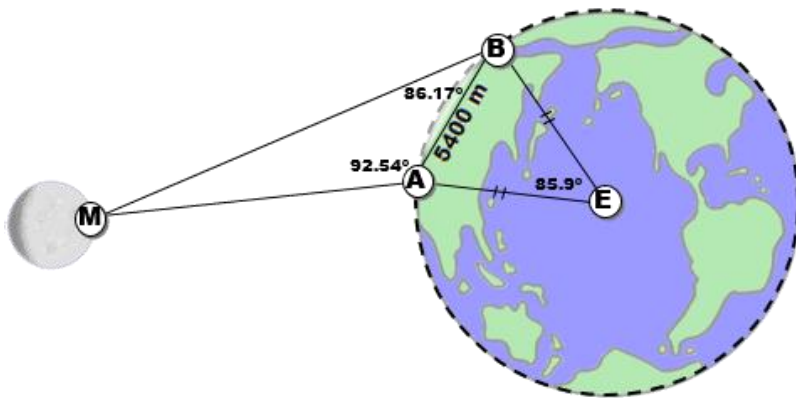


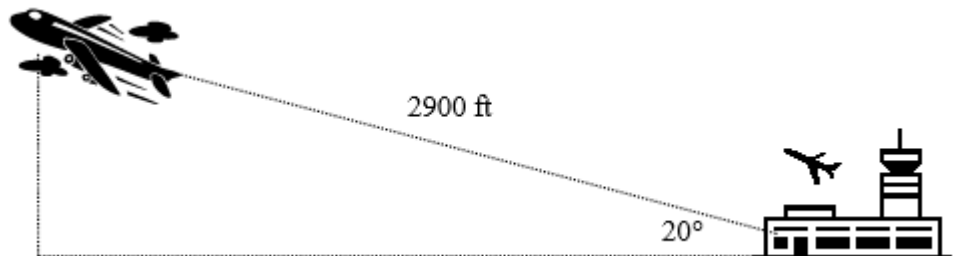
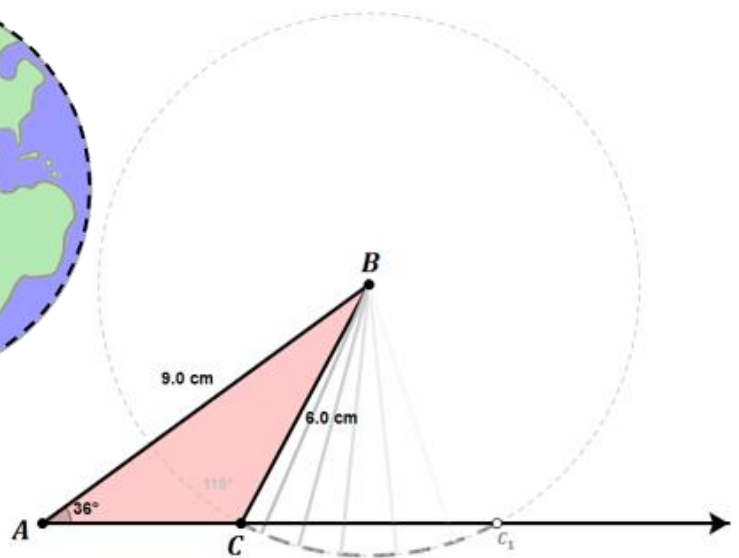
Pre-Calculus

Unit 1:

Trigonometry of General Triangles



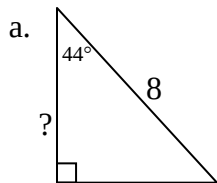
$$\sin \theta = \frac{O}{H}$$
$$\cos \theta = \frac{A}{H}$$
$$\tan \theta = \frac{O}{A}$$

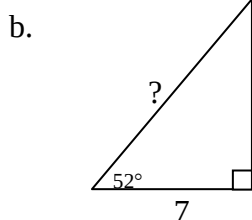


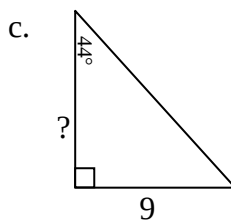
Sec 1.1 – Right Triangle Trigonometry
Right Triangle Trigonometry Sides

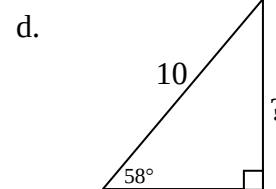
Name: _____

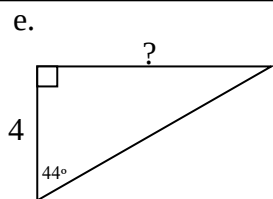
Find the requested unknown side of the following triangles.

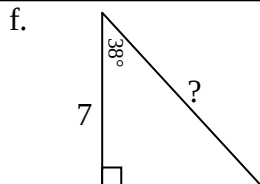


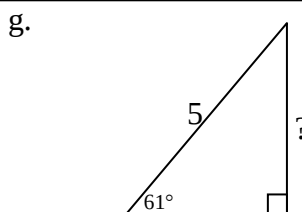


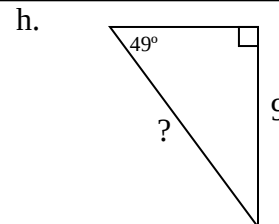




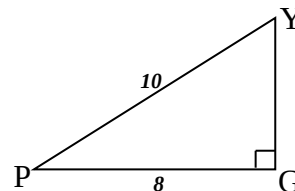




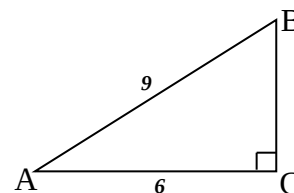




2. Find the EXACT value of $\sin(P)$.



3. Find the EXACT value of $\tan(B)$.



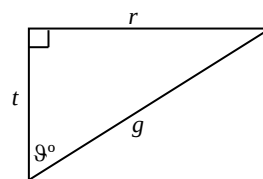
4. Which expression represents $\cos(9^\circ)$ for the triangle shown?

A. $\frac{g}{r}$

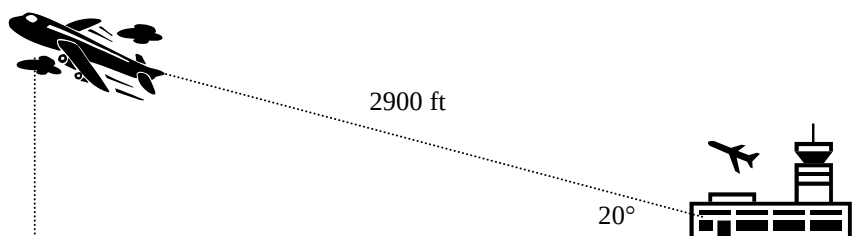
B. $\frac{r}{g}$

C. $\frac{g}{t}$

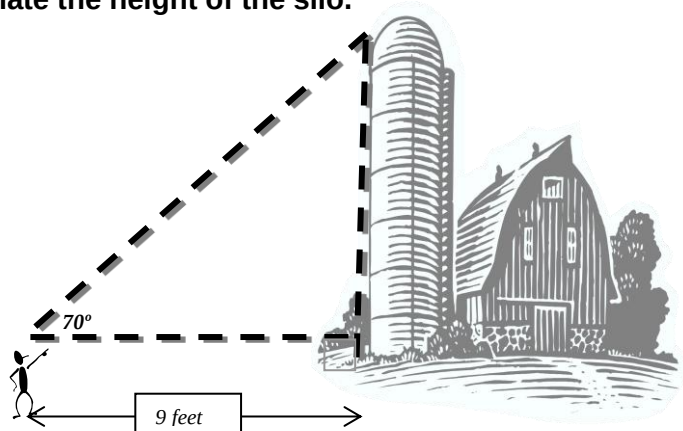
D. $\frac{t}{g}$



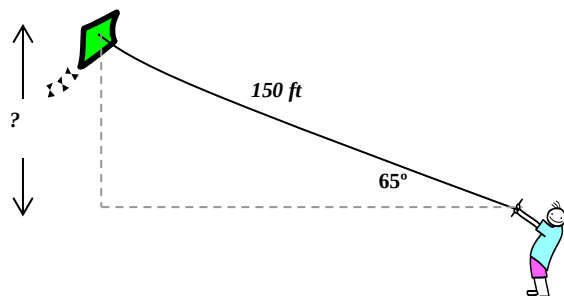
5. As a plane takes off it ascends at a 20° angle of elevation. If the plane has been traveling at an average rate of 290 ft/s and continues to ascend at the same angle, then how high is the plane after 10 seconds (the plane has traveled 2900 ft).



