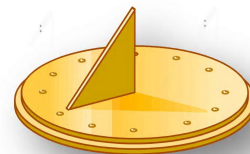
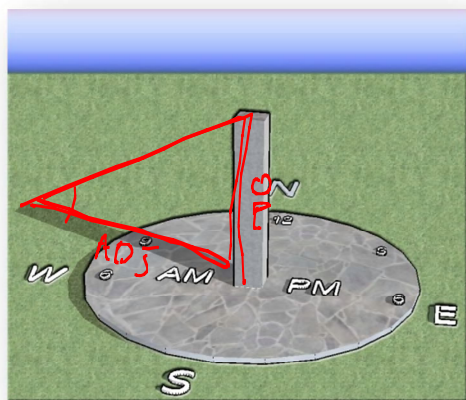


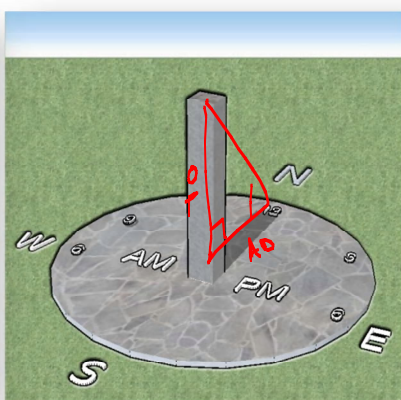
The word trigonometry is of Greek origin and literally translates to “Triangle Measurements”. Some of the earliest trigonometric ratios recorded date back to about 1500 B.C. in Egypt in the form of sundial measurements. They come in a variety of forms. The most basic sundials use a simple rod called a gnomon that simply sticks straight up out of the ground. Time is determined by the direction and length of the shadow created by the gnomon.



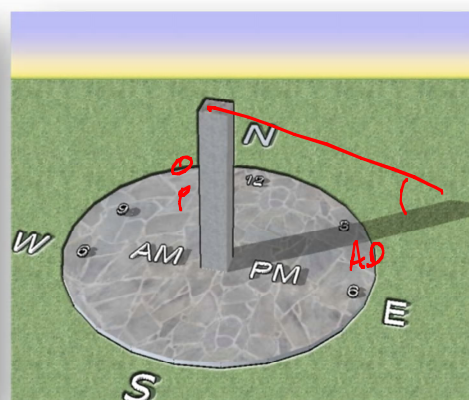
In the morning the sun **rises in the east** and alternately the shadow created by the gnomon points westerly. When the sun reaches its highest point in the sky it is known as ‘**High Noon**’. At 12:00 p.m. noon the shadow of a gnomon in a simple sundial is at its shortest length and points due north (at least it does so in the northern hemisphere). Then as the sun **sets in the west**, the shadow of the gnomon points east (as shown in the pictures below).



9:00 a.m.



12:00 p.m.

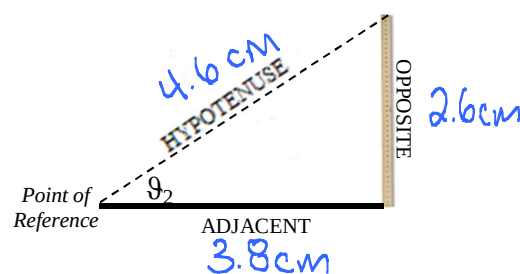
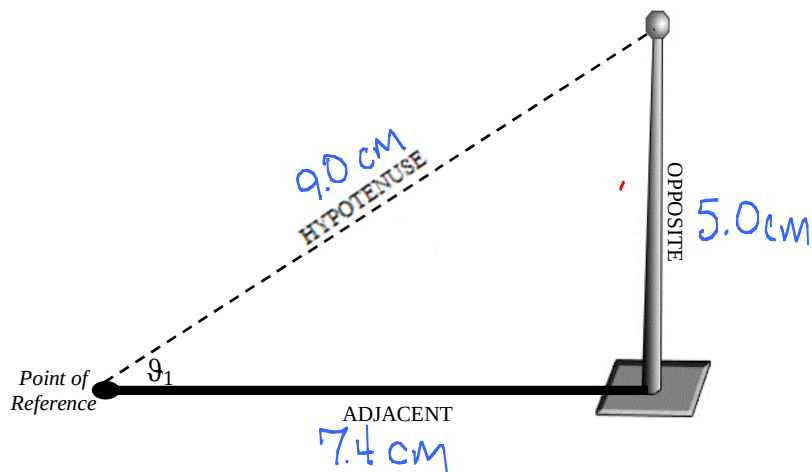


