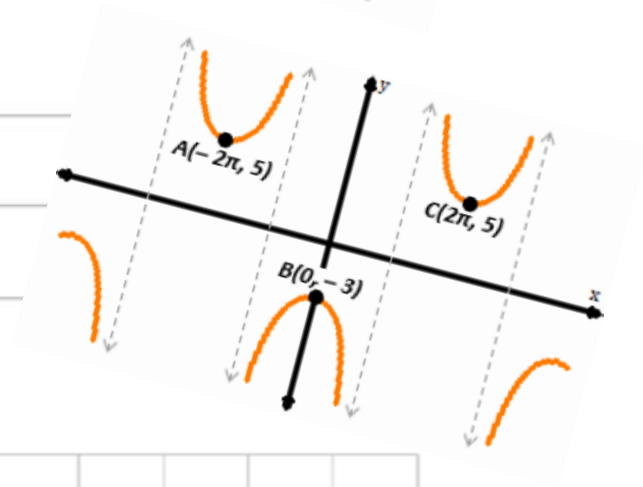
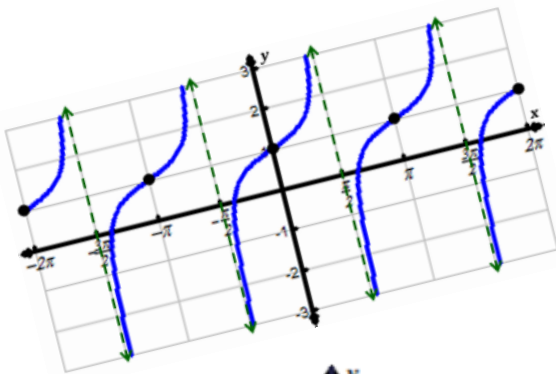


Pre-Calculus

Unit 2: Introduction to Trigonometric Functions



MAtThew M. Winking

Unit 2.1 - Introduction to Trig Functions

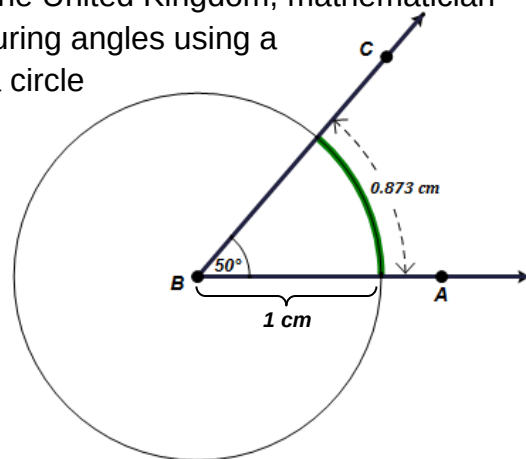
Degree & Radian Measures

Name: _____

- The Babylonian Degree method of measuring angles.** Around 1500 B.C. the Babylonians are credited with first dividing the circle up in to 360° . They used a base 60 (sexagesimal) system to count (i.e. they had 60 symbols to represent their numbers where as we only have 10 (a centesimal system of 0 through 9)). So, the number 360 was convenient as a multiple of 60. Additionally, according to Otto Neugebauer, an expert on ancient mathematics, there is evidence to support that the division of the circle in to 360 parts may have originated from astronomical events such as the division of the days of a year. So, that the earth moved approximately a degree a day around the sun. However, this would cause problems as years passed to keep the seasons accurately aligned in the calendar as there are 365.242 actual days in a year. Some ancient Persian calendars did actually use 360 days in their year further supporting this idea.

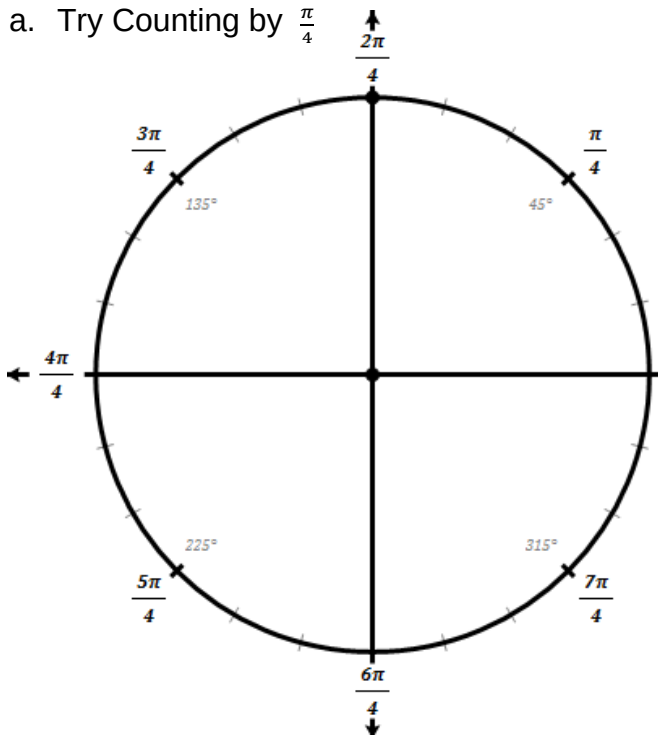


- The transition to Radian measure of angle:** Around 1700 in the United Kingdom, mathematician Roger Cotes saw some advantages in some situations to measuring angles using a radian system. A radian system simply put, drops a unit circle (a circle with a radius of 1) on to an angle such that the center is at the vertex and the length of the intercepted arc is the radian measure. So, a full circle of 360° is equivalent to $2\pi \cdot (1)$ radians. In the example at the right, an angle of 50° is shown. Then, a circle that has a radius of 1 cm is drawn with its center at the vertex. Finally, the intercepted arc length is determined to be approximately 0.873 or more precisely $\frac{5\pi}{18}$ radians.

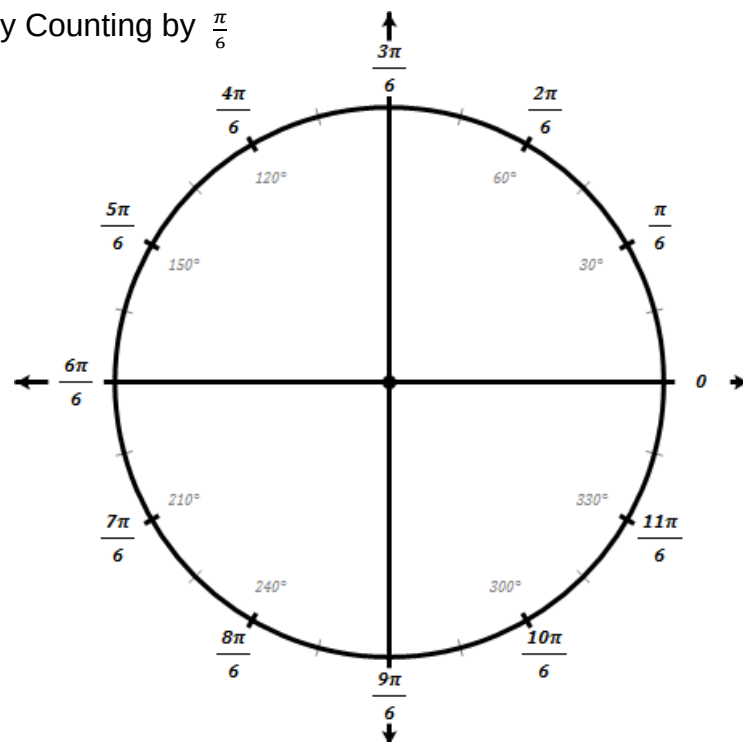


3. Common Radian Measures

- Try Counting by $\frac{\pi}{4}$



- Try Counting by $\frac{\pi}{6}$

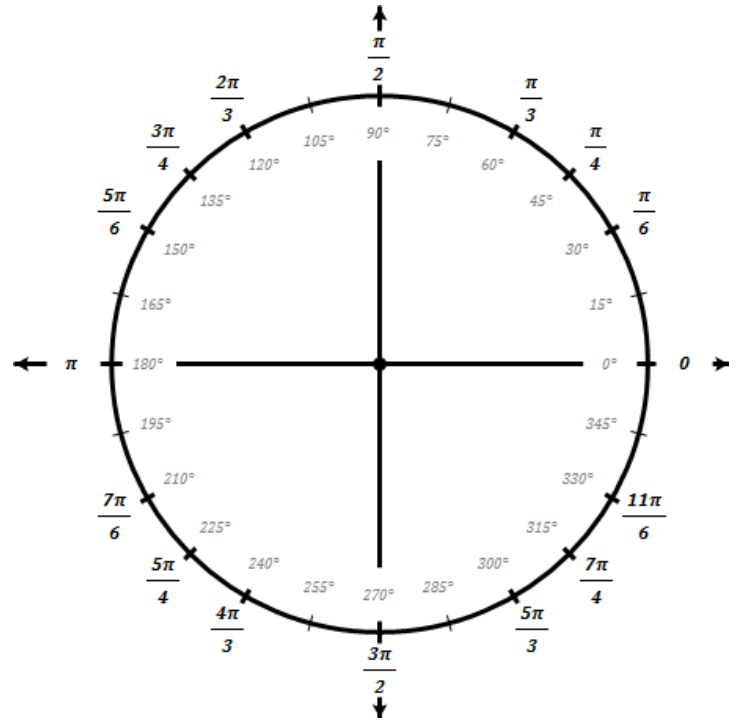


6. Determine the exact values of the following:

a. $\sin\left(\frac{\pi}{3}\right) =$

b. $\cos\left(\frac{3\pi}{4}\right) =$

c. $\tan\left(\frac{11\pi}{6}\right) =$



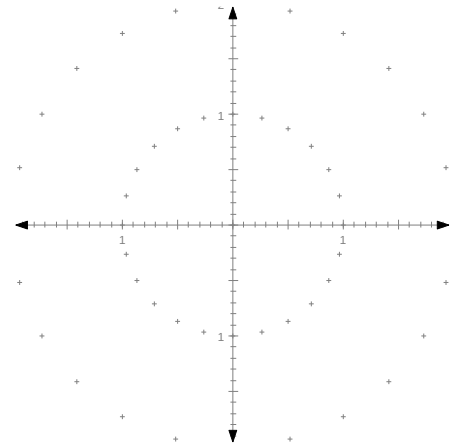
7. Determine the exact values of the following:

a. Given $\sin(x) = 0.361$ and x is in the first quadrant determine the following:

i. $\sin(x + \pi) =$

ii. $\sin(\pi - x) =$

iii. $\sin(2\pi - x) =$

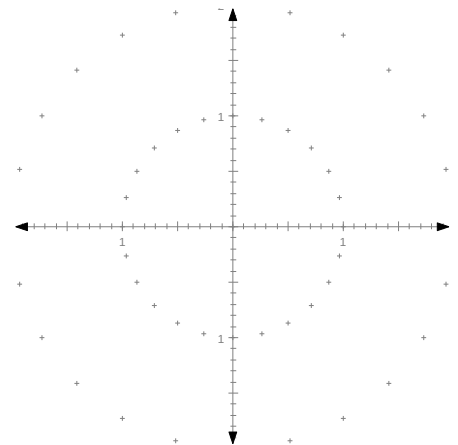


b. Given $\cos(x) = p$ and x is in the first quadrant determine the following:

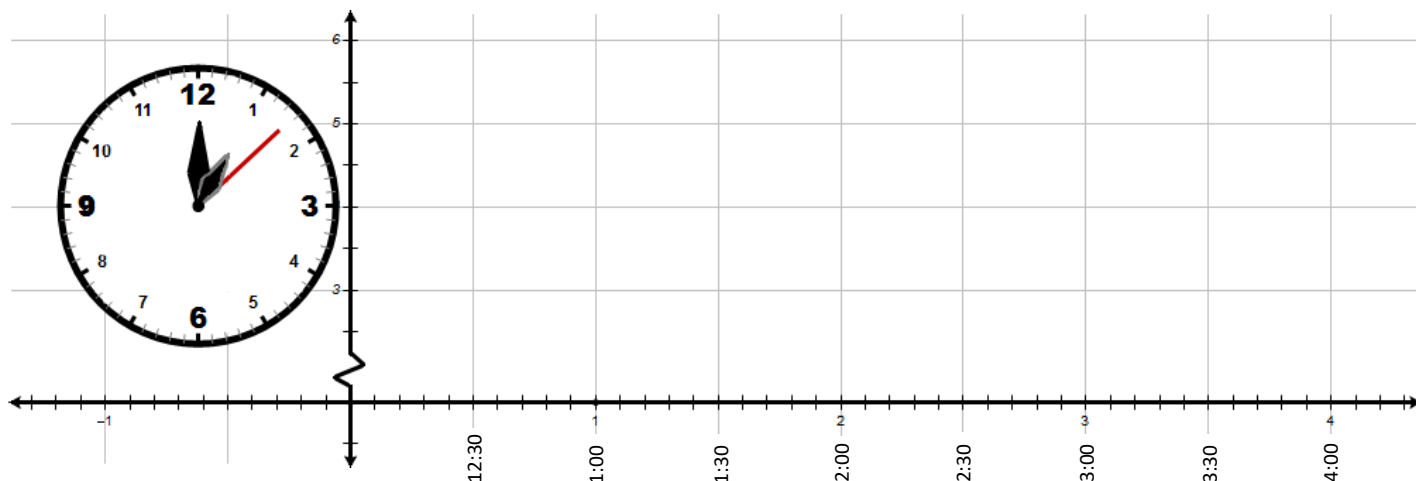
i. $\cos(x + \pi) =$

ii. $\cos (\pi - x) =$

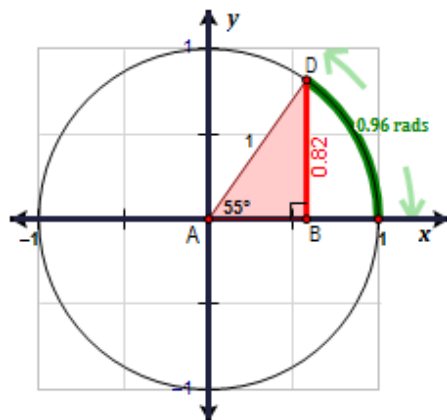
iii. $\cos(2\pi - x) =$



Let's consider a large clock that is hung on a wall such that the center of the clock is exactly 4 feet above the ground. The tip of the minute hand moves between a maximum of 5 feet and a minimum of 3 feet off the ground. Create a plot of the height of the tip of the minute hand starting from 12:00 am to 4:00 am. (The picture of the clock shows a time of roughly 1:00 am)