6. A person noted that the angle of elevation to the top of a silo was 65° at a distance of 9 feet from the silo. Using the diagram approximate the height of the silo.

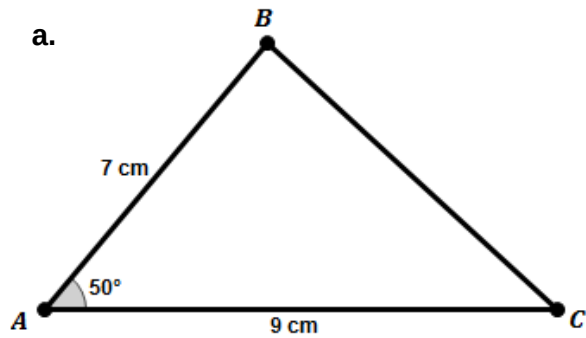


7. A kid is flying a kite and has reeled out his entire line of 150 ft of string. If the angle of elevation of the string is 65° then which expression gives the vertical height of the kite?

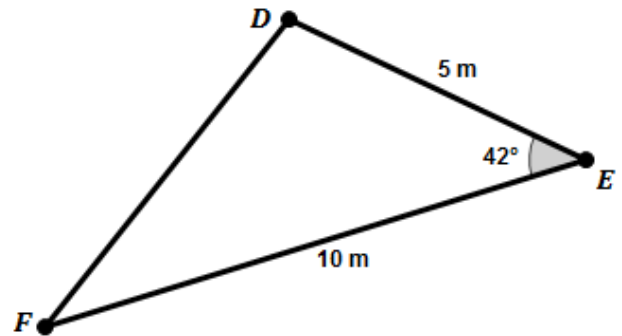


8. Find the approximate area of each of the following triangles. You may need to use trigonometry to assist in finding the area.

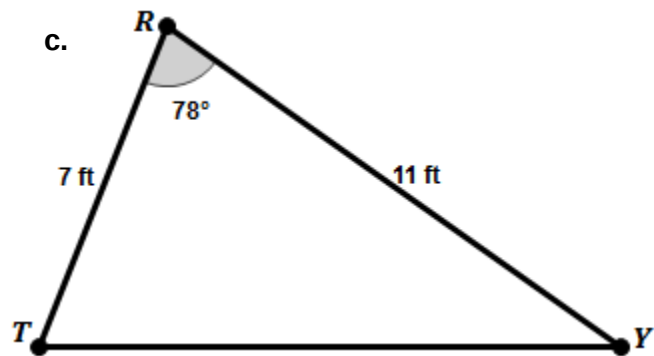
a.



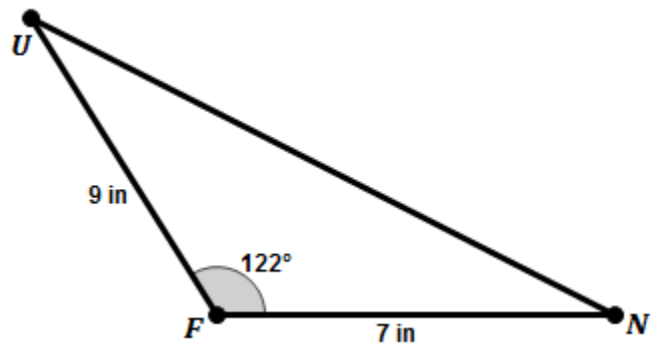
b.



c.



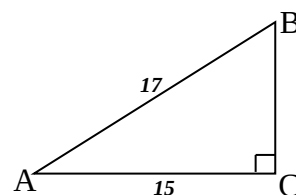
d.



9. Find the EXACT value of following based on the triangle shown:

a. $\sin(A) =$

d. $\csc(A) =$



b. $\cos(A) =$

e. $\sec(A) =$

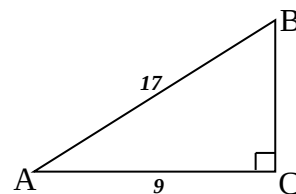
c. $\tan(A) =$

f. $\cot(A) =$

10. Find the EXACT value of following based on the triangle shown:

a. $\sin(A) =$

d. $\csc(A) =$



b. $\cos(A) =$

e. $\sec(A) =$

c. $\tan(A) =$

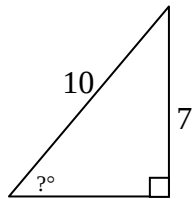
f. $\cot(A) =$

Sec 1-2 – Right Triangle Trigonometry
Right Triangle Trigonometry Angles

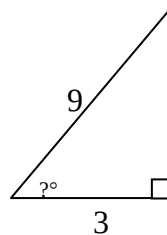
Name: _____

1. Find the requested unknown angles of the following triangles using a calculator.

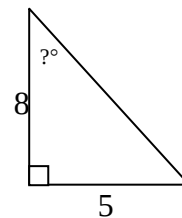
a.



b.



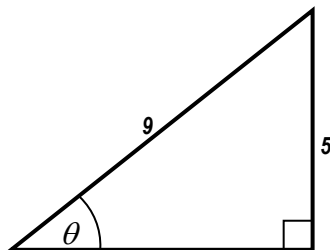
c.



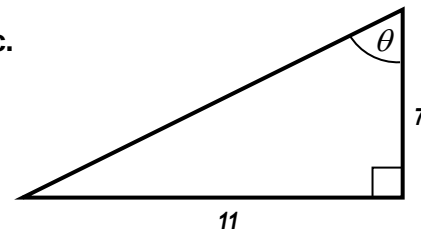
2. Find the approximate unknown angle, θ , using INVERSE trigonometric ratios ($\sin^{-1}$, $\cos^{-1}$, or $\tan^{-1}$).

a. $\cos \theta = 0.823$

b.



c.