3:30 p.m.

Notice how the shadow rotates throughout the day on the sundial shown. These were the earliest clocks. The shadows acts like the hand of a clock moving in a clockwise motion. This is the reason clock’s hands today move in the direction they do today.

By creating a segment from the top of the gnomon to the tip of the shadow a right triangle is formed. Some of the earliest mathematicians charted the placement of the shadows over time and seasons and they began to analyze the relationships of the measurements of the right triangle create by these sundials.

1. Consider the following diagrams of sundials. Let the vertex at the tip of the shadow be the point or angle of reference. Below show two examples of makeshift sundials using a **flagpole** and **meter stick**. Both diagrams represent 7:30 a.m. Using a ruler measure the length of each side of each triangle in the diagrams using centimeters to the nearest tenth.



2. Fill in the charts below with the measurements from problem #1. The ratios of the sides of right triangles have specific names that are used frequently in the study of trigonometry.

SINE is the ratio of Opposite to Hypotenuse (abbreviated '**sin**').

COSINE is the ratio of Adjacent to Hypotenuse (abbreviated '**cos**').

TANGENT is the ratio of Opposite to Adjacent (abbreviated '**tan**').

Flag Pole Triangle	
Opposite	5.0 cm
Adjacent	7.4 cm
Hypotenuse	9.0 cm
$\sin \theta_1 = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{5}{9} \approx .56$
$\cos \theta_1 = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{7.4}{9} \approx .82$
$\tan \theta_1 = \frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{5}{7.4} \approx .68$
θ_1 (using a protractor)	33.7°

Meter Stick Triangle	
Opposite	2.6 cm
Adjacent	3.8 cm
Hypotenuse	4.6 cm
$\sin \theta_2 = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{2.6}{4.6} \approx .57$
$\cos \theta_2 = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{3.8}{4.6} \approx .83$
$\tan \theta_2 = \frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{2.6}{3.8} \approx .68$
θ_2 (using a protractor)	34.4°

3. 40° and 50° are **complementary** angles because they have a sum of 90°.

a. What is an approximation of $\sin(40^\circ)$?

$$\sin(40) \approx .6427876097$$

b. What is an approximation of $\cos(50^\circ)$?

$$\cos(50) \approx .6427876097$$

c. What is an approximation of $\sin(30^\circ)$?

$$\sin(30) = .5$$

d. What is an approximation of $\cos(60^\circ)$?

$$\cos(60) = .5$$

e. What is an approximation of $\sin(55^\circ)$?

$$\sin(55) \approx .8191520443$$

f. What is an approximation of $\cos(35^\circ)$?

$$\cos(35) \approx .8191520443$$

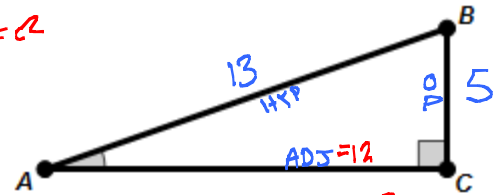
g. What do you think the "CO" in **COSINE** stands for?