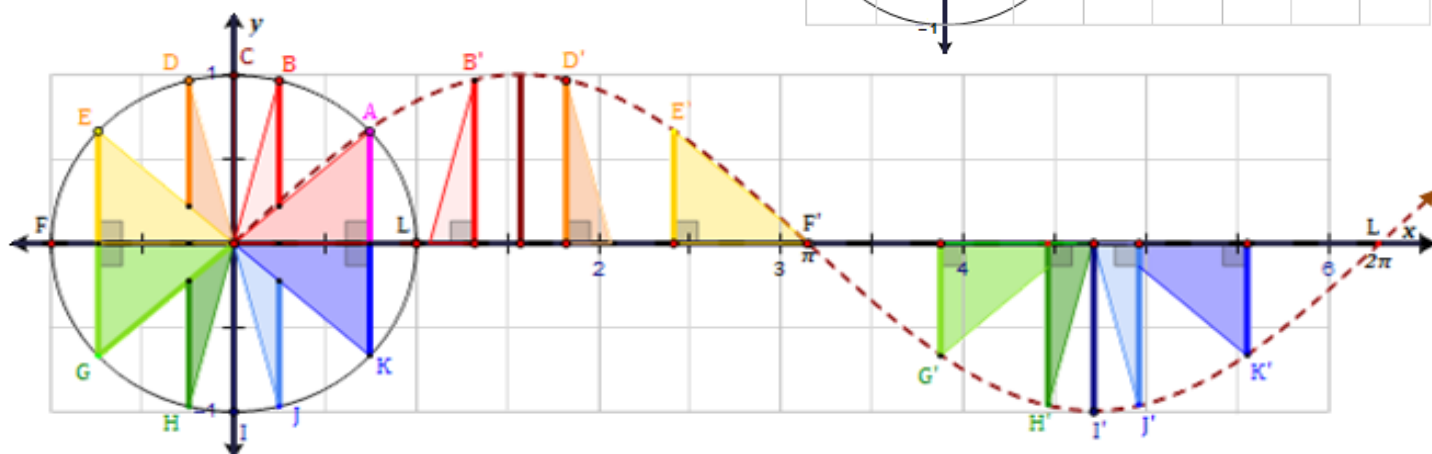
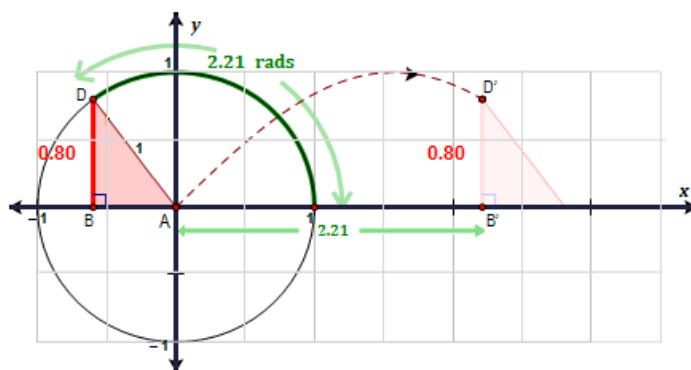


This plot should generate a **cosine wave** graph (see the animation on Matt's Math Labs). Hopefully, this should make you ask, "How do functions that represent ratios of triangles generate smooth waves?"

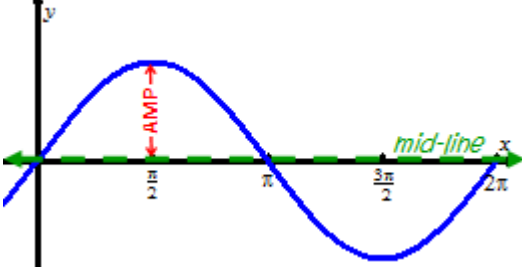
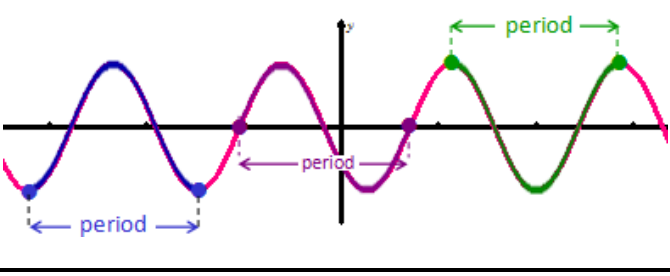
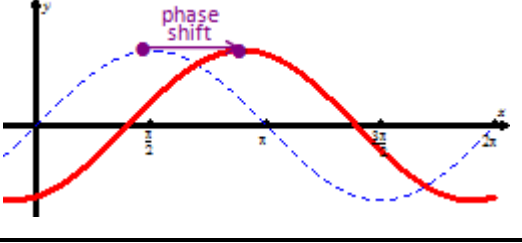
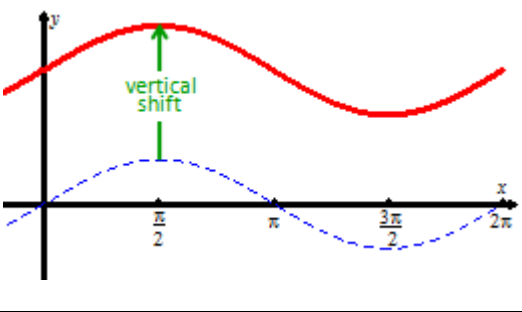


How Right Triangles create a WAVE: First consider a right triangle drawn inside of a unit circle like the one shown at the right. Determine the value of $\sin(A)$. Try the example at the right find $\sin(55^\circ)$ which is approximately the same as finding $\sin(0.96 \text{ rads})$.

Then, if we plot each of point where x is the measure of the angle of the unit circle in radians and y is value of $\sin(A)$ which is essentially the height of the right triangle for the given angle. The resulting locus of points generates a Sine Wave. The diagram below shows several more right triangles that map out more of the sine wave.

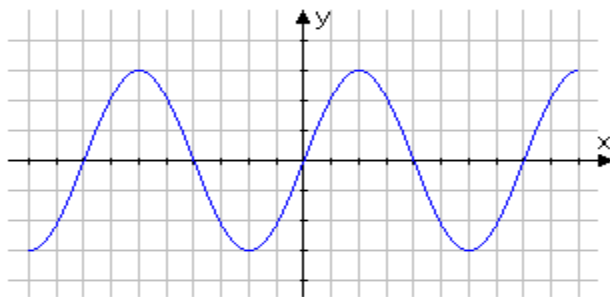


A Sine Wave is commonly described by 4 components.

Parts of a Sine Wave	Visual
Amplitude: The amplitude of a wave is the distance from the midline to highest point of the wave. (half of the vertical displacement). $\text{Amplitude} = \frac{(\text{Highest } Y - \text{Lowest } Y)}{2}$	
Period: The period of the wave is the horizontal distance of one complete cycle. $\text{Period} = (\text{Crest } X) - (\text{Previous Crest } X)$ $\text{Period} = (\text{Trough } X) - (\text{Previous Trough } X)$	
Phase Shift: The amount the wave is shifted right or left $\text{Phase Shift} = \frac{(\text{Crest } X) + (\text{Previous Trough } X)}{2}$	
Vertical Shift: The amount the wave is shifted up or down $\text{Vertical Shift} = \frac{(\text{Highest } Y + \text{Lowest } Y)}{2}$	

1. Find a possible equation of the following graphs.

a.



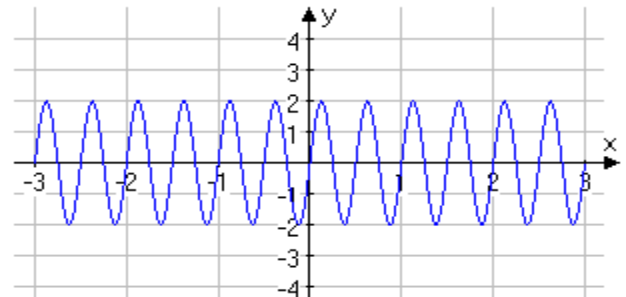
Amplitude =

Period =

Phase Shift =

Vertical Shift =

b.



Amplitude =

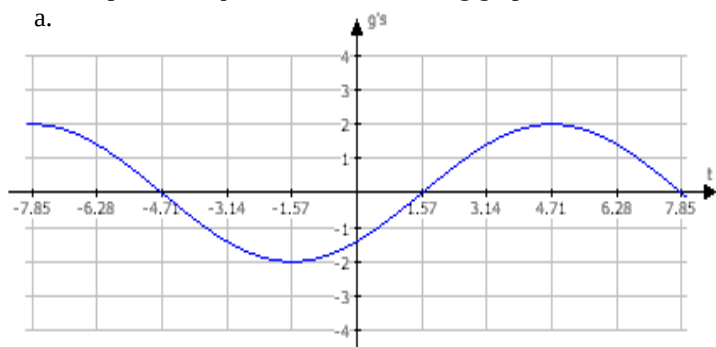
Period =

Phase Shift =

Vertical Shift =

2. Find a possible equation of the following graphs.

a.



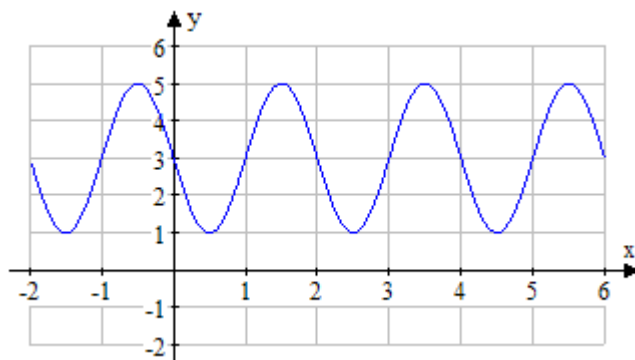
Amplitude =

Period =

Phase Shift =

Vertical Shift =

b.



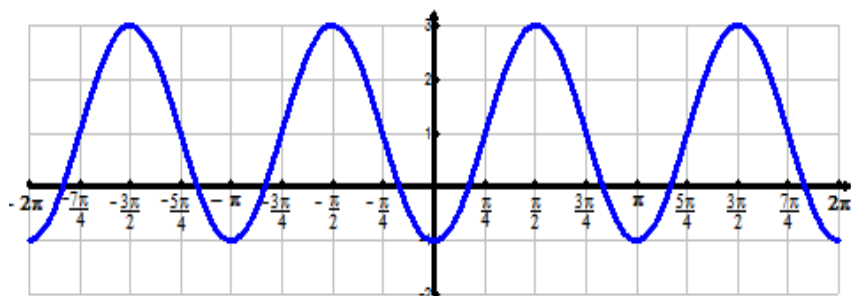
Amplitude =

Period =

Phase Shift =

Vertical Shift =

c.



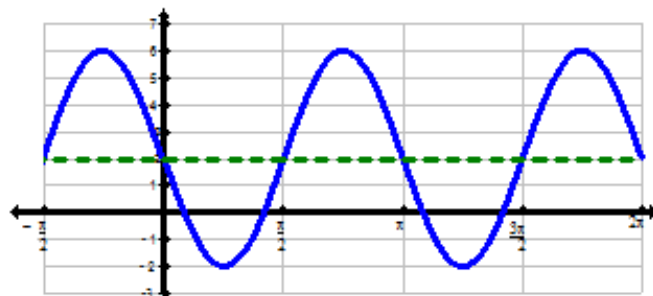
Amplitude =

Period =

Phase Shift =

Vertical Shift =

d.