$\theta =$

$\theta =$

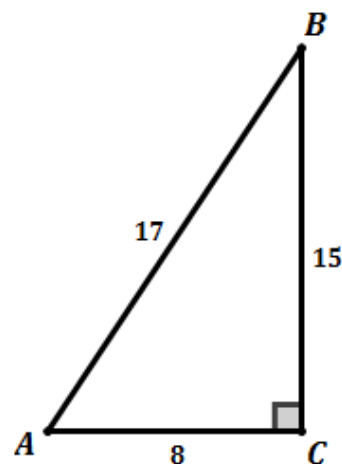
$\theta =$

3. Identify each of the following requested Trig Ratios.

A. $\sin A =$

B. $\cos B =$

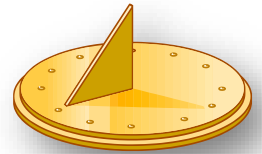
C. Measure of angle B =



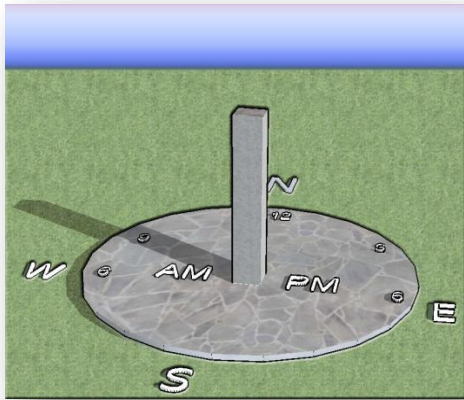
Sec 1-3 – Right Triangle Trigonometry
Solving with Trigonometry

Name: _____

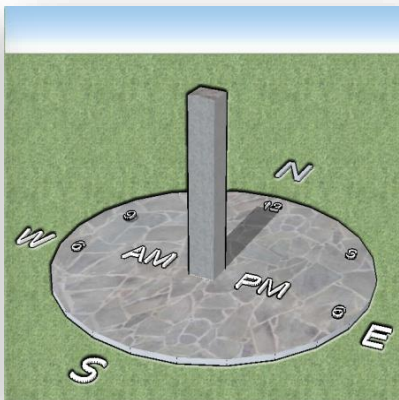
The word trigonometry is of Greek origin and literally translates to “Triangle Measurements”. Some of the earliest trigonometric ratios recorded date back to about 1500 B.C. in Egypt in the form of sundial measurements. They come in a variety of forms. The most basic sundials use a simple rod called a gnomon that simply sticks straight up out of the ground. Time is determined by the direction and length of the shadow created by the gnomon.



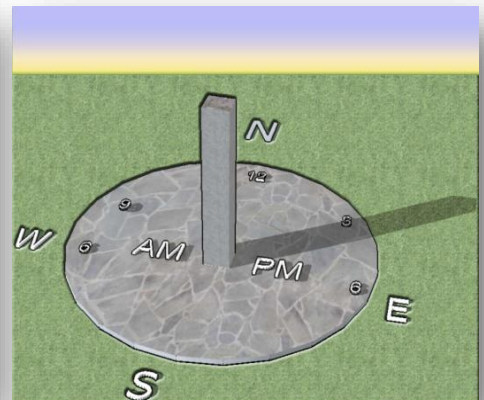
In the morning the sun **rises in the east** and alternately the shadow created by the gnomon points westerly. When the sun reaches its highest point in the sky it is known as ‘**High Noon**’. At 12:00 p.m. noon the shadow of a gnomon in a simple sundial is at its shortest length and points due north (at least it does so in the northern hemisphere). Then as the sun **sets in the west**, the shadow of the gnomon points east (as shown in the pictures below).



9:00 a.m.



12:00 p.m.

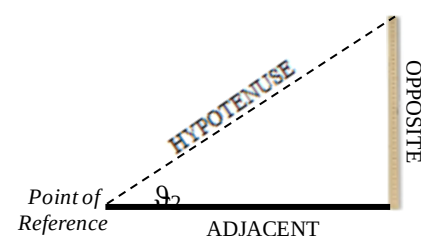
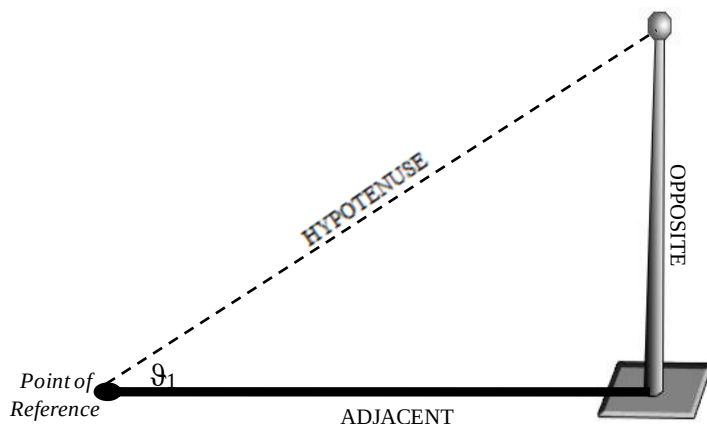


3:30 p.m.

Notice how the shadow rotates throughout the day on the sundial shown. These were the earliest clocks. The shadows acts like the hand of a clock moving in a clockwise motion. This is the reason clock’s hands today move in the direction they do today.

By creating a segment from the top of the gnomon to the tip of the shadow a right triangle is formed. Some of the earliest mathematicians charted the placement of the shadows over time and seasons and they began to analyze the relationships of the measurements of the right triangle create by these sundials.

1. Consider the following diagrams of sundials. Let the vertex at the tip of the shadow be the point or angle of reference. Below show two examples of makeshift sundials using a **flagpole** and **meter stick**. Both diagrams represent 7:30 a.m. Using a ruler measure the length of each side of each triangle in the diagrams using centimeters to the nearest tenth.



2. Fill in the charts below with the measurements from problem #1. The ratios of the sides of right triangles have specific names that are used frequently in the study of trigonometry.

SINE is the ratio of Opposite to Hypotenuse (abbreviated '**sin**').

COSINE is the ratio of Opposite to Hypotenuse (abbreviated '**cos**').

TANGENT is the ratio of Opposite to Hypotenuse (abbreviated '**tan**').

Flag Pole Triangle	
Opposite	
Adjacent	
Hypotenuse	
$\sin \theta_1 = \frac{\text{Opposite}}{\text{Hypotenuse}}$	
$\cos \theta_1 = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	
$\tan \theta_1 = \frac{\text{Opposite}}{\text{Adjacent}}$	
θ_1 (using a protractor)	

Meter Stick Triangle	
Opposite	
Adjacent	
Hypotenuse	
$\sin \theta_2 = \frac{\text{Opposite}}{\text{Hypotenuse}}$	
$\cos \theta_2 = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	
$\tan \theta_2 = \frac{\text{Opposite}}{\text{Adjacent}}$	
θ_2 (using a protractor)	

3. 40° and 50° are **complementary** angles because they have a sum of 90° .

a. What is an approximation of $\sin(40^\circ)$?

b. What is an approximation of $\cos(50^\circ)$?

c. What is an approximation of $\sin(30^\circ)$?

d. What is an approximation of $\cos(60^\circ)$?

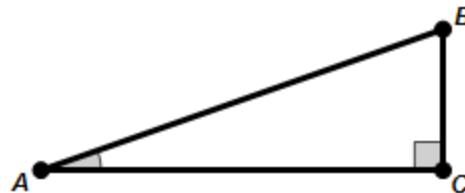
e. What is an approximation of $\sin(55^\circ)$?

f. What is an approximation of $\cos(35^\circ)$?

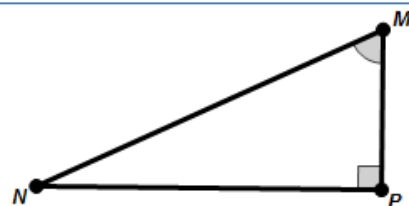
g. What do you think the "CO" in **COSINE** stands for?

4. Given the provided trig ratio find the requested trig ratio.

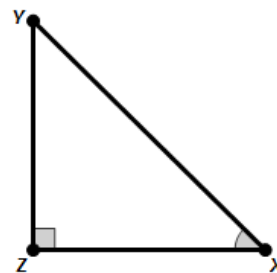
- a. Given: $\sin(A) = \frac{5}{13}$ and the diagram shown at the right, determine the value of $\tan(A)$.