↓
COMPLEMENT OF SINE

4. Given the provided trig ratio find the requested trig ratio.

$$a^2 + b^2 = c^2$$

- a. Given: $\sin(A) = \frac{5}{13}$ and the diagram shown at the right, determine the value of $\tan(A)$.



$$\sin A = \frac{5}{13}$$

← OPP
← HYP

$$\tan(A) = \frac{O}{A} = \frac{5}{12}$$

$$\begin{aligned} a^2 + 5^2 &= 13^2 \\ a^2 + 25 &= 169 \\ -25 &\quad -25 \\ \hline \sqrt{a^2} &= \sqrt{144} \\ a &= 12 \end{aligned}$$

- b. Given: $\cos(M) = \frac{8}{17}$ and the diagram shown at the right, determine the value of $\sin(M)$.

$$a^2 + b^2 = c^2$$

$$\cos(M) = \frac{8}{17}$$

← ADJ
← HYP

$$\sin(M) = \frac{OP}{HY} = \frac{15}{17}$$



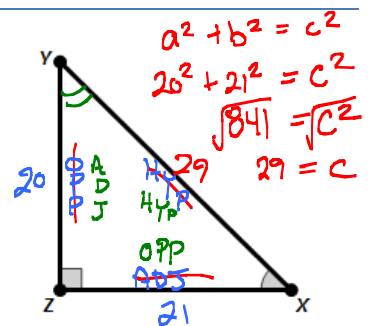
$$\begin{aligned} 8^2 + b^2 &= 17^2 \\ 64 + b^2 &= 289 \\ -64 &\quad -64 \\ \hline b^2 &= 225 \\ b &= 15 \end{aligned}$$

- c. Given: $\tan(X) = \frac{20}{21}$ and the diagram shown at the right, determine the value of $\sin(Y)$.

$$\tan(X) = \frac{20}{21}$$

← OPP
← ADJ

$$\sin(Y) = \frac{OP}{HY} = \frac{21}{29}$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 20^2 + 21^2 &= c^2 \\ \sqrt{841} &= \sqrt{c^2} \\ 29 &= c \end{aligned}$$

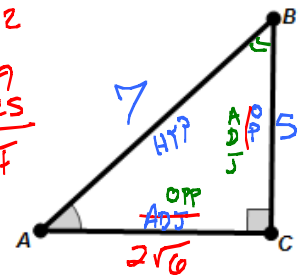
- d. Given: $\sin(A) = \frac{5}{7}$ and the diagram shown at the right, determine the value of $\cos(B)$.

$$\sin(A) = \frac{5}{7}$$

← OPP
← HYP

$$\cos(B) = \frac{AD}{HY} = \frac{5}{7}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 5^2 &= 7^2 \\ a^2 + 25 &= 49 \\ -25 &\quad -25 \\ \hline \sqrt{a^2} &= \sqrt{24} \\ a &= 2\sqrt{6} \end{aligned}$$



$$180 - 90 = 90^\circ$$

- e. Given: $\cos(R) = \frac{5}{14}$ and the diagram shown at the right, determine the value of $\tan(R)$.

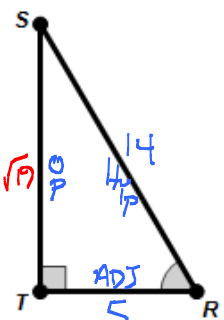
$$\cos(R) = \frac{5}{14}$$

← ADJ
← HYP

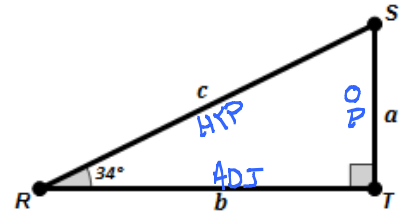
$$\tan(R) = \frac{OP}{AD} = \frac{3\sqrt{19}}{5}$$

$$\frac{3\sqrt{19}}{5}$$

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 5^2 &= 14^2 \\ a^2 + 25 &= 196 \\ -25 &\quad -25 \\ \hline \sqrt{a^2} &= \sqrt{171} \\ a &= 3\sqrt{19} \end{aligned}$$