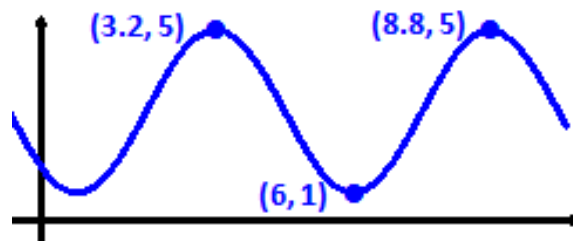
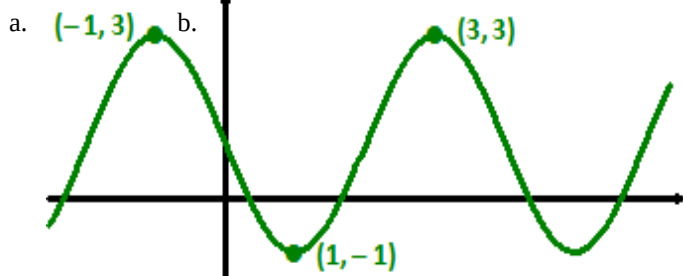
Amplitude =

Period =

Phase Shift =

Vertical Shift =

3. Find a possible equation of the following graphs.



Amplitude =

Period =

Phase Shift =

Vertical Shift =

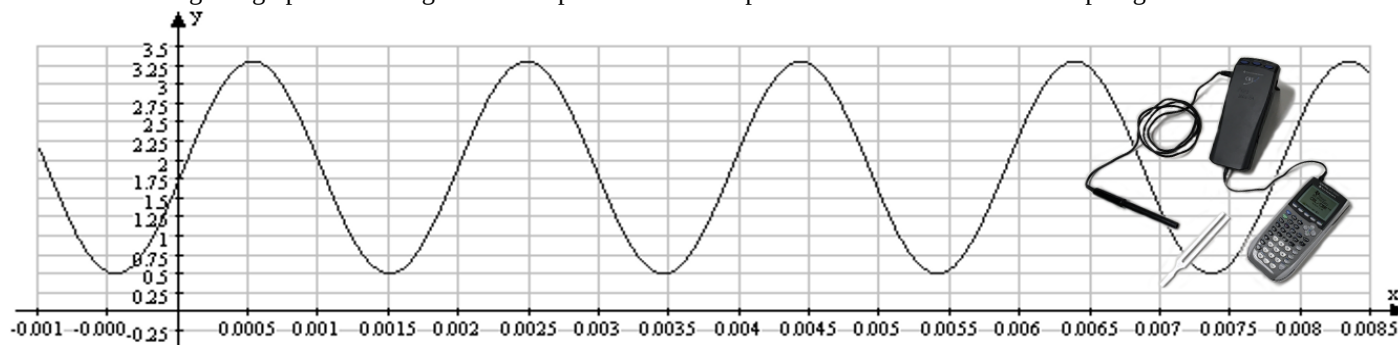
Amplitude =

Period =

Phase Shift =

Vertical Shift =

4. The following is a graph of a tuning fork held up to a CBL microphone connected to a TI-84 Graphing Calculator.



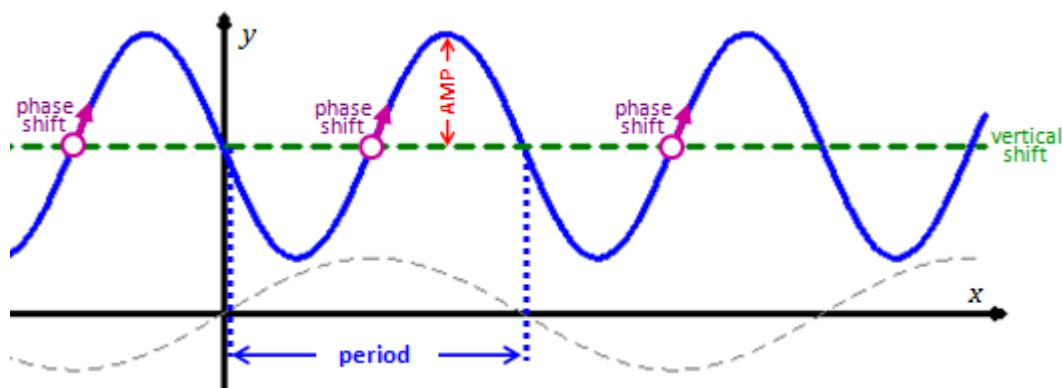
a. What is the amplitude?

b. What is the Vertical Shift?

c. What is the Period?

d. What is a possible Phase Shift?

e. What is a possible equation of the sound wave?



$$y = a \cdot \sin(b(x - c)) + d$$

a = Amplitude

c = Phase Shift

$$\frac{2\pi}{b} = \text{Period}$$

d = Vertical Shift

1. Determine the following parts of the graph described by following equation.

$$y = 4 \sin(\pi(x - 3)) + 5$$

2. Determine the following parts of the graph described by following equation.

$$y = 3 \sin\left(\frac{1}{2}\left(x - \frac{3\pi}{2}\right)\right) + 4$$

Amplitude =

Period =

Phase Shift =

Vertical Shift =

Amplitude =

Period =

Phase Shift =

Vertical Shift =

3. Determine the following parts of the graph described by following equation.

$$y = -5 \sin(2x + 6) - 1$$

Amplitude =

Period =

Phase Shift =

Vertical Shift =

4. Determine the following parts of the graph described by following equation.

$$y = 4 \sin(\pi x - 2) + 6$$

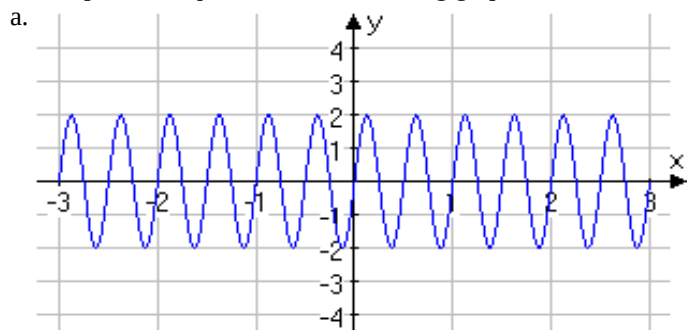
Amplitude =

Period =

Phase Shift =

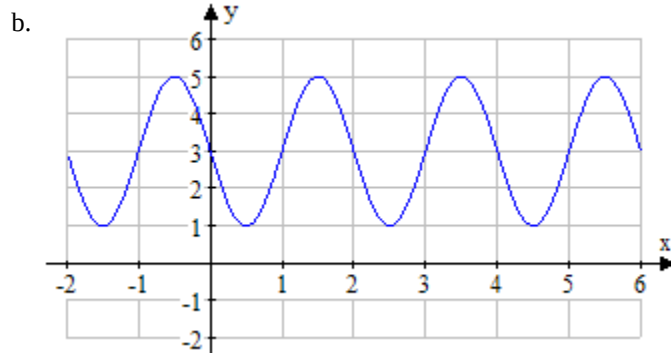
Vertical Shift =

1. Find a possible equation of the following graphs.



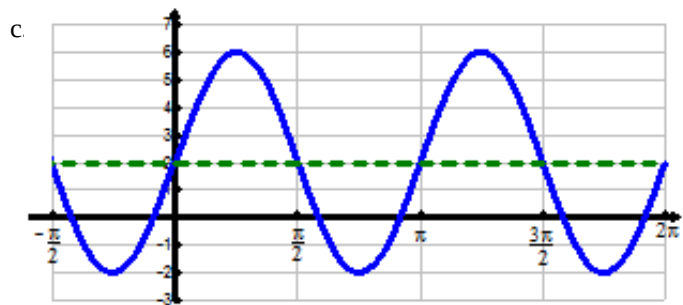
Amp = Per = Phase shift = Vert. Shift =

Equation as a Sine Wave:



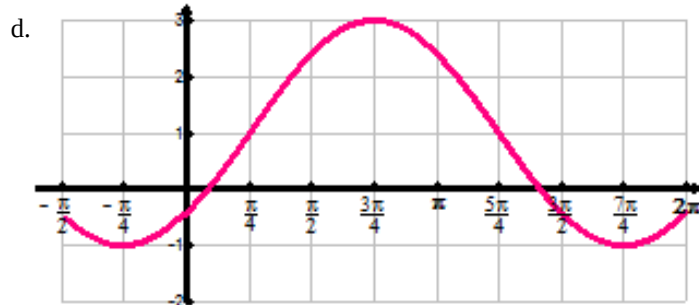
Amp = Per = Phase shift = Vert. Shift =

Equation as a Sine Wave:



Amp = Per = Phase shift = Vert. Shift =

Equation as a Sine Wave:



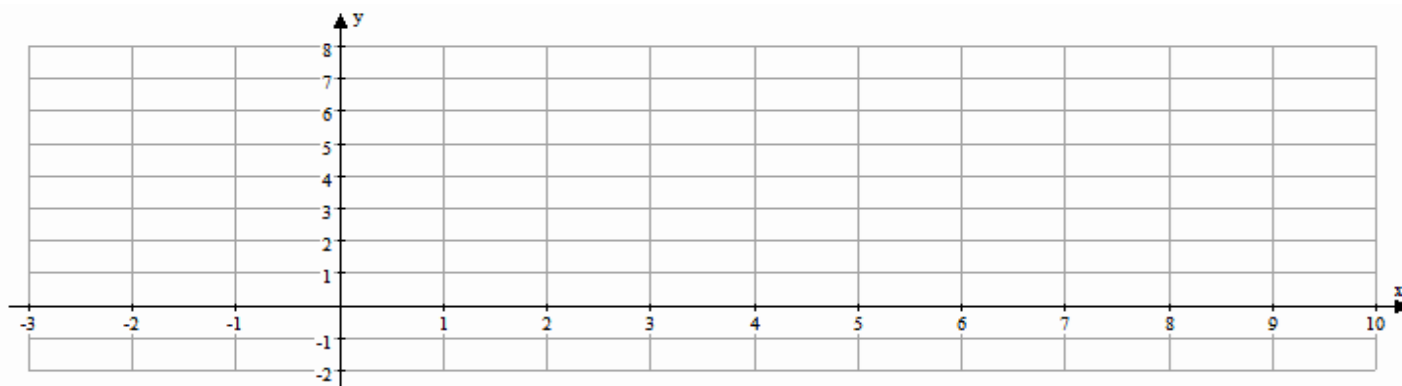
Amp = Per = Phase shift = Vert. Shift =

Equation as a Sine Wave:

2. Graph the following equation.

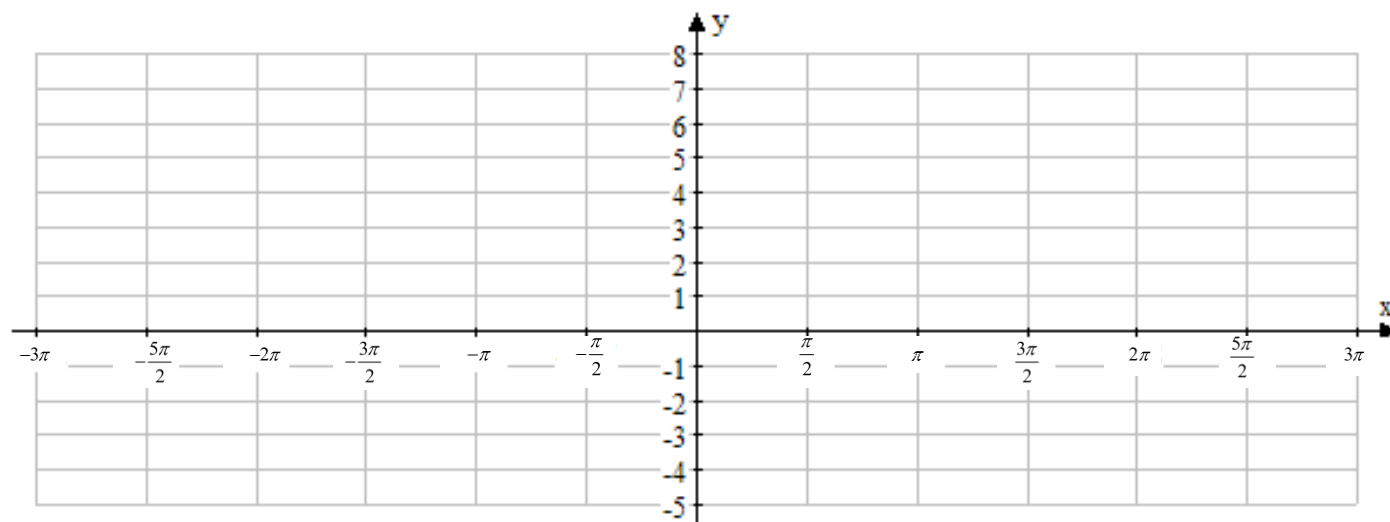
a.

$$y = 2 \sin\left(\frac{\pi}{2}(x - 3)\right) + 5$$

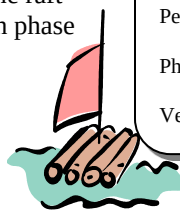


b.

$$y = 3 \sin\left(\frac{1}{2}\left(x - \frac{3\pi}{2}\right)\right) + 4$$



3. The Coast Guard observes a raft floating on the water bobbing up and down a total of 8 feet. Beginning at the top of the wave, the raft completes a full cycle every 5 seconds. Write an equation with phase shift 0 to represent the height of the raft after t seconds.



