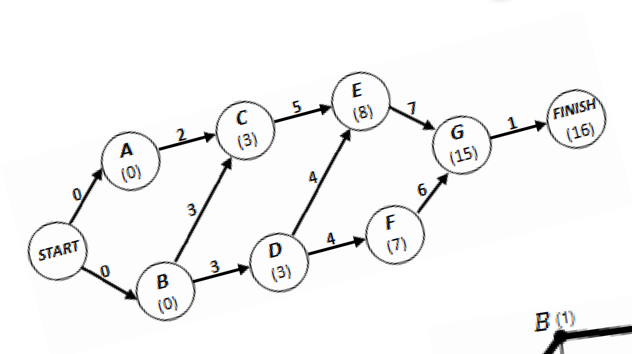
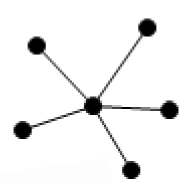
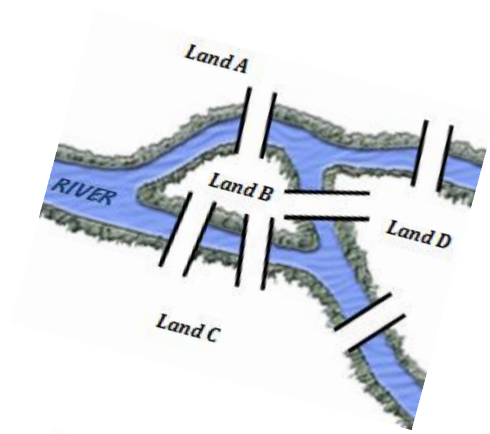
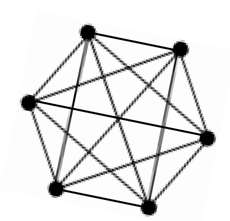
Adv. Math Decision Making

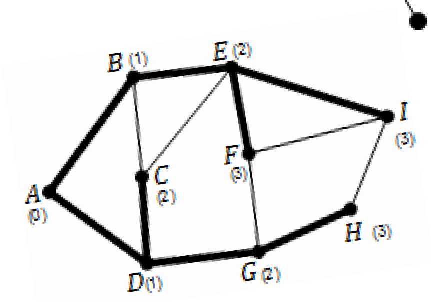
Unit 6: Networks & Graph Theory

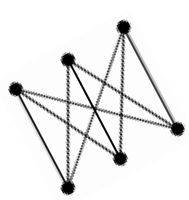


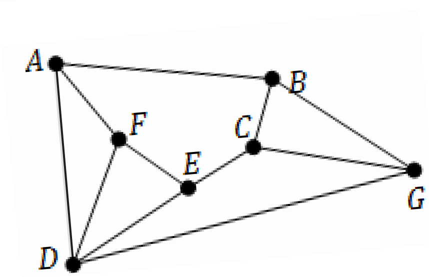


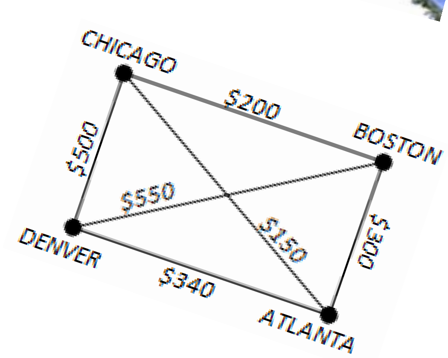












MAtTHew M. Winking

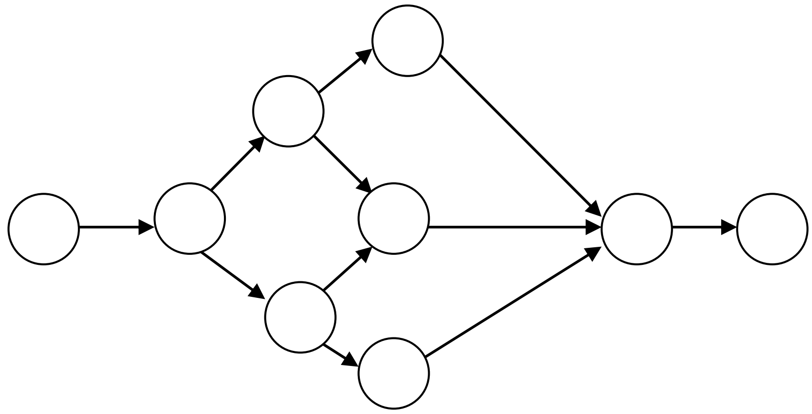
Sec 6.1 – EST Graphs
Networks & Graphs

Name:

These graphs are models to find the EARLIEST TIME any particular job can START. What do you think is meant in a team by the following statement? You are only as fast as your slowest link.

1. Label the provided graph (using the appropriate vertices and weighted edges).

Task	Time	Prerequisites
Start	0	
A	5	None
B	6	A
C	4	A
D	4	B
E	8	B,C
F	4	C
G	10	D,E,F
Finish		



a. What is the earliest time this entire graph can be completed?

b. Two tasks must be completed before "E" can start. Which task gets to "E" first and how long does that task have to wait on the other task?

2. Create a graph and label it appropriately.

Task	Time	Prerequisites
Start	0	
A	1	None
B	2	None
C	3	A, B
D	5	B
E	5	C
F	5	C, D
G	4	D,E
H	4	E, F
Finish		

What is the earliest time this entire graph can be completed?