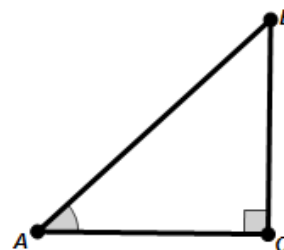
- b. Given: $\cos(M) = \frac{8}{17}$ and the diagram shown at the right, determine the value of $\sin(M)$.



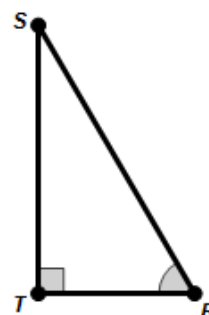
- c. Given: $\tan(X) = \frac{20}{21}$ and the diagram shown at the right, determine the value of $\sin(Y)$.



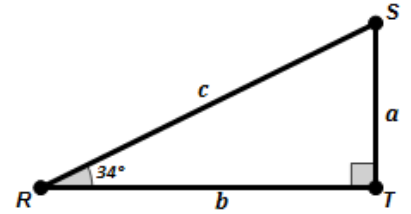
- d. Given: $\sin(A) = \frac{5}{7}$ and the diagram shown at the right, determine the value of $\cos(B)$.



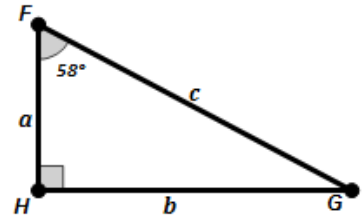
- e. Given: $\cos(R) = \frac{5}{14}$ and the diagram shown at the right, determine the value of $\tan(R)$.



5. In right triangle SRT shown in the diagram, angle T is the right angle and $m\angle R = 34^\circ$. Determine the approximate value of $\frac{a}{b}$.

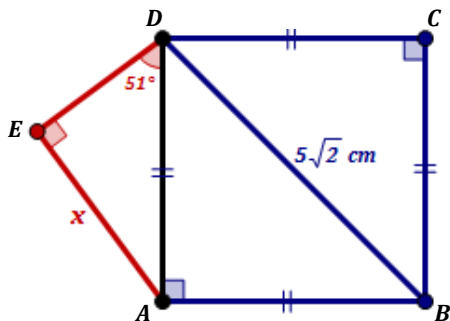


6. In right triangle FGH shown in the diagram, angle H is the right angle and $m\angle F = 58^\circ$. Determine the approximate value of $\frac{a}{c}$.

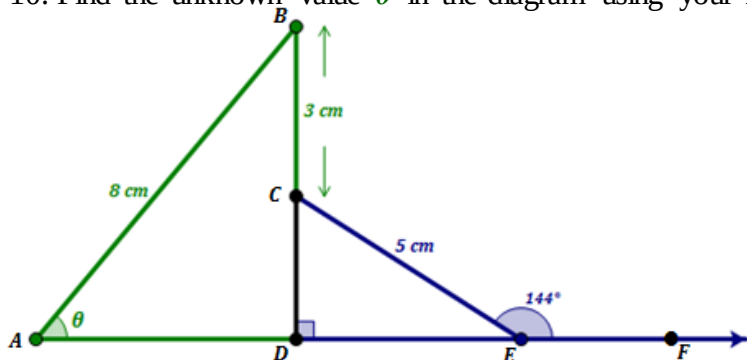


7. Consider a right triangle XYZ such that X is the right angle. Classify the triangle based on its sides provided that $\tan(Y) = 1$.
8. Consider a right triangle ABC such that C is the right angle. Classify the triangle based on its sides provided that $\tan(A) = \frac{3}{4}$.

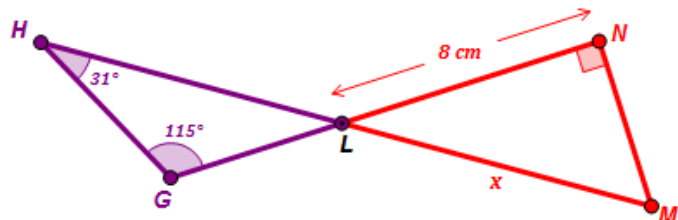
9. Find the unknown value x in the diagram using your knowledge of geometric figures and trigonometry.



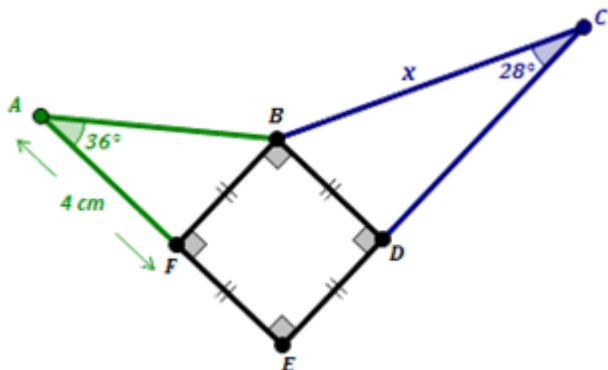
10. Find the unknown value θ in the diagram using your knowledge of geometric figures and trigonometry.



11. Find the unknown value x in the diagram using your knowledge of geometric figures and trigonometry.



12. Find the unknown value x in the diagram using your knowledge of geometric figures and trigonometry.

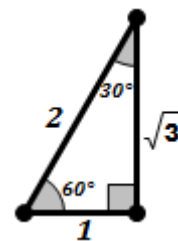
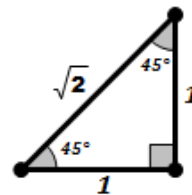
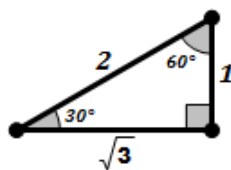


13. Using standard special right triangles find the exact value of the following.

a. $\sin(60^\circ) =$

b. $\cos(45^\circ) =$

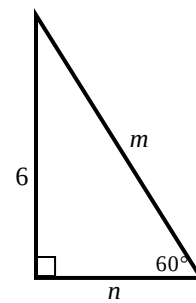
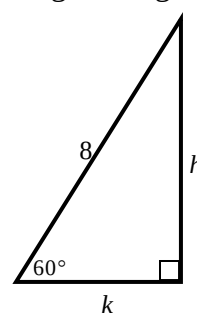
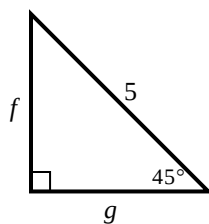
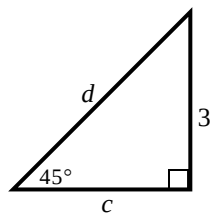
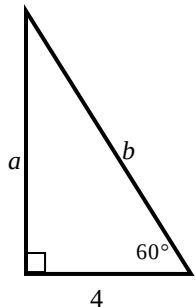
c. $\tan(30^\circ) =$