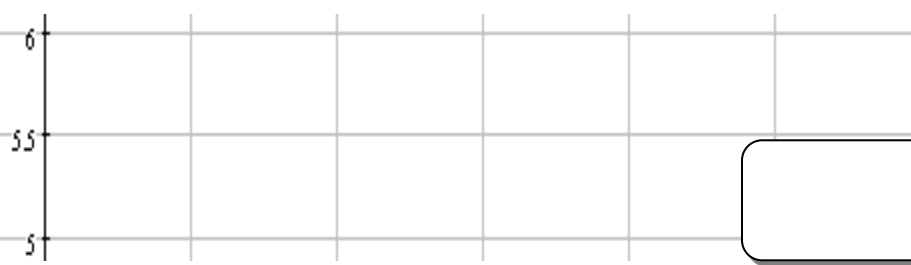
Amplitude =

Period =

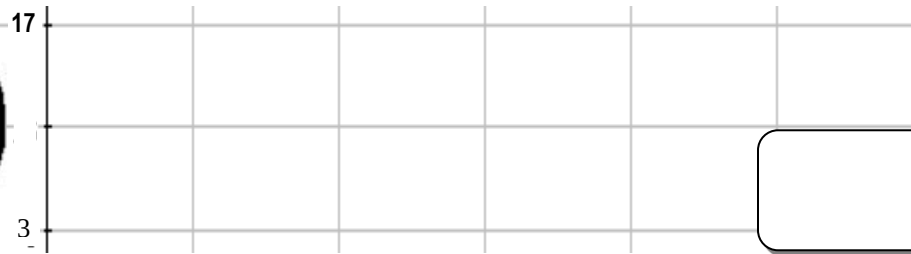
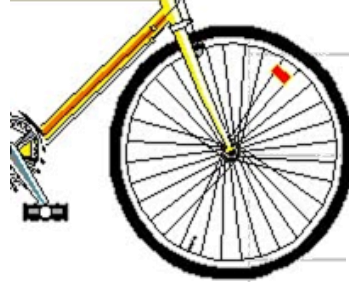
Phase Shift =

Vertical Shift =

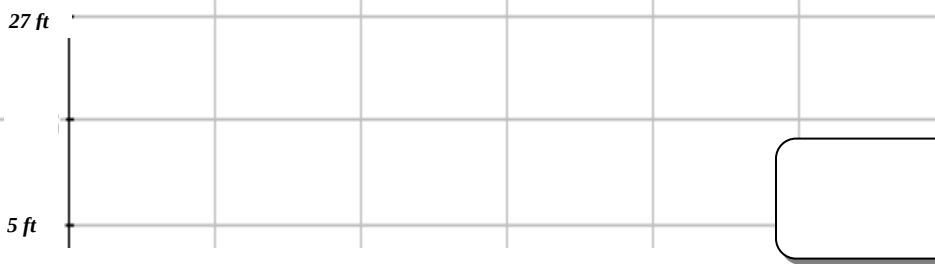
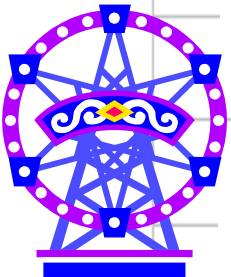
4. An insect is stuck on the very tip of a second hand of a wall clock for a couple of minutes. The tip of the second hand is 5 feet above the floor at its lowest point and 6 feet above the floor at its highest. The bug landed on the second hand at exactly 15 seconds after 10:10 pm. Describe the bug's height as a function of time. (remember a second hand takes exactly 60 seconds to complete a full cycle)



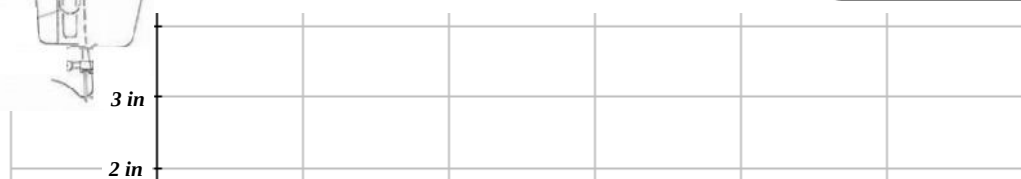
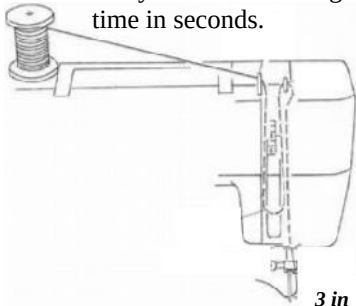
5. A reflector on a bicycle tire is going around with a bike tire one complete revolution every 0.4 seconds. At its highest point the reflector is 17 inches off the ground. At its lowest point it is 3 inches off the ground. Write an equation that describes the height of the reflector as a function of time if the reflector starts out at its highest point.



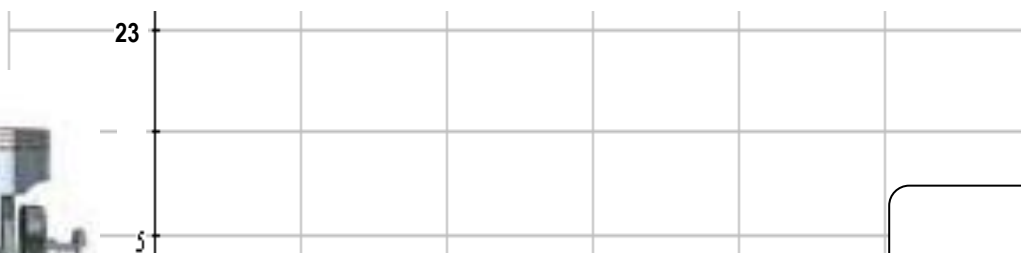
6. A person gets on a Ferris wheel that starts off 5 ft above ground and at its highest is 27 ft above ground. If the Ferris wheel completes a full rotation in 40 seconds. The person starts at the bottom. Write an equation that describes the height of the rider as a function of time.



7. A sewing machine needle is bouncing up and down between 3 and 2 inches off the table. If the needle completes a full cycle every 1 second and begins at the top of a cycle then write an equation that describes the height of the needle as a function of time in seconds.



8. A piston inside of an engine turns a crank shaft at 2000 times a second or once every .0005 seconds. The top of the piston is 20 inches above the ground at its lowest point and 23 inches above ground at its highest point. Create a function that describes the piston's height as a function of time in seconds (starting with the piston at its lowest point)

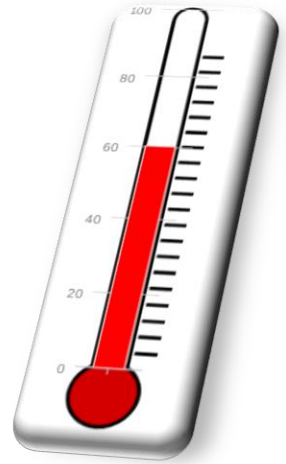


9. The average high temperature of a day in Atlanta can be modeled by the equation:

$$T = 20\sin(0.017(d + 1.816)) + 69$$

'**T**' represents the temperature in Fahrenheit and '**d**' is day number of the year (e.g. February 2nd would be day 33)

- Using the model what is the average high temperature on February 28th?
- Using the model what is the lowest high temperature of the year?
- Using the model what is the highest high temperature of the year?

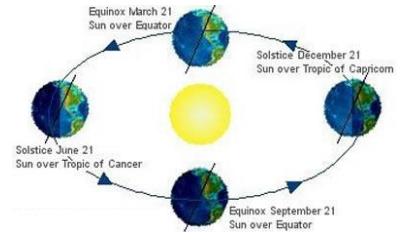


10. The number of minutes of sun each day in Louisiana can be modeled by the equation:

$$M = 113.3\sin(0.017(d + 1.319)) + 727$$

'**M**' represents the number of minutes of sunshine each day and '**d**' is day number of the year (e.g. February 2nd would be day 33)

- Using the model how many minutes of sunshine should there be on February 28th?
- Using the model how many minutes of sunshine are there on the longest day?



A Cosine Wave is commonly described by 4 components. The only difference between a Sine Wave and a Cosine Wave is the Phase Shift.

Parts of a Cosine Wave	Visual
Amplitude: The amplitude of a wave is the distance from the midline to highest point of the wave. (half of the vertical displacement.) $\text{Amplitude} = \frac{(\text{Highest } Y - \text{Lowest } Y)}{2}$	
Period: The period of the wave is the horizontal distance of one complete cycle. $\text{Period} = (\text{Crest } X) - (\text{Previous Crest } X)$ $\text{Period} = (\text{Trough } X) - (\text{Previous Trough } X)$	
Phase Shift: The amount the wave is shifted right or left $\text{Phase Shift} = \text{Crest } X$	
Vertical Shift: The amount the wave is shifted up or down $\text{Vertical Shift} = \frac{(\text{Highest } Y + \text{Lowest } Y)}{2}$	

$$y = a \cdot \cos(b(x - c)) + d$$

a = Amplitude

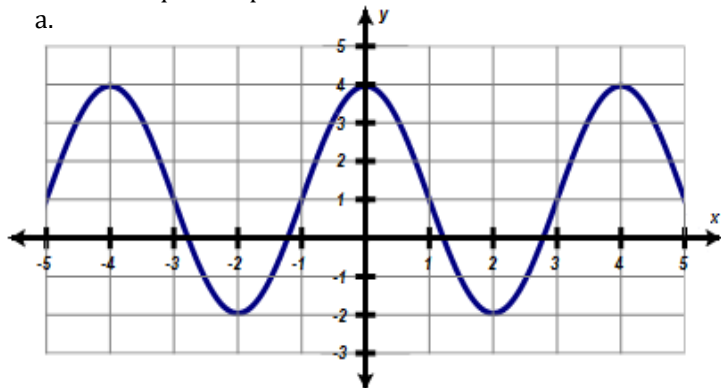
c = Phase Shift

$$\frac{2\pi}{b} = \text{Period}$$

d = Vertical Shift

1. Find the requested parameters of each COSINE wave shown in the graphs and then a possible equation.

a.



Amplitude =

Period =

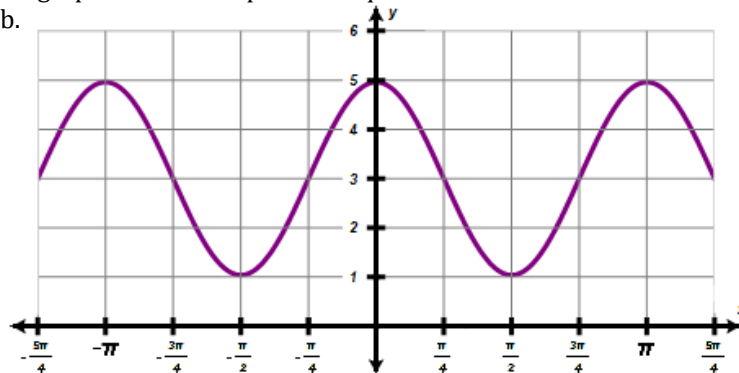
Phase Shift =

Vertical Shift =

Equation:

(Using Cosine)

b.



Amplitude =

Period =

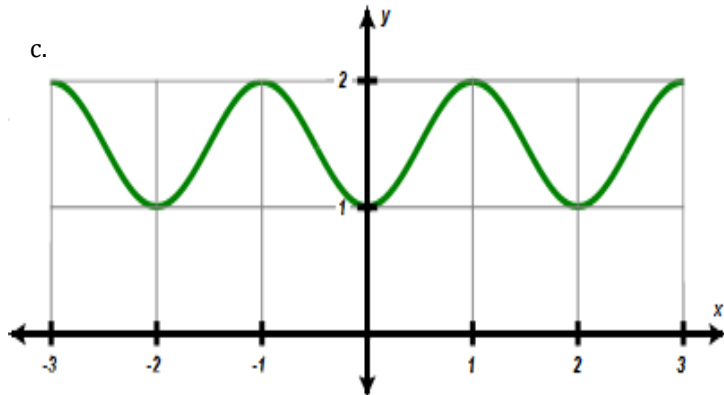
Phase Shift =

Vertical Shift =

Equation:

(Using Cosine)

c.



Amplitude =

Period =

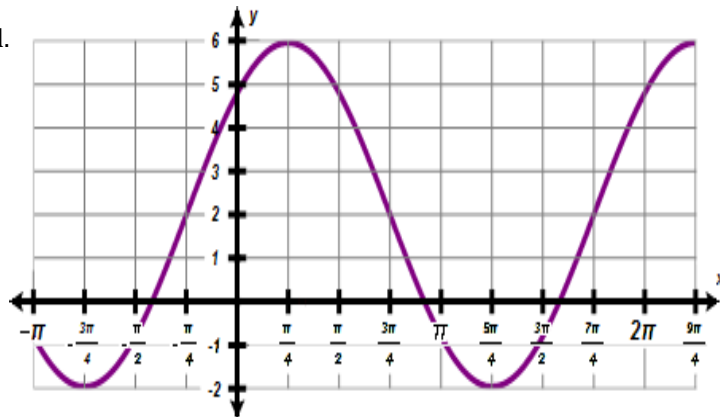
Phase Shift =

Vertical Shift =

Equation:

(Using Cosine)

d.