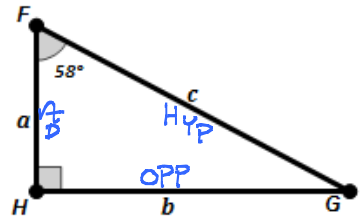


5. In right triangle SRT shown in the diagram, angle T is the right angle and $m\angle R = 34^\circ$. Determine the approximate value of $\frac{a}{b}$.



$$\tan(34) = \frac{a}{b} \approx \boxed{0.675}$$

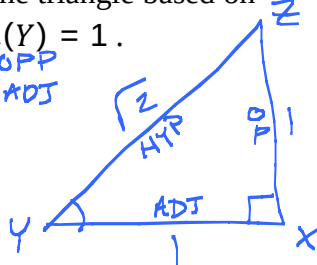
6. In right triangle FGH shown in the diagram, angle H is the right angle and $m\angle F = 58^\circ$. Determine the approximate value of $\frac{a}{c}$.



$$\cos(58) = \frac{a}{c} \approx \boxed{.530}$$

7. Consider a right triangle XYZ such that X is the right angle. Classify the triangle based on its sides provided that $\tan(Y) = 1$.

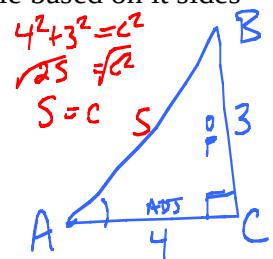
$$\tan(Y) = \frac{1}{1}$$



THE TRIANGLE MUST BE ISOSCELES.

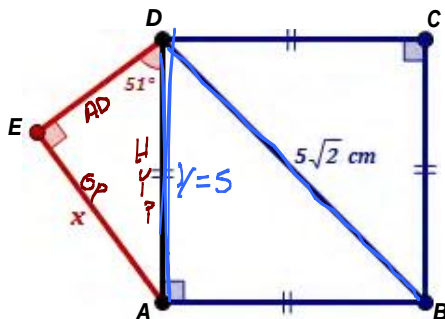
8. Consider a right triangle ABC such that C is the right angle. Classify the triangle based on its sides provided that $\tan(A) = \frac{3}{4}$.

$$\tan(A) = \frac{3}{4}$$



THE TRIANGLE MUST BE SCALENE.

9. Find the unknown value x in the diagram using your knowledge of geometric figures and trigonometry.



$$5 \cdot \sin(51) = 3.885729807$$

$$\frac{Y}{1} = \frac{5\sqrt{2}}{\sqrt{2}}$$

$$Y = 5$$

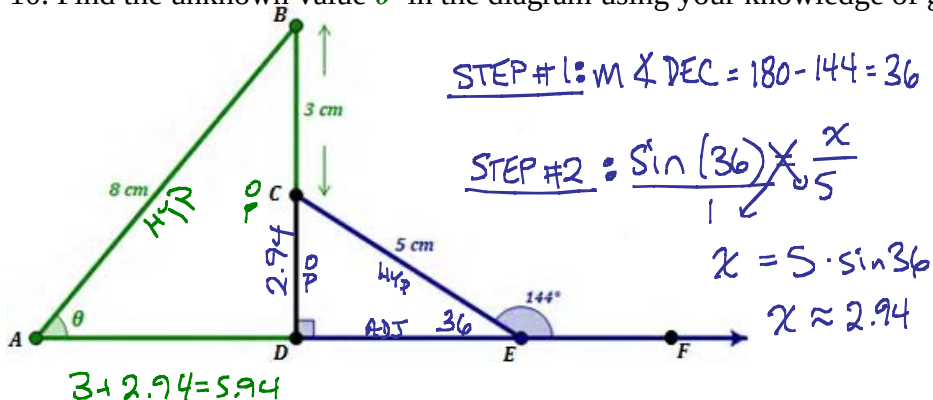
HAVE: HY
WANT: OP

$$\sin(51) = \frac{x}{5}$$

$$x = 5 \cdot \sin(51)$$

$$x \approx \boxed{3.89}$$

10. Find the unknown value θ in the diagram using your knowledge of geometric figures and trigonometry.



STEP #3:

