Sec 4.3 – Trigonometric Identities
Double Angle Identities

Name:

Starting with the sum and difference identities, create the double angle identities:

$\boxed{\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta}$

1. Simplify $\sin(\theta + \theta)$

$$\underbrace{\sin(\theta+\theta)} = \underbrace{\sin\theta\cos\theta + \cos\theta\sin\theta}$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

2. Simplify $\cos(\theta + \theta)$

$$\underbrace{\cos(\theta+\theta)} = \underbrace{\cos\theta\cos\theta} - \underbrace{\sin\theta\sin\theta}$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

3. Using the Pythagorean Identities, find 2 new ways to write the double angle formula for cosine.

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos(2\theta) = 1 - \sin^2\theta - \sin^2\theta$$

$$\cos(2\theta) = 1 - 2\sin^2\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos(2\theta) = \cos^2\theta - (1 - \cos^2\theta)$$

$$\cos(2\theta) = \cos^2\theta - 1 + \cos^2\theta$$

$$\cos(2\theta) = 2\cos^2\theta - 1$$

4. Simplify $\tan(\theta + \theta)$ using the sum identity $\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}}$

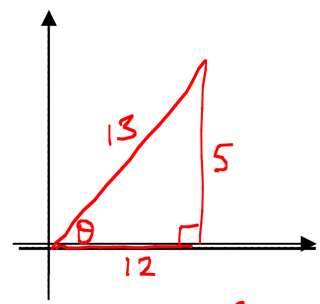
$$\underbrace{\tan(\theta+\theta)} = \underbrace{\frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Double-Angle Identities

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
- $\cos(2\theta) = 1 - 2\sin^2\theta$
- $\cos(2\theta) = 2\cos^2\theta - 1$
- $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

5. Given that $\sin(\theta) = \frac{5}{13}$ and angle A lies in the first quadrant, find the exact value of each of the following:



$a^2 + 5^2 = 13^2$
$a^2 + 25 = 169$
$a^2 = 144$
$a = 12$

a. $\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) = \frac{120}{169}$

$\boxed{\frac{120}{169}}$

b. $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$

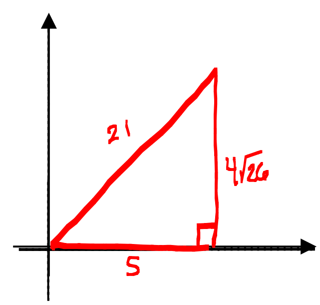
$\boxed{\frac{119}{169}}$

c. $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2(5/12)}{1-(5/12)^2} = \frac{10/12}{1 - 25/144} = \frac{5/6}{119/144} = \frac{5}{6}\cdot\frac{144}{119} = \frac{120}{119}$

$\boxed{\frac{120}{119}}$

6. Given that $\cos(\theta) = \frac{5}{21}$ and angle A lies in the first quadrant, find the exact value of each of the following:



$5^2 + b^2 = 21^2$
$25 + b^2 = 441$
$b^2 = 416$
$b = 4\sqrt{26}$

a. $\sin(2\theta) = 2\sin\theta\cos\theta$

$= 2\left(\frac{4\sqrt{26}}{21}\right)\left(\frac{5}{21}\right) = \frac{40\sqrt{26}}{441}$

$\boxed{\frac{40\sqrt{26}}{441}}$

b. $\cos(2\theta) = \cos^2\theta - \sin^2(\theta)$

$= \left(\frac{5}{21}\right)^2 - \left(\frac{4\sqrt{26}}{21}\right)^2$

$= \frac{25}{441} - \frac{416}{441} = \frac{-391}{441}$

$\boxed{-\frac{391}{441}}$

c. $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2(4\sqrt{26}/5)}{1-(4\sqrt{26}/5)^2}$

$= \frac{\frac{8\sqrt{26}}{5}}{1-\frac{416}{25}} = \frac{\frac{8\sqrt{26}}{5}}{\frac{-391}{25}} = \frac{8\sqrt{26}}{5}\cdot\frac{25}{-391} = \frac{-40\sqrt{26}}{391}$

$\boxed{\frac{-40\sqrt{26}}{391}}$

Simplify the following trigonometric expressions using the sum and difference identities.

7. $\cos\theta\sin\theta = \frac{\sin(2\theta)}{2}$

$$\cos\theta\sin\theta = \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}$$: REPLACE $\sin(2\theta)$ WITH $2\sin(\theta)\cos(\theta)$

$$\cos\theta\sin\theta = \sin\theta\cos\theta$$: COMMUTATIVE PROPERTY

$$\cos\theta\sin\theta = \cos\theta\sin\theta \checkmark$$

8. $\sin(2x) + 1 = (\sin x + \cos x)^2$

$$\sin(2x)+1 = (\sin x + \cos x)(\sin x + \cos x)$$: EXPAND or F.O.I.L.

$$\sin(2x)+1 = \sin^2 x + \sin x\cos x + \sin x\cos x + \cos^2 x$$

$$\sin(2x)+1 = \underbrace{2\sin x\cos x} + \underbrace{\sin^2 x + \cos^2 x}$$: COLLECT LIKE TERMS AND REARRANGE

$$\sin(2x)+1 = \sin(2x) + 1 \checkmark$$: REPLACE $2\sin x\cos x$ WITH $\sin(2x)$ AND $\sin^2 x + \cos^2 x$ WITH 1

9. $\cos(2x) = (\cos x + \sin x)(\cos x - \sin x)$

$$\cos(2x) = \cos^2 x - \sin x\cos x + \sin x\cos x - \sin^2 x$$: EXPAND/ F.O.I.L.

$$\cos(2x) = \cos^2 x - \cancel{\sin x\cos x} + \cancel{\sin x\cos x} - \sin^2 x$$: SIMPLIFY

$$\cos(2x) = \cos^2 x - \sin^2 x$$: REPLACE $\cos^2 x - \sin^2 x$ WITH $\cos(2x)$

$$\cos(2x) = \cos(2x) \checkmark$$

10. $2\cos^2\theta = \cot\theta\sin 2\theta$

$$2\cos^2\theta = \frac{\cos\theta}{\sin\theta}\; 2\sin\theta\cos\theta$$: REPLACE $\cot\theta$ WITH $\cos\theta/\sin\theta$ AND REPLACE $\sin(2\theta)$ WITH $2\sin\theta\cos\theta$

$$2\cos^2\theta = \frac{\cos\theta}{\cancel{\sin\theta}}\;\frac{2\cancel{\sin\theta}\cos\theta}{1}$$: SIMPLIFY

$$2\cos^2\theta = 2\cos^2\theta \checkmark$$