Amplitude =

Period =

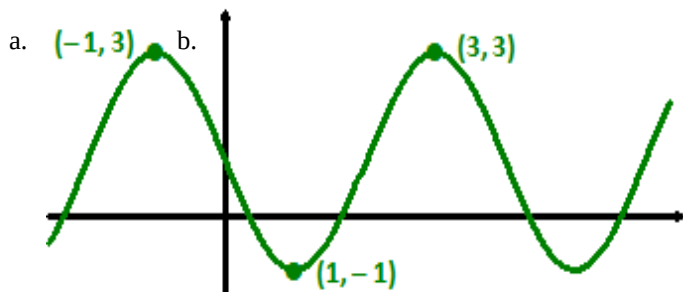
Phase Shift =

Vertical Shift =

Equation:

(Using Cosine)

2. Find the requested parameters and a possible equation of the following graphs using Cosine.



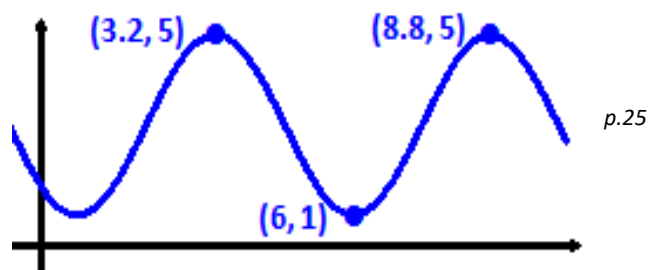
Amplitude =

Period =

Phase Shift =

Vertical Shift =

Equation:
(Using Cosine)



Amplitude =

Period =

Phase Shift =

Vertical Shift =

Equation:
(Using Cosine)

3. Find the requested parameters based on the following equations.

a. $y = 6\cos(2(x+3))$

Amplitude =

Period =

Phase =

Vertical =

c. $y = 2\cos(3x-12)-5$

Amplitude =

Period =

Phase =

Vertical =

b. $y = \pi \cos\left(\frac{\pi}{3}(x-5)\right) + 1$

Amplitude =

Period =

Phase =

Vertical =

d. $y = -3\cos(\pi x + 5) + 4$

Amplitude =

Period =

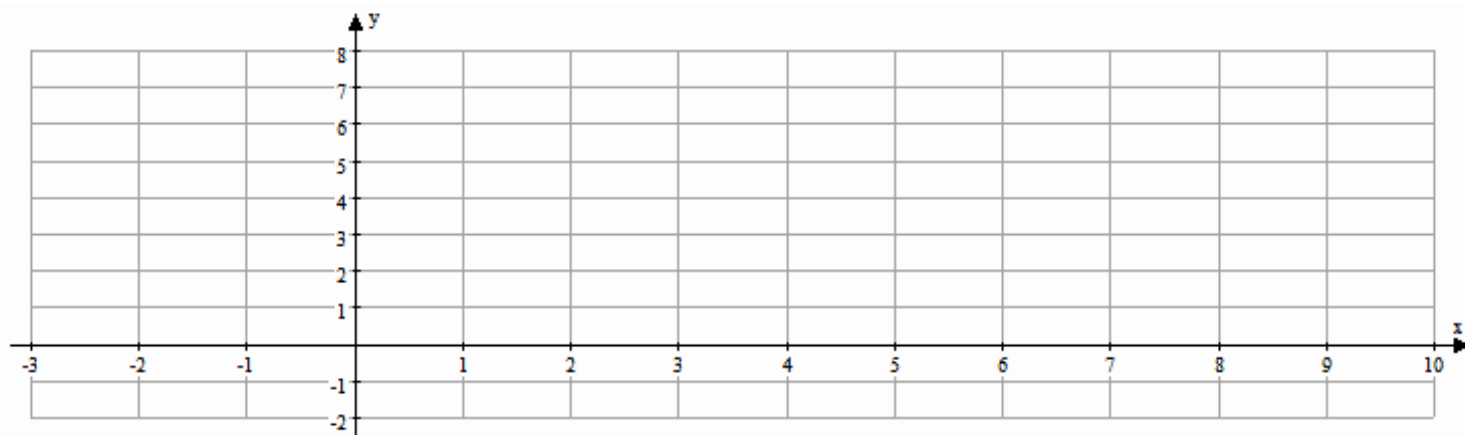
Phase =

Vertical =

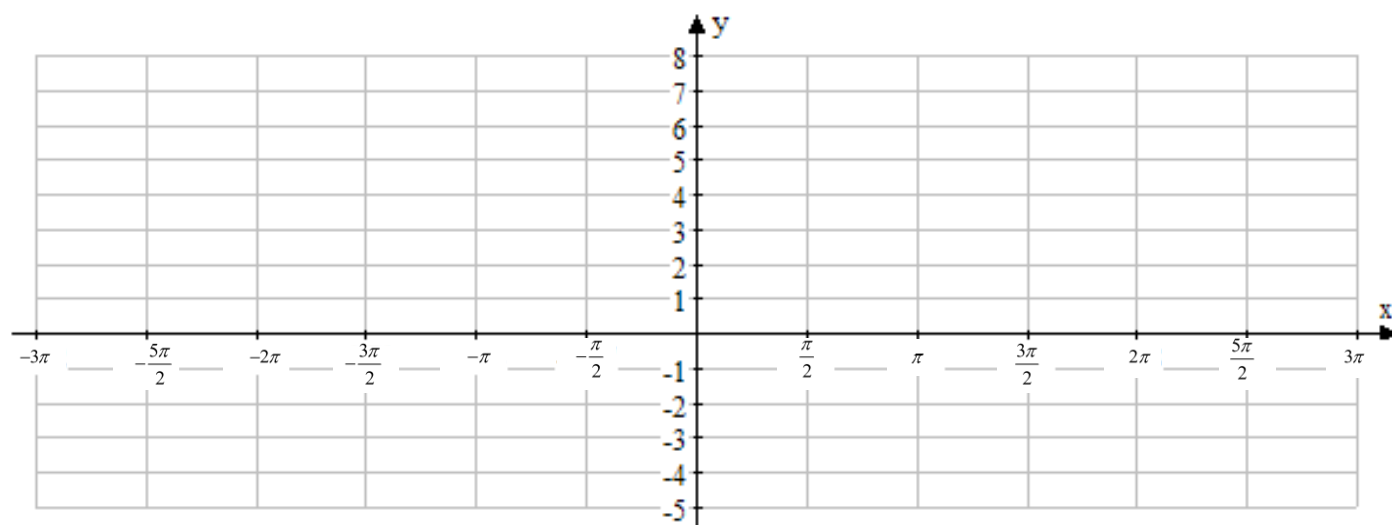
4. Graph the following equation.

a.

$$y = 3\cos\left(\frac{\pi}{2}(x-1)\right) + 6$$



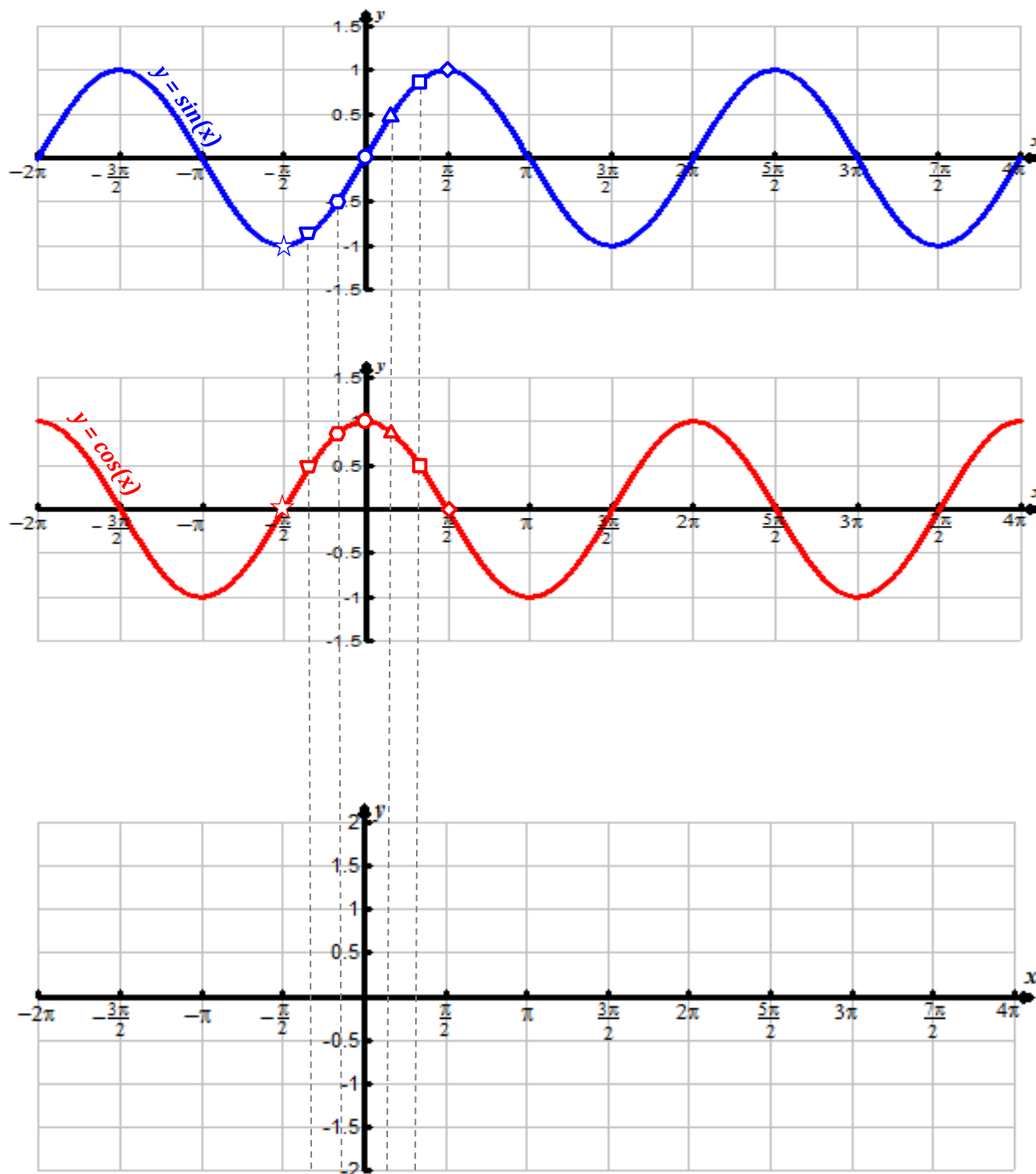
b. $y = 2\cos\left(2\left(x - \frac{\pi}{2}\right)\right) + 5$



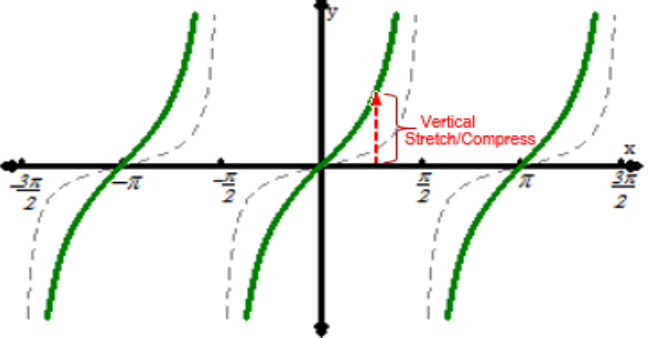
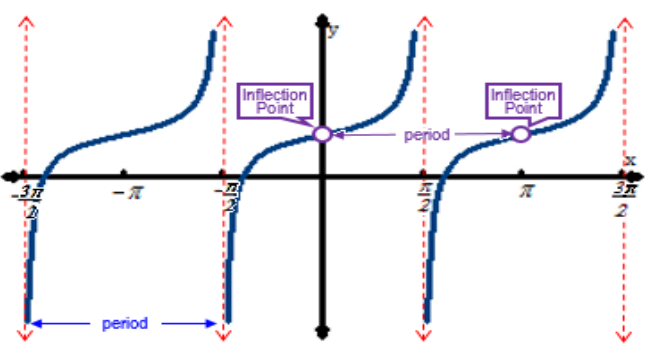
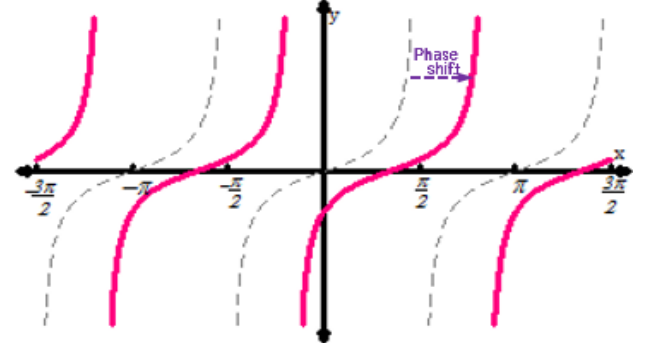
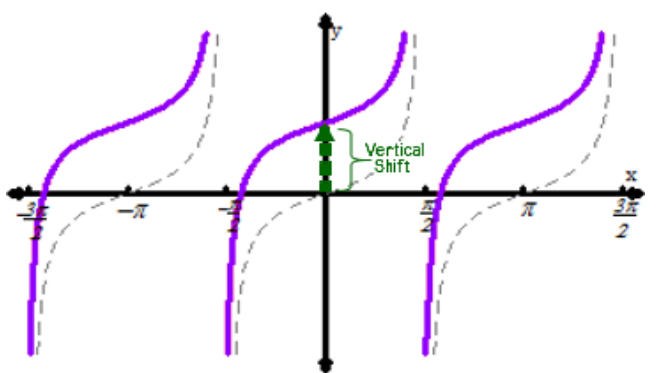
Recall, $\sin(\theta) = \frac{opp}{hyp}$, $\cos(\theta) = \frac{adj}{hyp}$, and $\tan(\theta) = \frac{opp}{adj}$. Use this to show:

$$\frac{\sin(\theta)}{\cos(\theta)} = \quad \quad \quad = \tan(\theta)$$

Use the above relationship to help you graph $y = \tan(x)$.



A tangent graph is commonly described by 4 components.