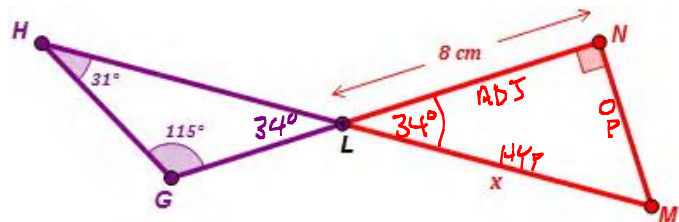
$$\sin \theta = \frac{5.94}{8}$$

$$\sin(\theta) = \sin\left(\frac{5.94}{8}\right)$$

$$\theta \approx 47.9^\circ$$

5 * sin(36)
2.938926261
Ans + 3
5.938926261
sin(5.94/8)
47.94481491

11. Find the unknown value x in the diagram using your knowledge of geometric figures and trigonometry.



STEP #1 $m \angle GLN = 180 - 31 - 115$
 $= 34^\circ$

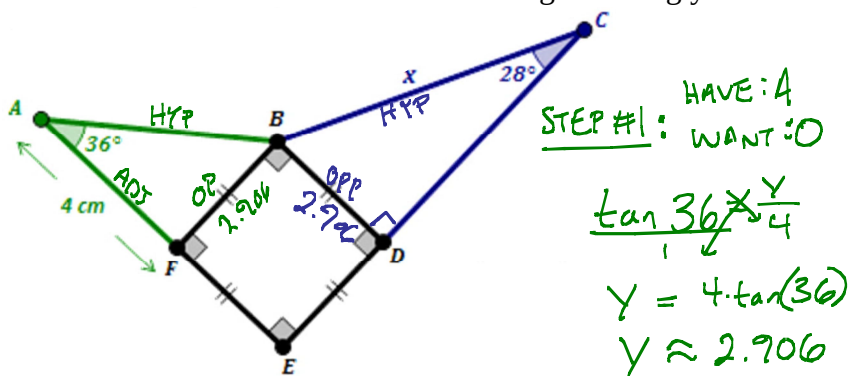
STEP #2: $\cos(34) = \frac{8}{x}$

$$8 = \frac{x \cdot \cos 34}{\cos 34}$$

$$9.65 \approx x$$

180-31-115
8/cos(34)
9.649743588

12. Find the unknown value x in the diagram using your knowledge of geometric figures and trigonometry.



STEP #2:

HAVE: OP
WANT: HYP

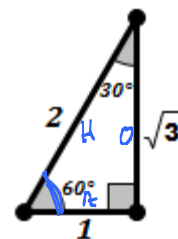
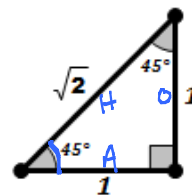
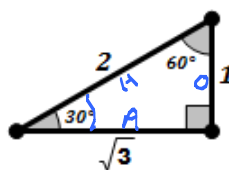
$$\sin(28) = \frac{2.906}{x}$$

