$\ln(3a^x) - \ln(b^2) \longrightarrow$ USE ADD PROP.

$\ln(3) + \ln(a^x) - \ln(b^2) \longrightarrow$ USE MULT. PROP

$\ln(3) + x\ln(a) - 2\ln(b)$

$$\boxed{\ln(3) + x\ln(a) - 2\ln(b)}$$