d. $\sin(30^\circ) =$

e. $\tan(45^\circ) =$

14. Find the unknown sides without using trigonometry but with special right triangles.



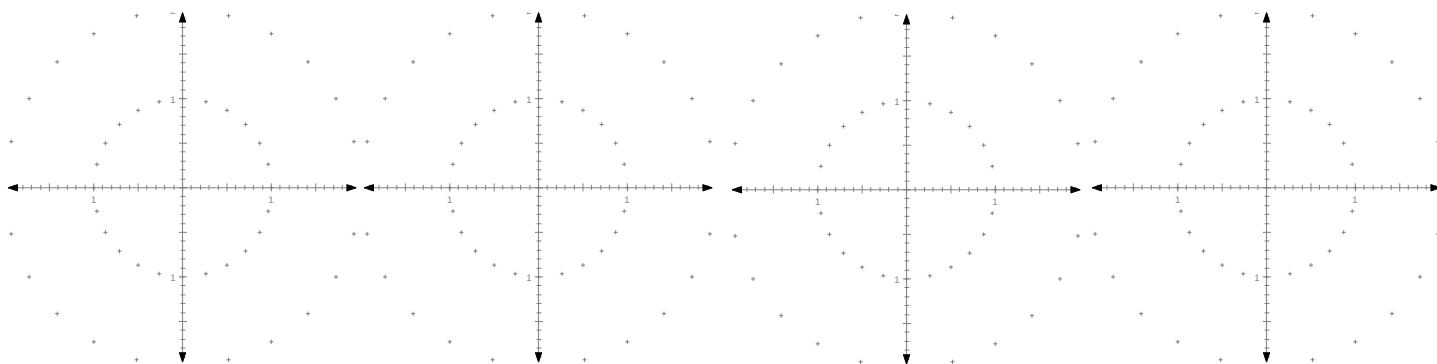
15. Find the reference angle & use special right triangles to determine the EXACT value of the following.

a. $\sin(45^\circ) =$

b. $\sin(120^\circ) =$

c. $\cos(225^\circ) =$

d. $\cos(300^\circ) =$

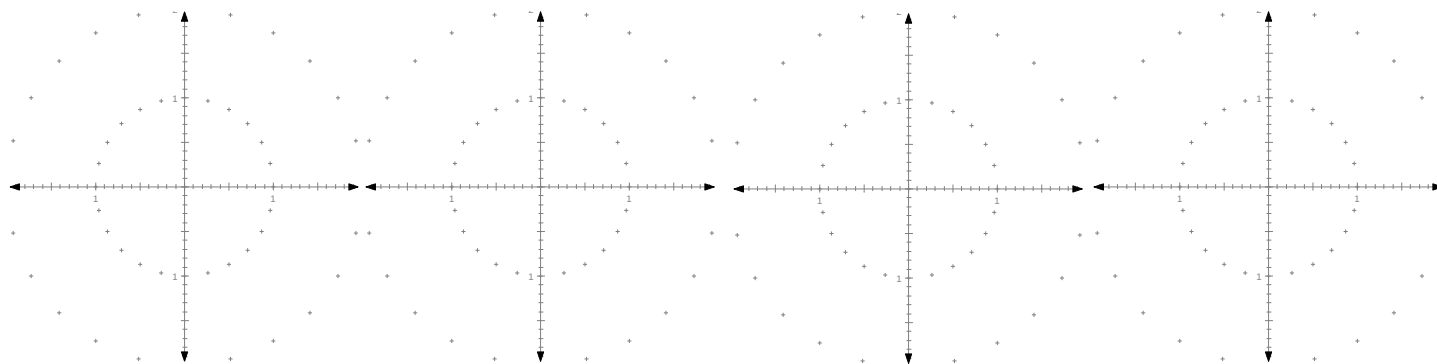


e. $\tan(330^\circ) =$

f. $\sin(405^\circ) =$

g. $\cos(480^\circ) =$

h. $\sin(180^\circ) =$

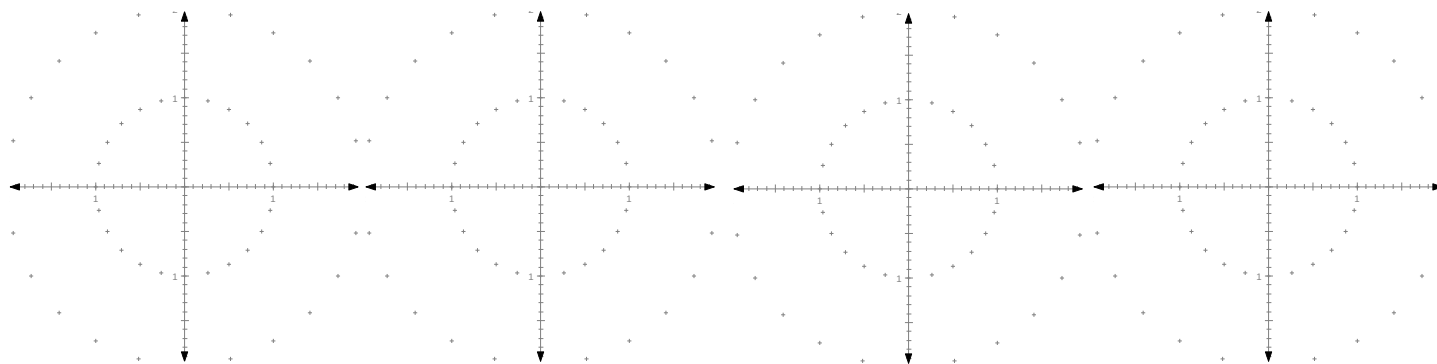


i. $\csc(330^\circ) =$

j. $\sec(225^\circ) =$

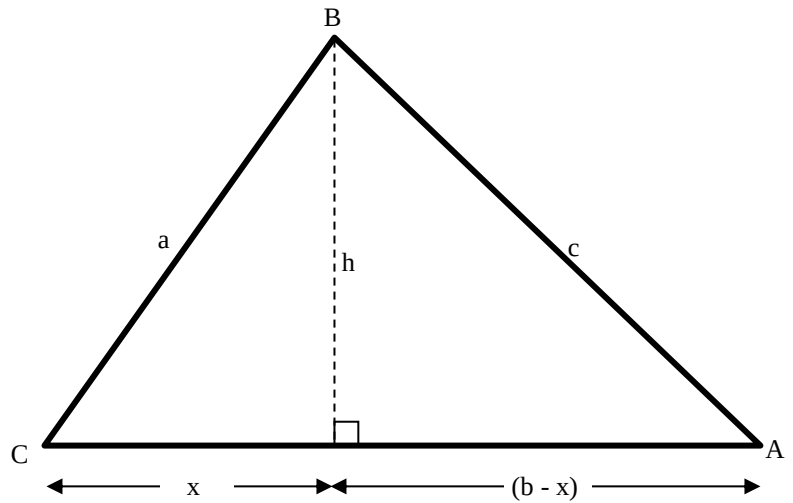
k. $\cot(840^\circ) =$

l. $\cos(-450^\circ) =$

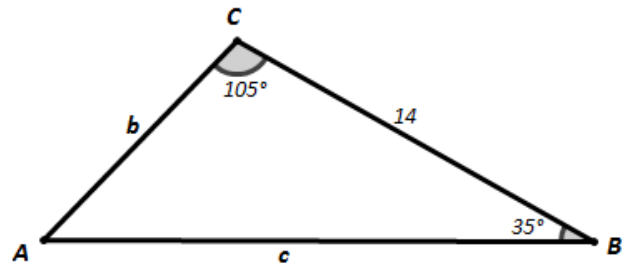
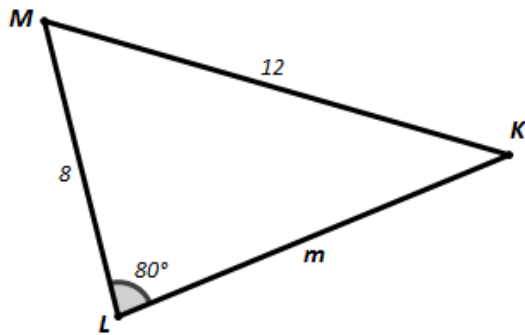


Law of Sines: Start with $\sin(A)$ and $\sin(C)$.