Parts of a Tangent and Cotangent Graph	Visual
Vertical Stretch/Compress: Tangent and Cotangent do not have an amplitude because their range is all real numbers. However, they can be transformed by being vertically stretched or compressed.	
Period: The period of the graph is the horizontal distance of one complete cycle. $\text{Period} = (\text{Asymptote } X) - (\text{Previous Asymptote } X)$ $\text{Period} = (\text{Inflection } X) - (\text{Previous Inflection } X)$	
Phase Shift: The amount the graph is shifted right/ left $\text{Phase Shift} = \text{Inflection } X$	
Vertical Shift: The amount the graph is shifted up/down $\text{Vertical Shift} = \text{Inflection } Y$	

$$y = a \cdot \tan(b(x - c)) + d$$

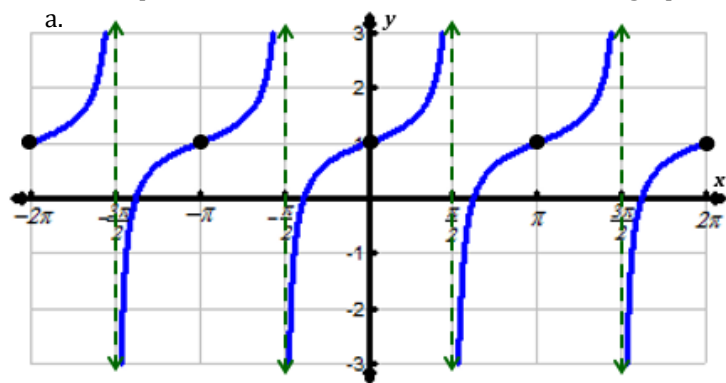
a = Vertical Stretch/Compress

c = Phase Shift

$\frac{\pi}{b} = \text{Period}$

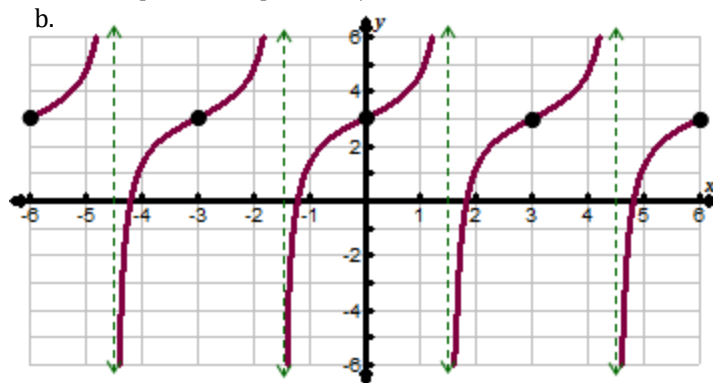
d = Vertical Shift

1. Find the parameters of each **TANGENT/COTANGENT** graph and determine a possible equation. (assume there is no vertical stretch)



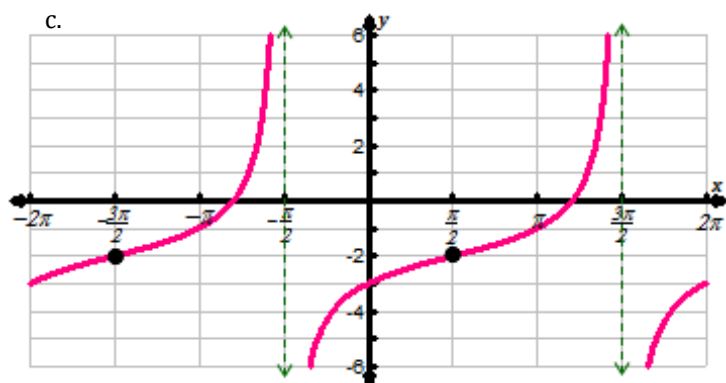
Period = Phase = Vertical =
 Shift Shift

Possible
Equation:
(Using Tangent)



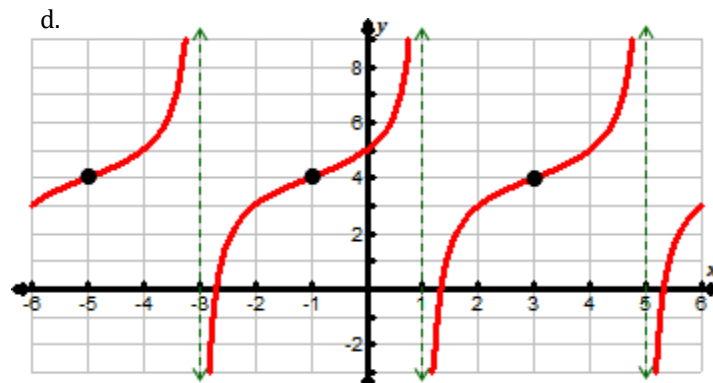
Period = Phase = Vertical =
 Shift Shift

Possible
Equation:
(Using Tangent)



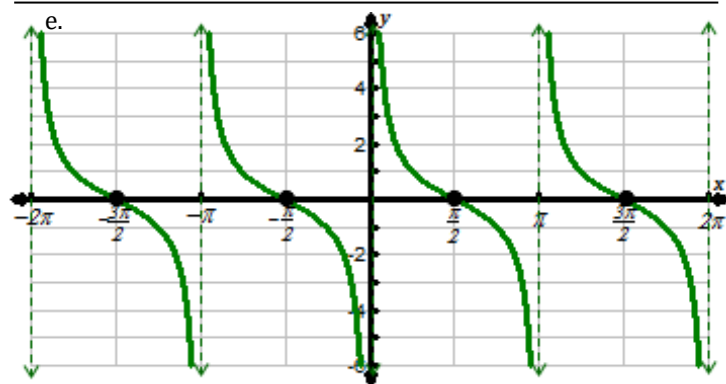
Period = Phase = Vertical =
 Shift Shift

Possible
Equation:
(Using Tangent)



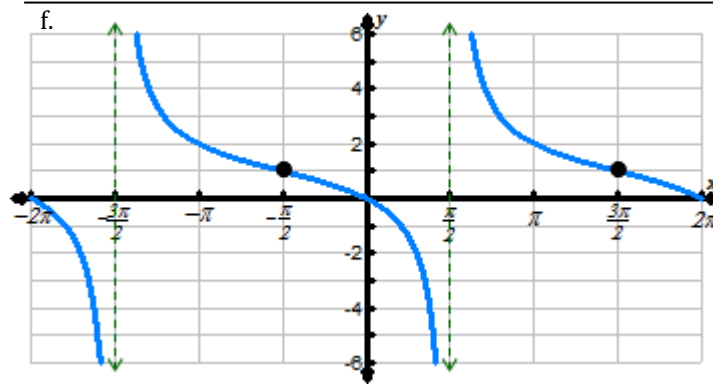
Period = Phase = Vertical =
 Shift Shift

Possible
Equation:
(Using Tangent)



Period = Phase = Vertical =
 Shift Shift

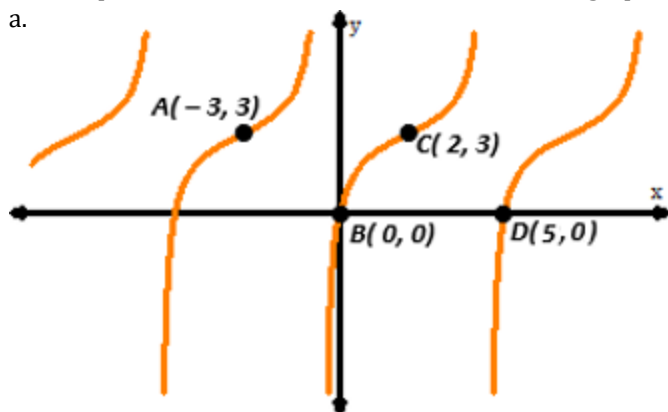
Possible
Equation:
(Using Cotangent)



Period = Phase = Vertical =
 Shift Shift

Possible
Equation:
(Using Cotangent)

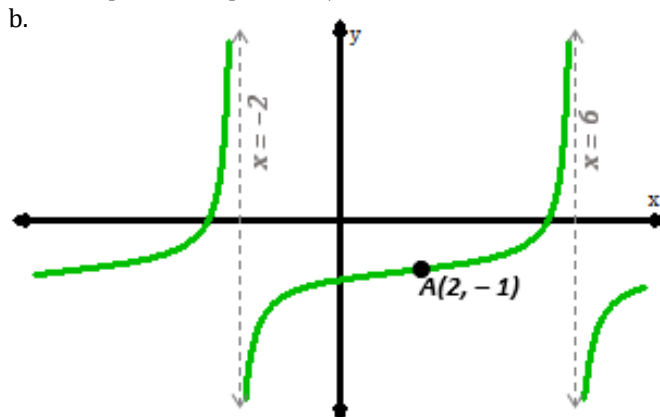
2. Find the parameters of each **TANGENT/COTANGENT** graph and determine a possible equation. (assume there is no vertical stretch)



Period = Phase = Vertical =
 Shift Shift

Possible
Equation:

(Using Tangent)



Period = Phase = Vertical =
 Shift Shift

Possible
Equation:

(Using Tangent)

3. Find the requested **TANGENT/COTANGENT** parameters based on the following equations.

a. $y = 6 \tan(4(x + 2)) - 1$

Vertical =
Stretch/Compress

Period =

Phase =
Shift

Vertical =
Shift

c. $y = 4 \tan(4x - 12) - 5$

Vertical =
Stretch/Compress

Period =

Phase =
Shift

Vertical =
Shift

b. $y = \pi \cot\left(\frac{\pi}{3}(x - 5)\right) + 4$

Vertical =
Stretch/Compress

Period =

Phase =
Shift

Vertical =
Shift

d. $y = -2 \tan(\pi x + 3) + 6$

Vertical =
Stretch/Compress

Period =

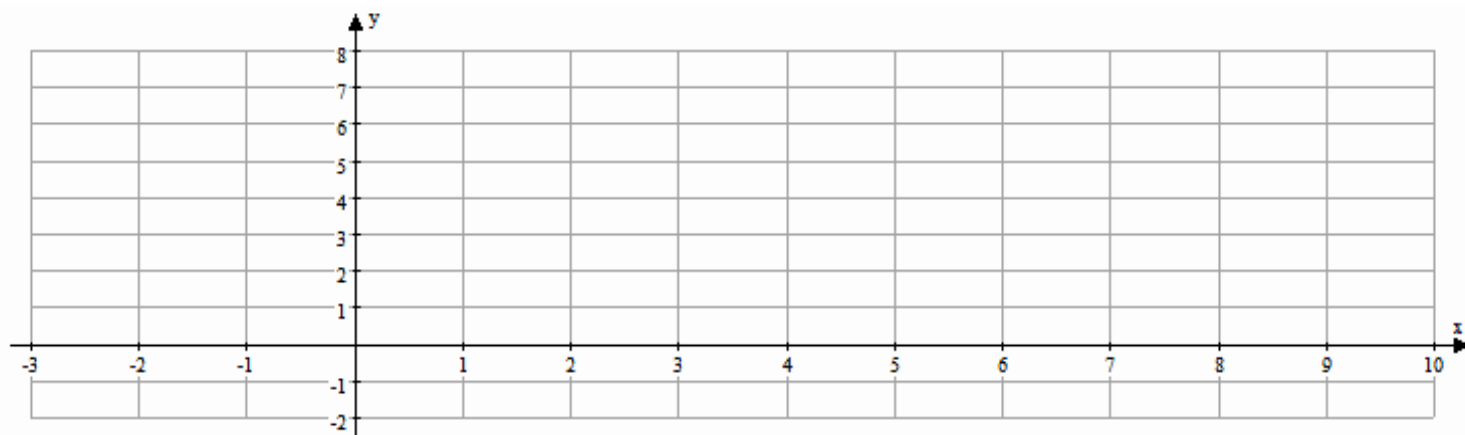
Phase =
Shift

Vertical =
Shift

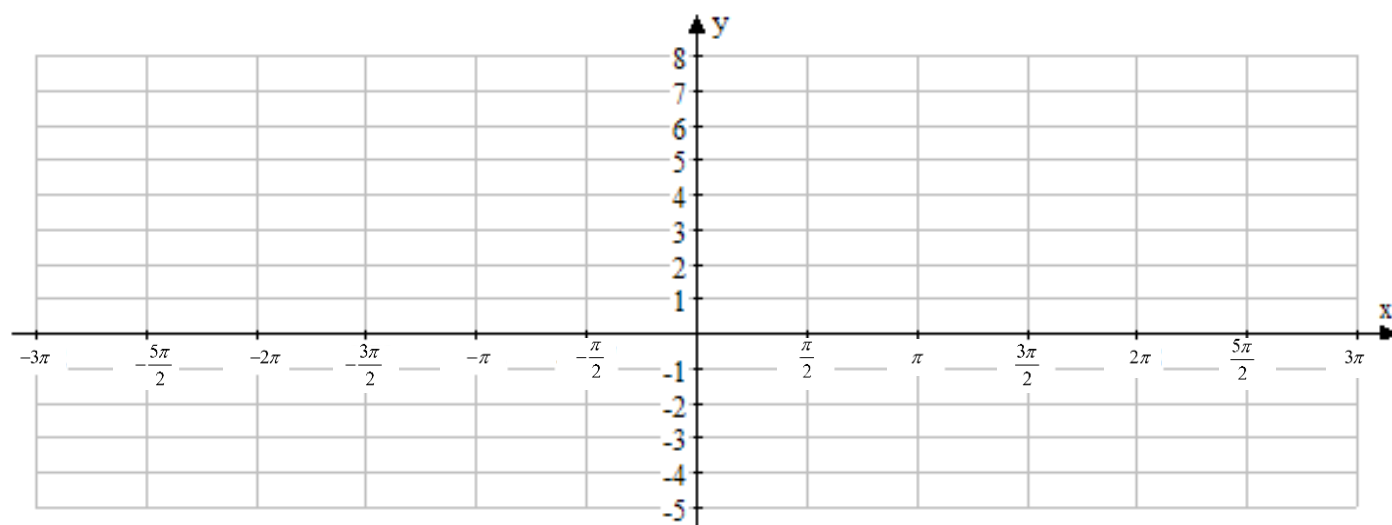
4. Graph the following equation.

