

It is important to consider saving for retirement and many people set up an Individual Retirement Account (IRA) with a financial planning company (e.g. Charles Schwab, J.P. Morgan, etc.). Usually, it is common that a person contributes an amount each month and then the financial planning company invests it in something like a mutual fund (Oppenheimer, Ariel Fund, T. Rowe, etc.). For example, the Ariel Mutual Fund has averaged almost a 9% annual return over the last decade.

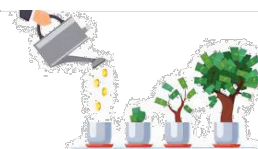
Let's consider a monthly account statement of a retirement account in which a person varies in the amount they contribute to their IRA each month but the mutual fund they invest in stays at a constant rate of 9% annually.

Similar to monthly compound interest, a person investing would only receive $\frac{1}{12}$ of the 9% each month. So each month the person would earn $\frac{9\%}{12}$ or 0.75% per month. Let's also suggest the deposits into the IRA are made at the end of the month.

Month	IRA Value on the 1 st of the Month	Interest Rate (APR = 9%)	Interest Earned by End of the Month	Amount to be Deposited in IRA at the End of the Month
Jan.	\$0	0.75%	0	\$250
Feb.	\$250	0.75%	$250 * .0075 =$ \$1.88	\$250
Mar.	$250 + 1.88 + 250 =$ \$501.88	0.75%	$501.88 * .0075 =$ \$3.76	\$250
Apr.	$501.88 + 3.76 + 250 =$ \$755.64	0.75%	$755.64 * .0075 =$ \$5.67	\$250
May	$755.64 + 5.67 + 250 =$ \$1011.31	0.75%	$1011.31 * .0075 =$ \$7.58	\$250
Jun.	$1011.31 + 7.58 + 250 =$ \$1268.89	0.75%	$1268.89 * .0075 =$ \$9.52	\$250
Jul.	$1268.89 + 9.52 + 250 =$ \$1528.41	0.75%	$1528.41 * .0075 =$ \$11.46	\$250
Aug.	$1528.41 + 11.46 + 250 =$ \$1789.87	0.75%	$1789.87 * .0075 =$ \$13.42	\$250
Sep.	$1789.87 + 13.42 + 250 =$ \$2053.29	0.75%	$2053.29 * .0075 =$ \$15.40	\$250
Oct.	$2053.29 + 15.40 + 250 =$ \$2318.69	0.75%	$2318.69 * .0075 =$ \$17.39	\$250
Nov.	$2318.69 + 17.39 + 250 =$ \$2586.08	0.75%	$2586.08 * .0075 =$ \$19.40	\$250
Dec.	$2586.08 + 19.40 + 250 =$ \$2855.48	0.75%	$2855.48 * .0075 =$ \$21.42	\$250
TOTAL	$2855.48 + 21.42 + 250 =$ \$3126.90	9%	\$126.90	\$3000

Using a monthly ledger like this could easily allow for varying return rates and varying deposits made. However, if equal payments and a consistent interest rate are used we can use a formula to find the total value.

Many people invest in an annuity for retirement. An annuity is a sequence of equal payments made at equal time periods. The value of an annuity is the sum of all deposits and interest accrued.



The value of an annuity can be determined using the following formula.

$\text{Value of Annuity} = \frac{P \left[\left(1 + \frac{r}{n} \right)^{nt} - 1 \right]}{\left(\frac{r}{n} \right)}$	$P =$ Payment $r =$ rate as a decimal $t =$ number of years $n =$ compounds/payments per year	<i>Annually:</i> $n = 1$ <i>Semi-Annually:</i> $n = 2$ <i>Quarterly:</i> $n = 4$ <i>Monthly:</i> $n = 12$ <i>Weekly:</i> $n = 52$
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It's important to start investing early. Let's try an example and say that at age 26, you started investing \$250 every month and continued to do so until age 56 and the retirement investment yields 9.5% annually (possibly using a mutual fund like T-Rowe, Oppenheimer, or Ariel). How much would the account be worth at age 56.