PROOF:



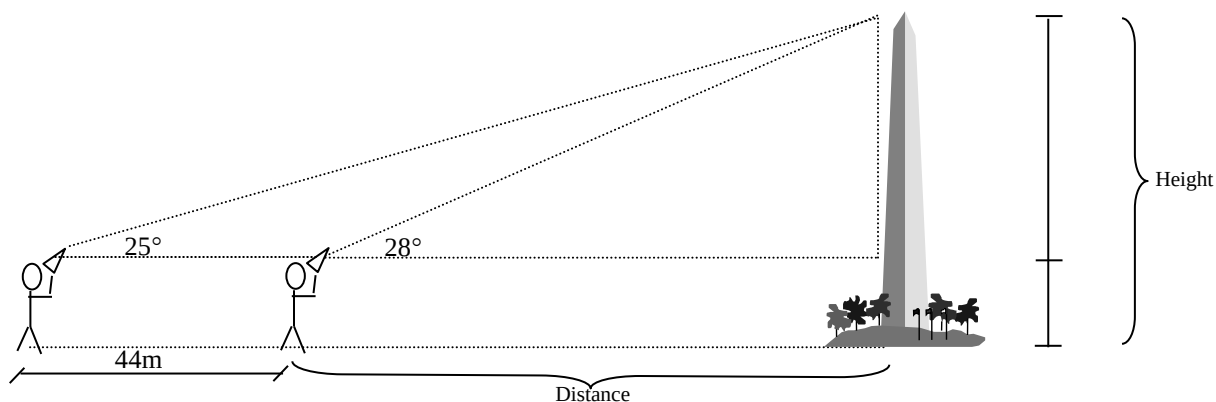
1. Find the unknown sides and angles of each triangle using the **Law of Sines**.



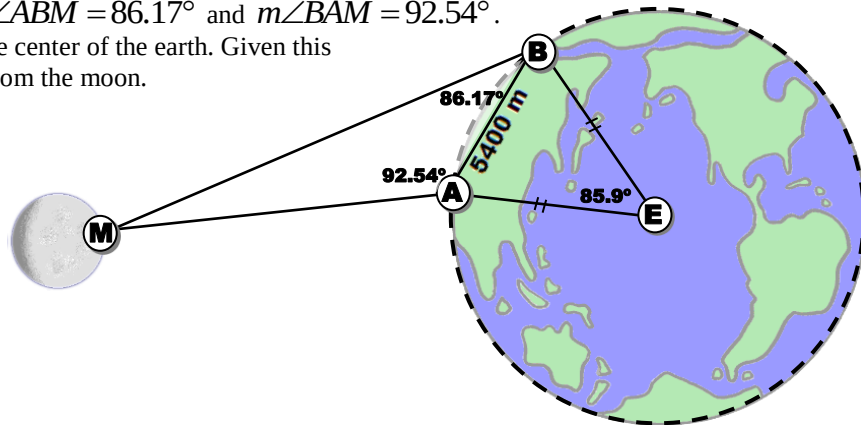
m	
$m\angle M$	
$m\angle K$	

c	
b	
$m\angle A$	

2. A student was trying to determine the height of the Washington monument from a distance. So, he measured two angles of elevation 44 meters apart. The angle of elevation the furthest away from the monument measured to be 25° and the closest angle of elevation measured 28° . The student determining the angles is 1.6 Meters tall from his feet to his eyeballs. Find the Height =



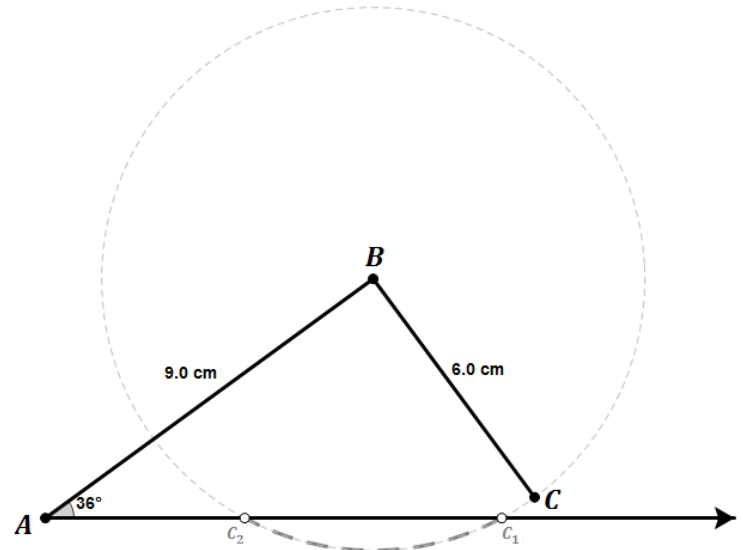
3. Two students that are on the same longitudinal line are approximately 5400 miles apart. They used an inclinometer, a little geometry, and a tangent line to determine that $m\angle ABM = 86.17^\circ$ and $m\angle BAM = 92.54^\circ$. The two students form a central angle of 85.9° with the center of the earth. Given this information determine how far each student is away from the moon.



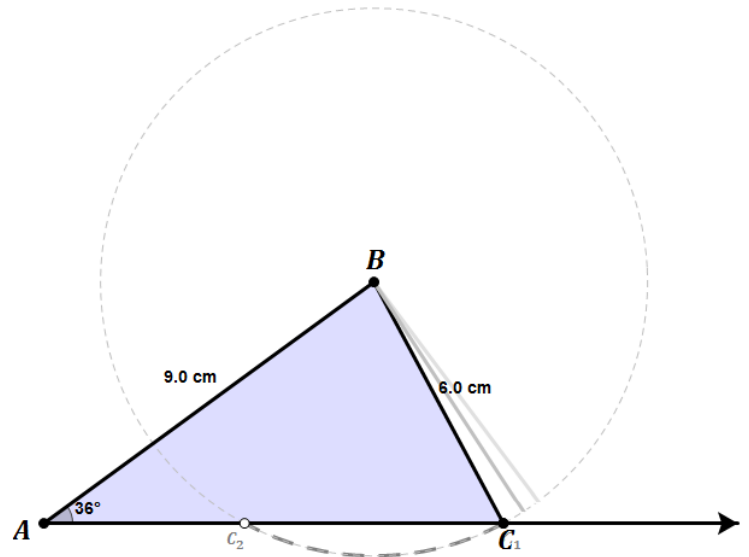
Use this information to find the radius of the Earth and then the circumference ($C = 2\pi r$).

In some examples you may be provided with 3 measures of a triangle that can potentially describe more than one triangle. Consider triangle ABC with the following measures $m\angle A = 36^\circ$, $AB = 9 \text{ cm}$, and $BC = 6 \text{ cm}$.