a.

$$y = \tan\left(\frac{\pi}{4}(x-1)\right) + 4$$



b. $y = \cot\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right) + 3$

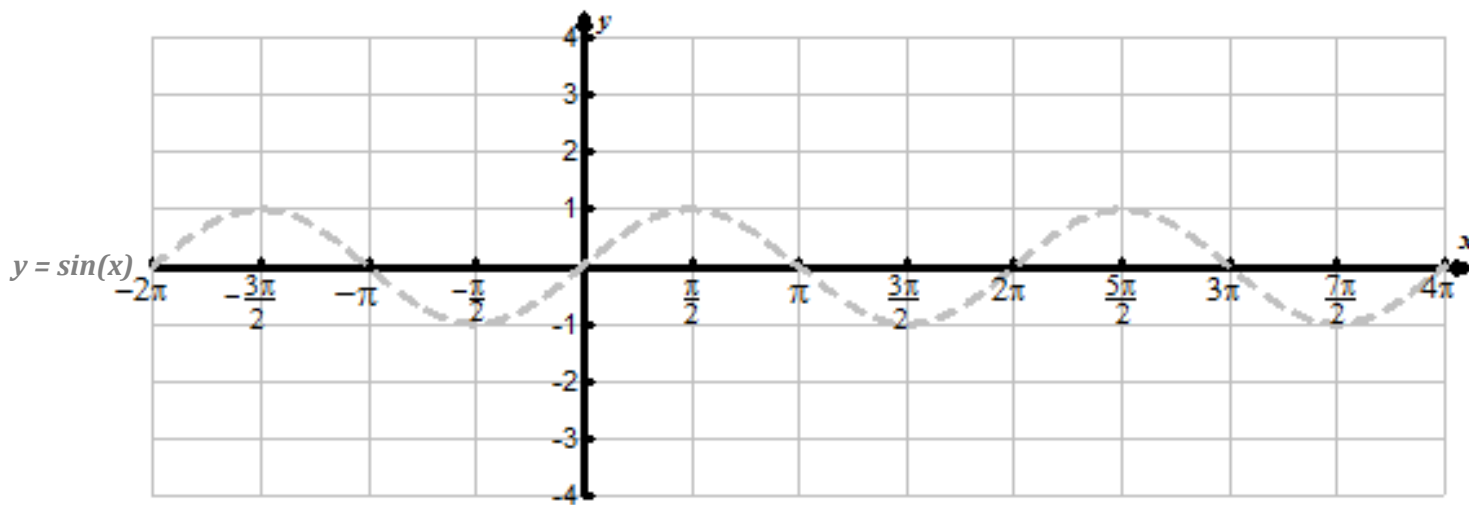


Recall, $\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$, $\csc(\theta) = \frac{\text{hyp}}{\text{opp}}$, $\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$, and $\sec(\theta) = \frac{\text{hyp}}{\text{adj}}$. Use this to show:

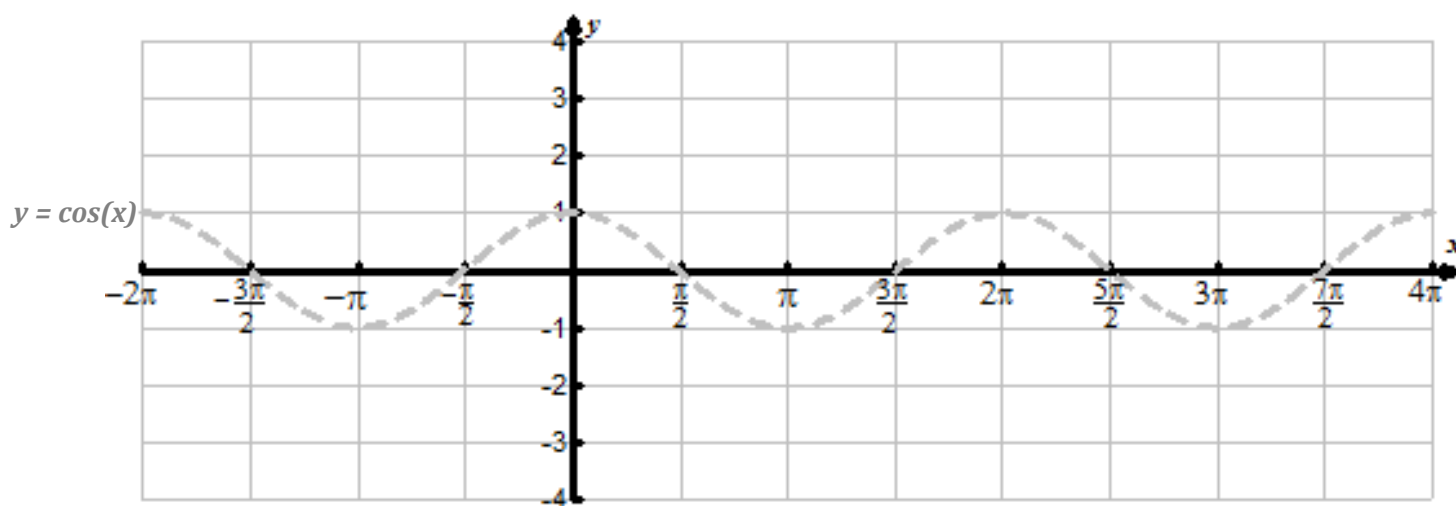
$$\frac{1}{\sin(\theta)} =$$

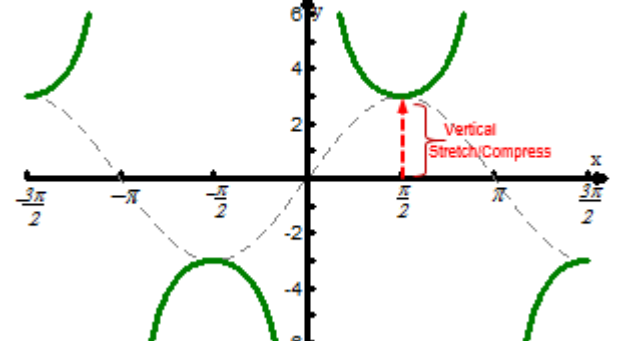
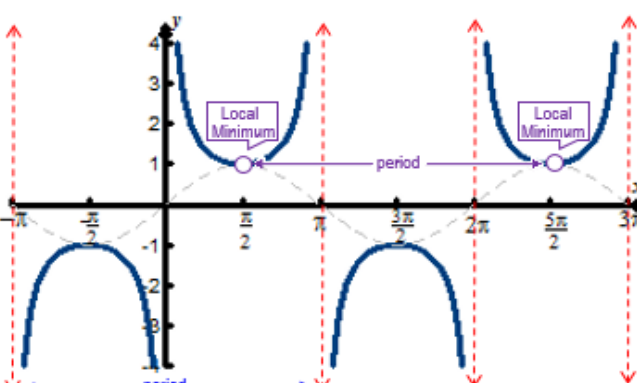
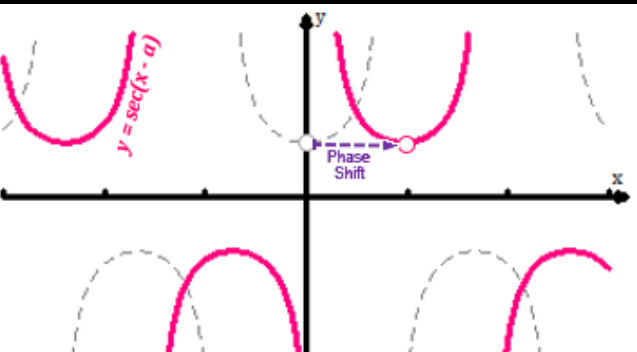
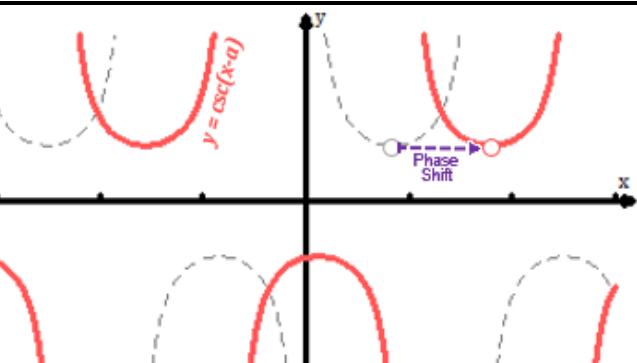
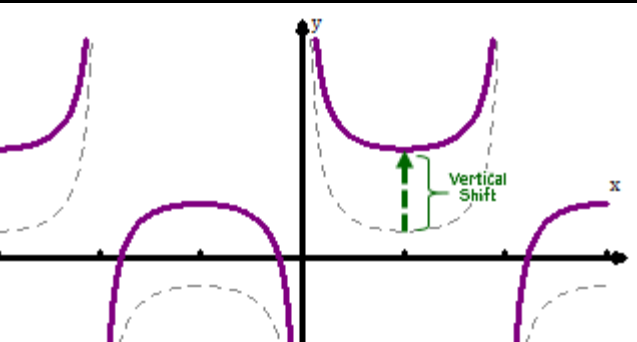
$$\frac{1}{\cos(\theta)} =$$

Use the above relationship to help you graph $y = \csc(x)$.



Use the above relationship to help you graph $y = \sec(x)$.



Parts of a Tangent and Cotangent Graph	Visual
Vertical Stretch/Compress: Secant and Cosecant graphs have a vertical stretch that would be equivalent to the amplitude of their reciprocal trig function. Vertical Factor = $\frac{(\text{Local Max } Y) - (\text{Local Min } Y)}{2}$	
Period: The period of the graph is the horizontal distance of one complete cycle. Period = $(\text{Local Min } X) - (\text{Previous Local Min } X)$ Period = $(\text{Local Max } X) - (\text{Previous Local Max } X)$ Period = $(\text{Asymptote } X) - (\text{2nd Previous Asymptote } X)$	
Phase Shift (Secant): The amount the graph is shifted right/ left Secant Phase Shift = $\text{Local Min } X$	
Phase Shift (Cosecant): The amount the graph is shifted right/ left Cosecant Phase Shift = $\frac{\text{Local Min } X + \text{Next Local Max } X}{2}$	
Vertical Shift: The amount the graph is shifted up/down Vertical Shift = $\frac{\text{Local Max } Y + \text{Local Min } Y}{2}$	

$$y = a \cdot \sec(b(x - c)) + d$$

$$y = a \cdot \csc(b(x - c)) + d$$

**a = Vertical
Stretch/Compress**

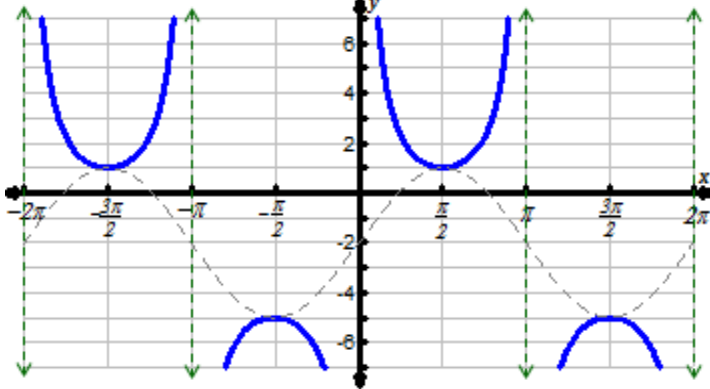
**c = Phase
Shift**

$$\frac{2\pi}{b} = \text{Period}$$

**d = Vertical
Shift**

1. Find the parameters of each SECANT/COSECANT graph and determine a possible equation.

a.

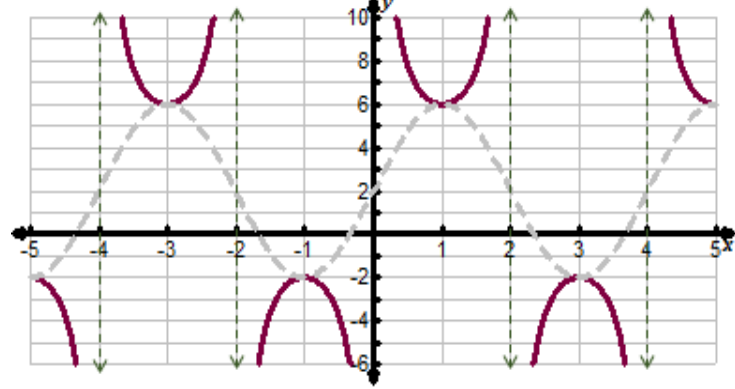


Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:

(Using Cosecant)

b.