$$2.906 = \frac{x \cdot \sin 28}{\sin 28}$$

$$6.19 \approx x$$

13. Using standard special right triangles find the exact value of the following.

a. $\sin(60^\circ) = \frac{OP}{HY} = \frac{\sqrt{3}}{2}$



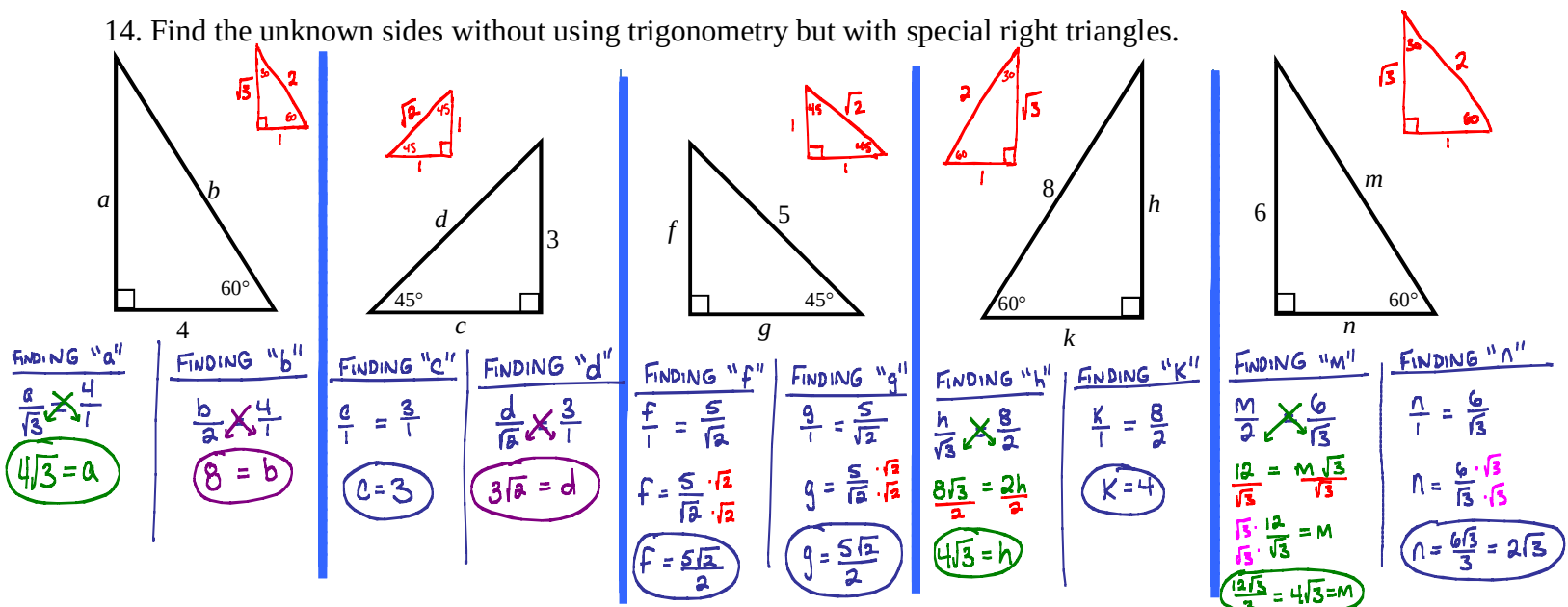
b. $\cos(45^\circ) = \frac{AD}{HY} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$

d. $\sin(30^\circ) = \frac{OP}{HY} = \frac{1}{2}$

c. $\tan(30^\circ) = \frac{OP}{AD} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3}$

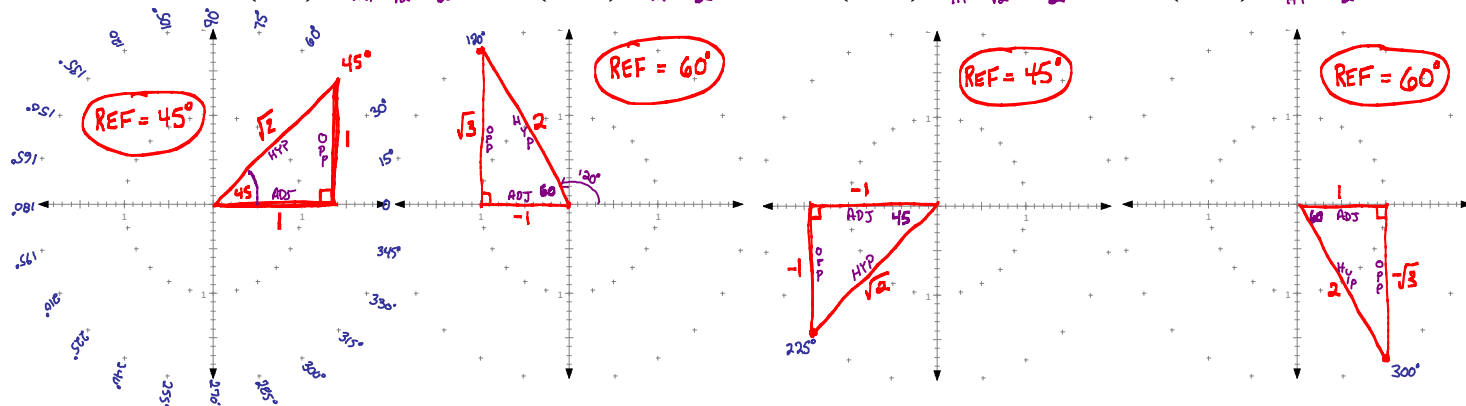
e. $\tan(45^\circ) = \frac{OP}{AD} = \frac{1}{1} = 1$