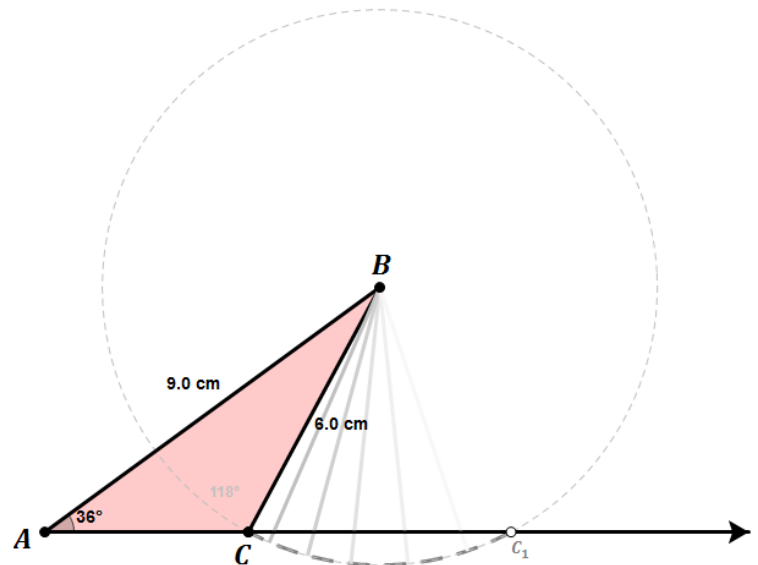
- First we could create a ray emanating from point A. Then, create a new ray emanating from A but at an angle of 36° from the first. Next, we could measure 9cm along the newly created ray and label the point B. Finally, create a circle with a center at B and a radius of 6 cm. If more than one triangle is possible the circle should intersect the original first ray twice. Each intersection represents a potential vertex C of a triangle with the given measures.



- The Standard or Regular Case.** Usually this is represented by creating the triangle that has the longest possible side for the only side measure that was not given (in this example AC). We can use Law of Sines regularly in this example to find the unknown measures of the triangle.



- The Ambiguous Case.** Usually this is represented by creating the triangle that has the shortest possible side for the only side measure that was not given (in this example AC). To find the unknown measures of the ambiguous case, we will need to use geometry and law of sines.



When provided with **Side-Side-Angle (SSA)** measures of a triangle with an acute angle, 1 of 4 situations can occur.

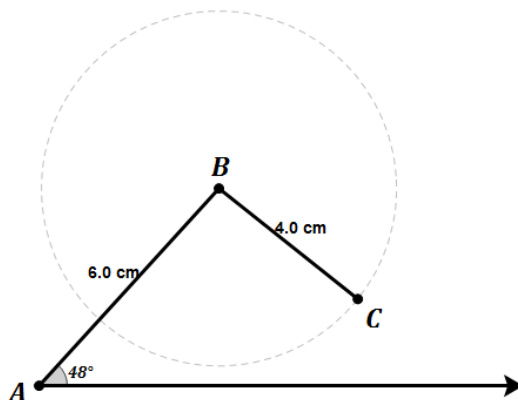
Case #1: (No Triangle Possible)

Find the height of the triangle from point B to the ray emanating from point A.

$$AB \cdot \sin(A) > BC$$

$$6 \cdot \sin(48^\circ) > 4$$

$$4.46 > 4$$



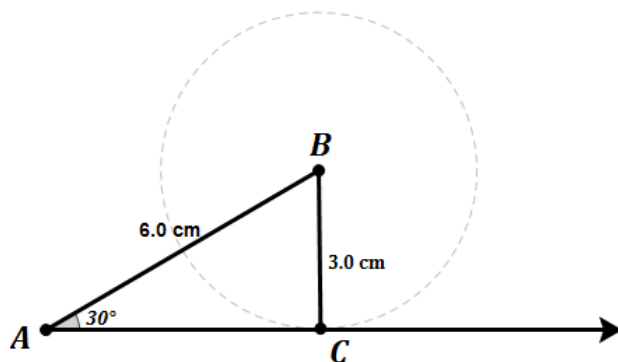
Case #2: (One Right Triangle Possible)

Find the height of the triangle from point B to the ray emanating from point A.

$$AB \cdot \sin(A) = BC$$

$$6 \cdot \sin(30^\circ) = 3$$

$$3 = 3$$



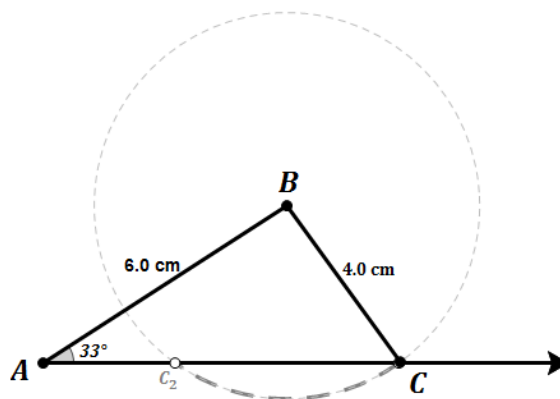
Case #3: (Two Triangles Possible)

Find the height of the triangle from point B to the ray emanating from point A.

$$AB \cdot \sin(A) < BC < AB$$

$$6 \cdot \sin(33^\circ) < 4 < 6$$

$$3.27 < 4 < 6$$

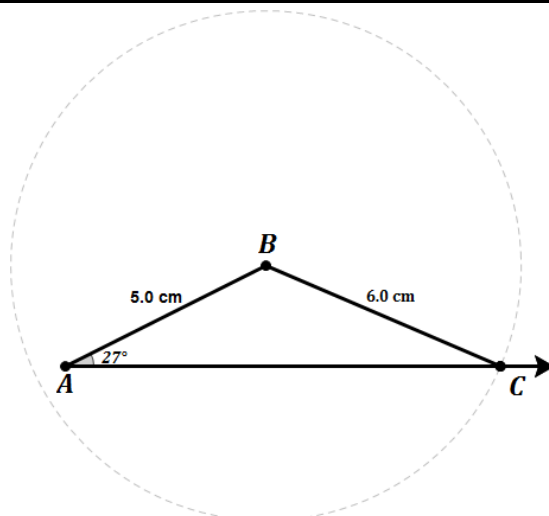


Case #4: (One Triangle Possible)

Find the height of the triangle from point B to the ray emanating from point A.

$$BC \geq AB$$

$$6 \geq 5$$



4. Sketch the potential triangle for each set of measures and determine which case of SSA is presented

- a. Triangle $\triangle ABC$ with measures
 $A = 38^\circ$, $c = 8$, $a = 10$

- d. Triangle $\triangle TRY$ with measures
 $T = 45^\circ$, $y = 3\sqrt{2}$, $t = 3$

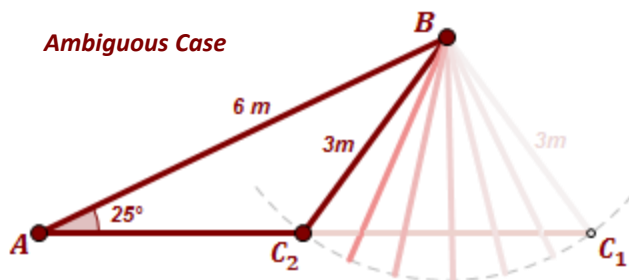
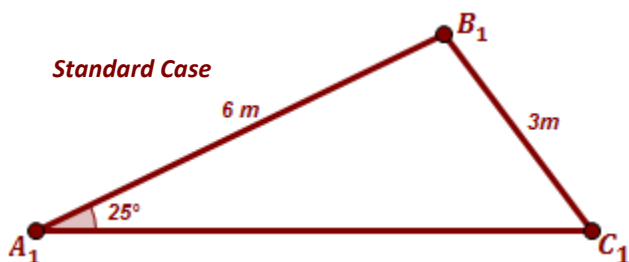
- b. Triangle $\triangle MED$ with measures
 $M = 40^\circ$, $d = 9$, $m = 6$