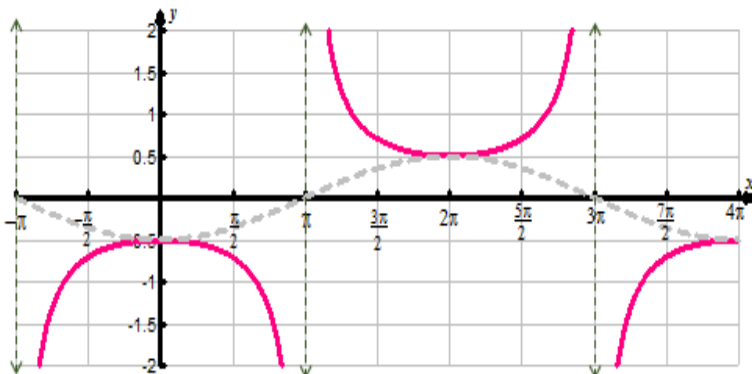


Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:

(Using Cosecant)

c.

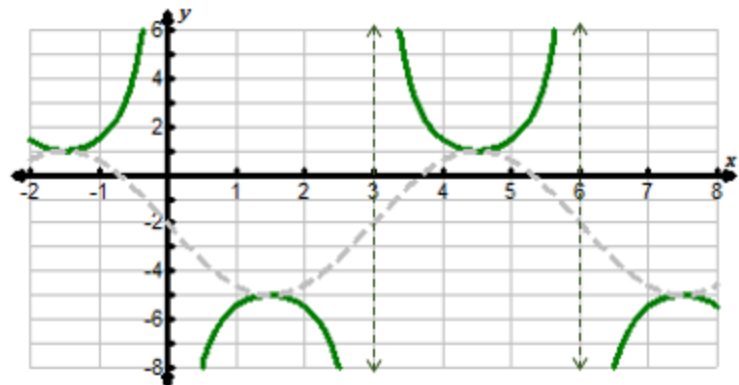


Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:

(Using Cosecant)

d.

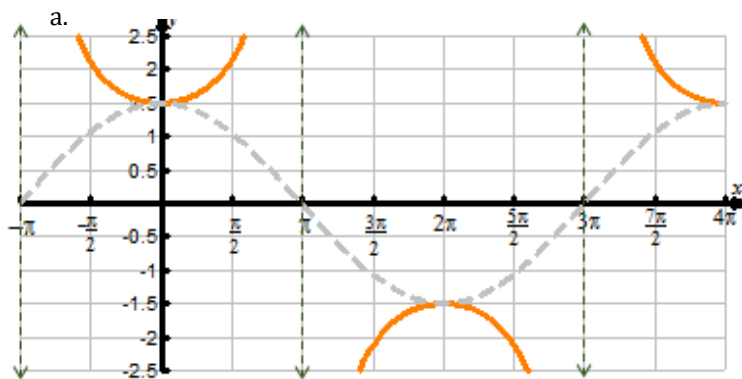


Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:

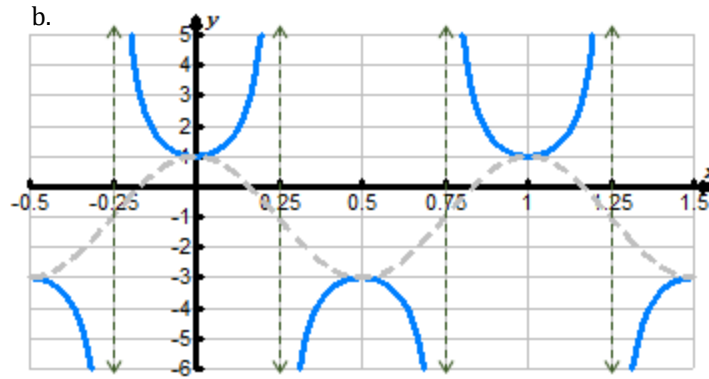
(Using Cosecant)

2. Find the parameters of each **SECANT/COSECANT** graph and determine a possible equation.



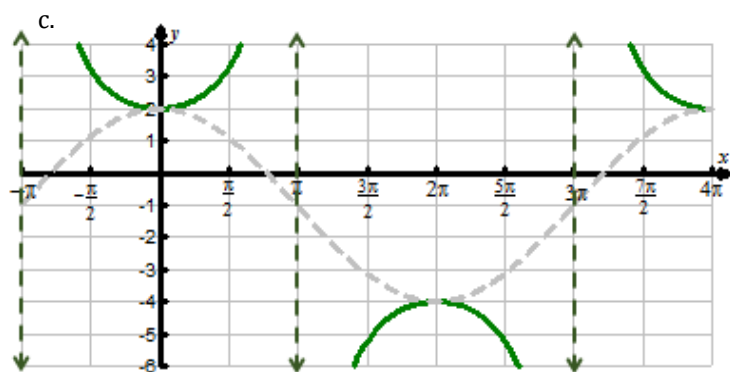
Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Cosecant)



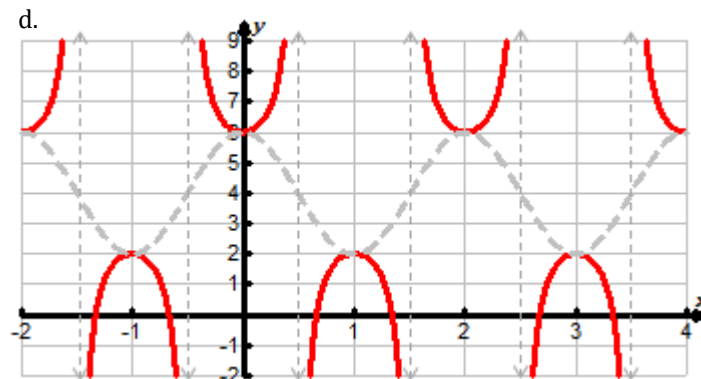
Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Secant)



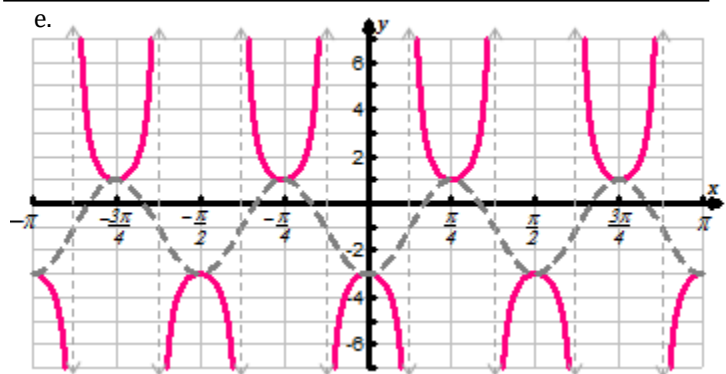
Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Secant)



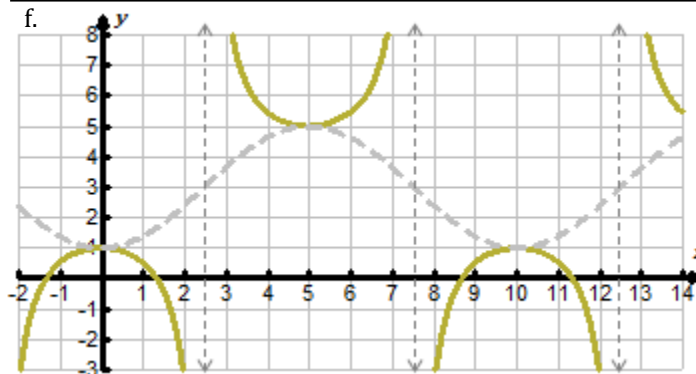
Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Secant)



Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Secant)

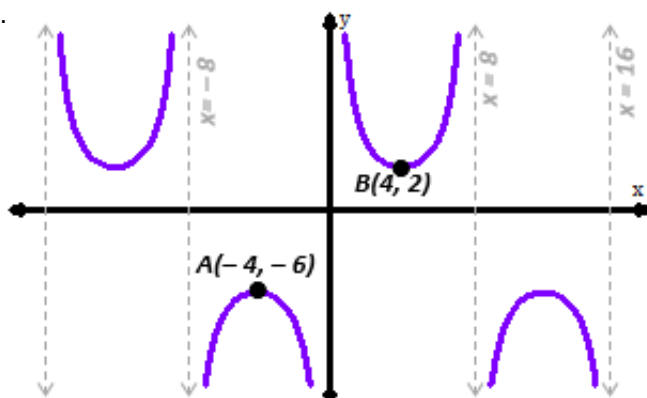


Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Secant)

3. Find the parameters of each **SECANT/COSECANT** graph and determine a possible equation.

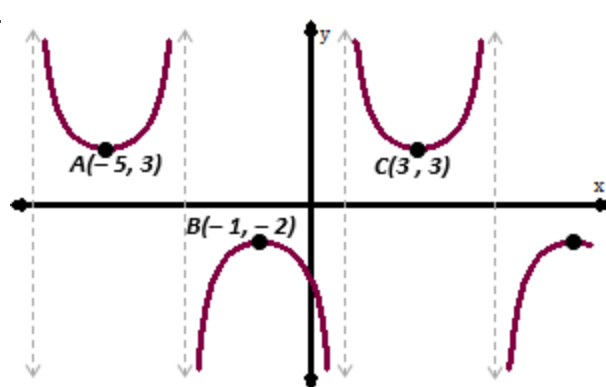
a.



Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Secant)

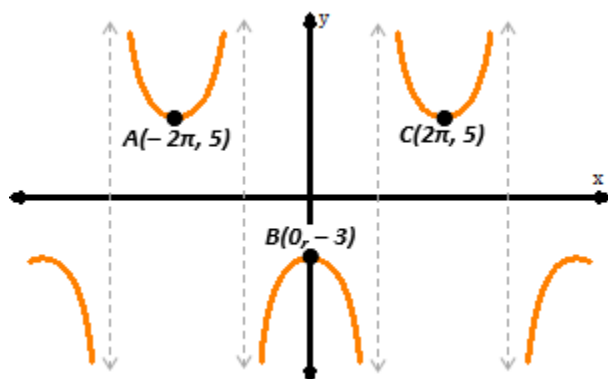
b.



Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Secant)

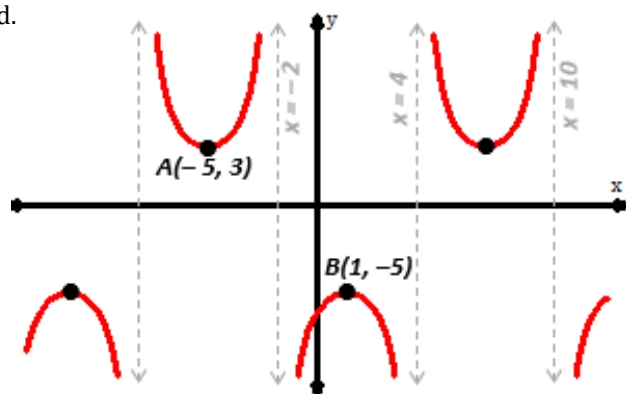
c.



Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Cosecant)

d.



Vertical : Period: Phase: Vertical:
Stretch/Compress Shift Shift

Possible
Equation:
(Using Cosecant)

4. Find the requested **SECANT/COSECANT** parameters based on the following equations.

a.

$$y = 3\sec(2x - 6) + 1$$

Vertical =
Stretch/Compress

Period =

Phase =
Shift

Vertical =
Shift

b.

$$y = 0.5\csc\left(\frac{2\pi}{3}(x+1)\right) - 3$$

Vertical =
Stretch/Compress

Period =

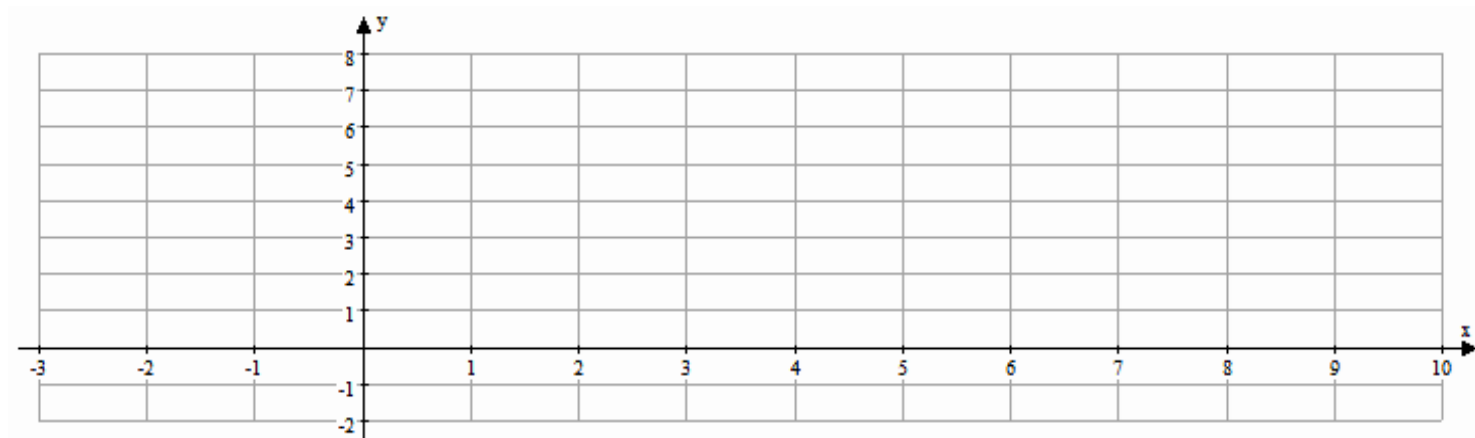
Phase =
Shift

Vertical =
Shift

5. Graph the following equation.

a.

$$y = 2 \csc\left(\frac{\pi}{2}(x-1)\right) + 5$$



b. $y = 2 \sec\left(2\left(x - \frac{\pi}{2}\right)\right) + 6$