14. Find the unknown sides without using trigonometry but with special right triangles.

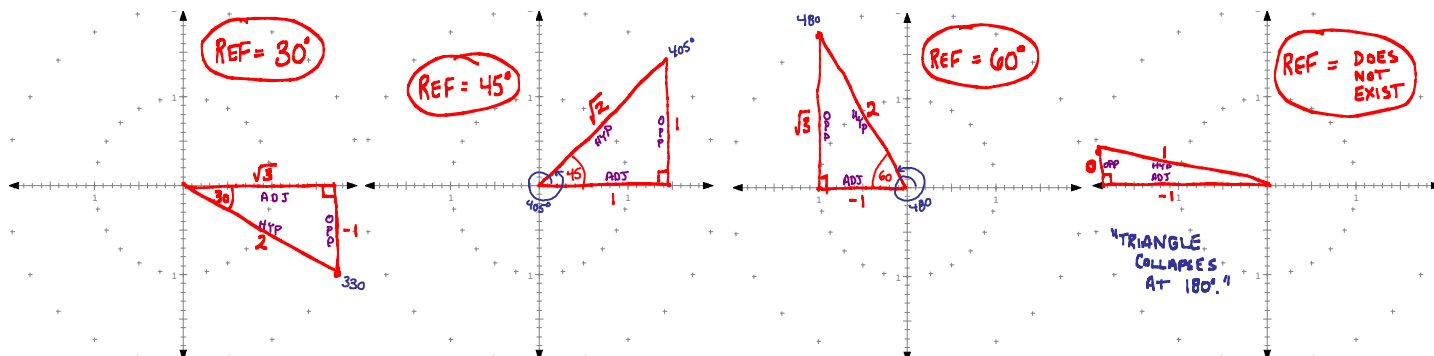


15. Find the reference angle & use special right triangles to determine the EXACT value of the following.

a. $\sin(45^\circ) = \frac{OPP}{HYP} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ b. $\sin(120^\circ) = \frac{OPP}{HYP} = \frac{\sqrt{3}}{2}$ c. $\cos(225^\circ) = \frac{ADJ}{HYP} = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$ d. $\cos(300^\circ) = \frac{ADJ}{HYP} = \frac{1}{2}$



e. $\tan(330^\circ) = \frac{OPP}{ADJ} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$ f. $\sin(405^\circ) = \frac{OPP}{HYP} = \frac{1}{2}$ g. $\cos(480^\circ) = \frac{ADJ}{HYP} = \frac{-1}{2}$ h. $\sin(180^\circ) = \frac{OPP}{HYP} = \frac{0}{1} = 0$



i. $\csc(330^\circ) = \frac{HYP}{OPP} = \frac{2}{-1} = -2$ j. $\sec(225^\circ) = \frac{HYP}{ADJ} = \frac{2}{-1} = -2$ k. $\cot(840^\circ) = \frac{ADJ}{OPP} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$ l. $\cos(-450^\circ) = \frac{ADJ}{HYP} = \frac{0}{1} = 0$