From this statement we can see that $P = \$250$, $r = 0.095$, $t = 30$, $n = 12$.

If we substitute each value in to the formula, we can determine how much the account would be worth when you retire at 56.

$$\text{Value of Annuity} = \frac{\$250 \left[\left(1 + \frac{0.095}{12} \right)^{12 \cdot 30} - 1 \right]}{\left(\frac{0.095}{12} \right)} \approx \$508,259$$



If you saved just \$250 every month for 30 years in such an account, the account would be worth over a half of a million dollars.

It is also worth investigating how much you contributed and how much interest you earned. If we put in \$250 every month for 30 years, then we invested a total of $\text{Contributions} = \$250 \cdot 12 \cdot 30 = \$90,000$.

$$\text{Contributions} = P \cdot n \cdot t$$

So, the rest came from interest which we could subtract from the account's value to find the interest.

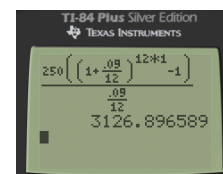
$\text{Interest} = \$508,259 - \$90,000 = \$418,259$. So, we would have invested \$90,000 and made \$418,259 in interest.

$$\text{Interest} = \text{Value of the Annuity} - \text{Contributions}$$

The important thing is to invest early as the retirement account grows exponentially and the increase in value of the account is most noticeable towards at the latter part of the term of your investment.

Can you use the formula to verify the monthly ledger of the IRA on the previous page?

$$A = \frac{P \left(\left(1 + \frac{r}{n} \right)^{nt} - 1 \right)}{\left(\frac{r}{n} \right)} = \frac{\$250 \left[\left(1 + \frac{.09}{12} \right)^{12 \cdot 1} - 1 \right]}{\left(\frac{.09}{12} \right)} = \$3126.90$$



1. On your own, determine how much an Annuity would be worth if you invested \$300 per month in a investment fund had a 10% APR return starting when you are 18 years old and continuing to do so until age 58

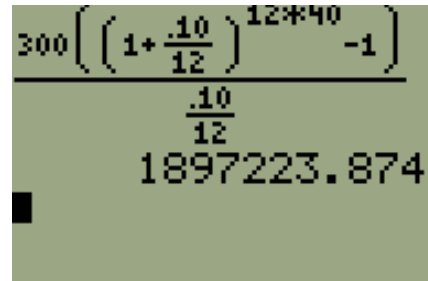
$$\text{Value of Annuity} = \frac{P \left[\left(1 + \frac{r}{n} \right)^{nt} - 1 \right]}{\left(\frac{r}{n} \right)}$$

$$P = 300 \quad r = .10$$

$$n = 12 \quad t = 40$$

How much would the annuity be worth when you are 58?

\$1,897,223.87



How much did you contribute out of pocket?

$$(\underbrace{\$300}_{\text{AMOUNT PER YEAR}} * \underbrace{12}_{\text{YEARS}}) * \underbrace{.40}_{\text{YEARS}} = \underbrace{\$144,000}_{\text{AMOUNT CONTRIBUTED}}$$

How much did you earn in total interest?

$$\underbrace{\$1,897,223.87}_{\text{VALUE OF ACCOUNT}} - \underbrace{\$144,000}_{\text{AMOUNT CONTRIBUTED}} = \underbrace{\$1,753,223.87}_{\text{AMOUNT EARNED}}$$

2. Determine how you would need to save each month in an annuity that has an 12% APR return so that the annuity would be worth \$1,000,000 in 25 years?

$$\text{Value of Annuity} = \frac{P \left[\left(1 + \frac{r}{n} \right)^{nt} - 1 \right]}{\left(\frac{r}{n} \right)}$$

$$1000000 = P * \frac{\left[\left(1 + \frac{.12}{12} \right)^{12 * 25} - 1 \right]}{\left(\frac{.12}{12} \right)}$$

$$\frac{1000000}{1878.8466} = \frac{1878.8466 \cdot P}{1878.8466}$$

$$\$532.24 \approx P$$

\$1,000,000 = INVESTING \$532.24 PER MONTH FOR 25 YEARS WITH A 12% APR