- e. Triangle $\triangle LOG$ with measures
 $L = 50^\circ$, $g = 30$, $l = 18$

- c. Triangle $\triangle FUN$ with measures
 $F = 32^\circ$, $n = 12$, $f = 8$

- f. Triangle $\triangle HIP$ with measures
 $P = 30^\circ$, $p = 6$, $h = 12$

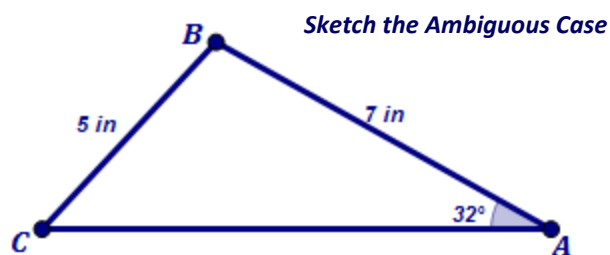
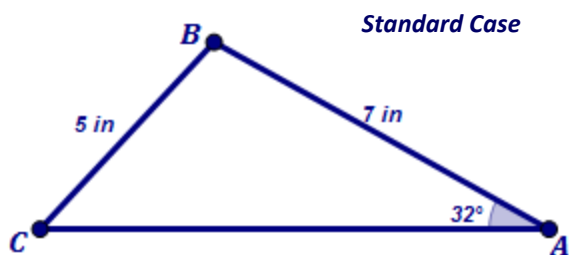
5. Consider $\triangle ABC$ with the measures $A = 25^\circ$, $c = 6$, $a = 3$. Determine all of the unknown sides of both triangles that could be created with those measures.



$C_1 \approx$
$B_1 \approx$
$b_1 \approx$

$C_2 \approx$
$B_2 \approx$
$b_2 \approx$

5. Consider $\triangle ABC$ with the measures $A = 32^\circ$, $c = 7$, $a = 5$. Determine all of the unknown sides of both triangles that could be created with those measures.

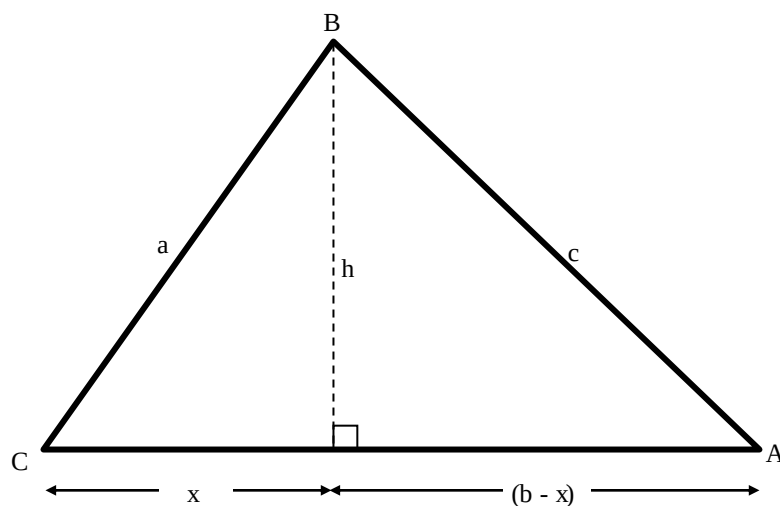


$C_1 \approx$
$B_1 \approx$
$b_1 \approx$

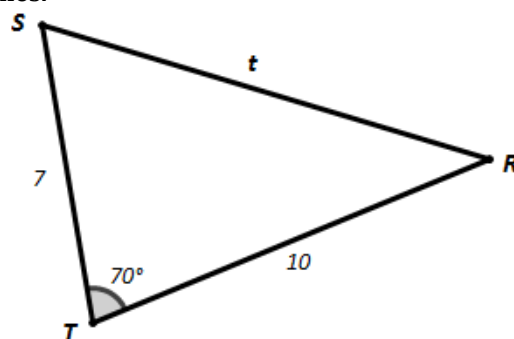
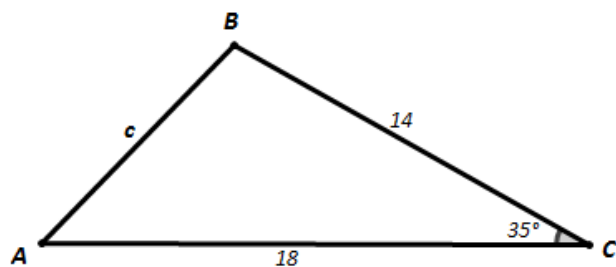
$C_2 \approx$
$B_2 \approx$
$b_2 \approx$

Law of Cosines: Start with $\cos(C)$ and the Pythagorean theorem for both of the right triangles.

PROOF :



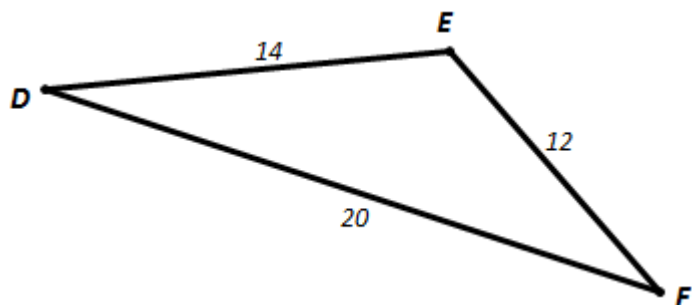
1. Find the unknown sides and angles of each triangle using the **Law of Cosines**.



f	
d	
$m\angle D$	

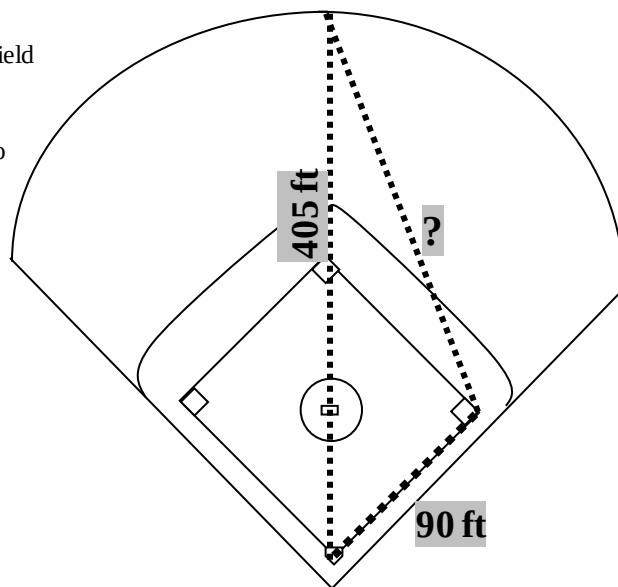
t	
$m\angle S$	
$m\angle R$	

2. Find the unknown sides and angles of each triangle using the **Law of Sines**.



$m\angle D$	
$m\angle E$	
$m\angle F$	

3. A centerfield baseball player caught a ball right at the deepest part of center field against the wall. From home plate to where the player caught the ball is 405 feet. The outfielder is trying to complete a double play by throwing the ball to first base. Using the diagram, how far did the outfielder need to throw the ball. (The bases are all laid out in a perfect square with each base 90 feet away from the next. Since it is a square you should be able to determine the angle created by 1st base – home plate – 2nd base)



4. On one night, a scientist needs to determine the distance she is away from the International Space Station. At the specific time she is determining this the space station distance they are both on the same line of longitude 77° E. Furthermore, she is on a latitude of 29° N and the space station is orbiting just above a latitude of 61.4° N. In short, the central angle between the two is 32.4° . If the Earth's radius is 3959 miles and the space station orbits 205 miles above the surface of the Earth, then how far is the scientist away from the space station?