3. Create a graph and label it appropriately.

Task	Time	Prerequisites
Start	0	
A	3	None
B	2	None
C	2	A
D	3	B
E	4	B
F	1	C, D, E
G	3	D
H	1	F,G
I	3	F
Finish		

What is the earliest time this entire graph can be completed?

4. Fill in and graph: What is the least amount of time needed to prepare dinner? ______

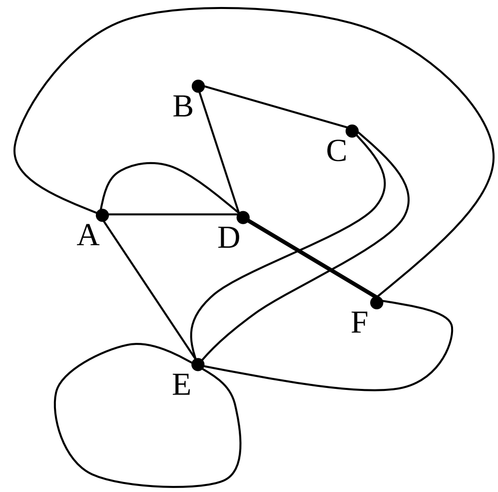
Task	Time (min)	Prerequisite Task
Start	0	None
A Wash hands	1	None
B Defrost hamburger	120	A
C Shape meat into patties	15	B
D Cook hamburgers	14	C
E Peel and slice potatoes	15	A
F Fry potatoes	40	E
G Make salad	15	A
H Set table	5	A
I Serve food	8	D,F,G,H

Sec 6.2 – Definitions Networks & Graphs

Name:

In the previous problems you were using visual graphs of problems.

Now we need to define some common terms that will be helpful.



VERTICES are the points on the graph and the lines connecting the vertices are called **EDGES**.

1. The **DEGREE** of a vertex is defined by the number of edges that have that vertex as an endpoint. Find the degree of each vertex.

Vertex	Degree
A	4
B	
C	

Vertex	Degree
D	
E	
F	

2. Two vertices are **ADJACENT**, if they are connected by an edge.
   a. List all of the vertices adjacent to vertex A:___________
   b. List all of the vertices adjacent to vertex B:___________
   c. List all of the vertices adjacent to vertex C:___________

d. Create an **Adjacency Matrix** of the graph.

$$\begin{matrix} & A & B & C & D & E & F \\ A & & & & & & \\ B & & & & & & \\ C & & & & & & \\ D & & & & & & \\ E & & & & & & \\ F & & & & & & \end{matrix}$$

3. A **LOOP** occurs when an edge has both endpoints being the same vertex.
   a. List any vertices that have a loop:______________________

4. **MULTIPLE EDGES** occur when 2 adjacent vertices have more than one edge connecting the same 2 vertices.
   a. List any pair of adjacent vertices that have multiple edges:___________________

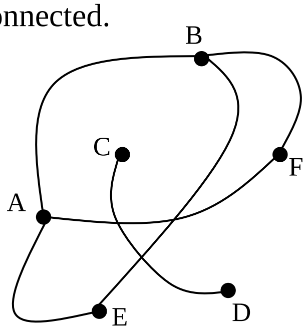
5. A **WALK** is a route you can take from one point to another and you can use multiple edges
   a. List all of the 2 step walks starting from vertex B (a 2 step walk uses 2 edges).

   BDB, BDA, ____

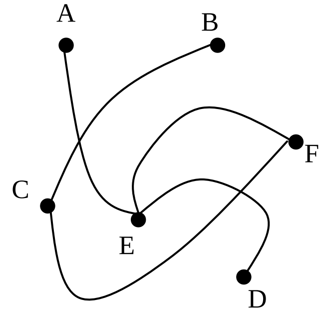
6. Square the Adjacency Matrix using your calculator and show your answer below. See if the row representing vertex B sums to the same number of walks in #5.

7. A graph is **CONNECTED** if at least one path exists between any two points. Circle the graphs below that are connected.

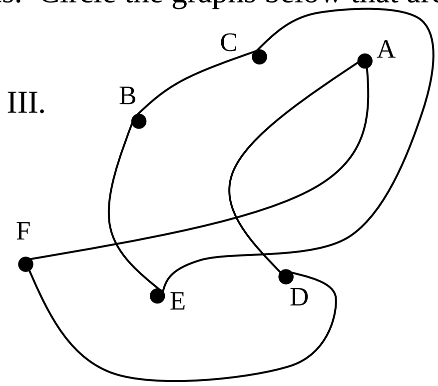
I.



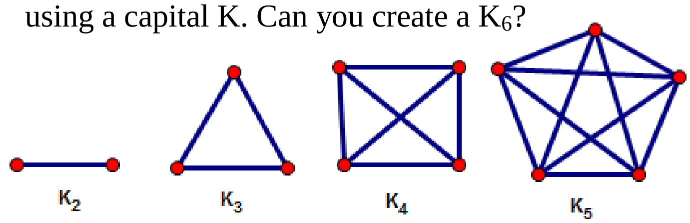
II.



III.



8. A complete graph is a graph in which all vertices are adjacent to one another and they are usually denoted using a capital K. Can you create a K_6?

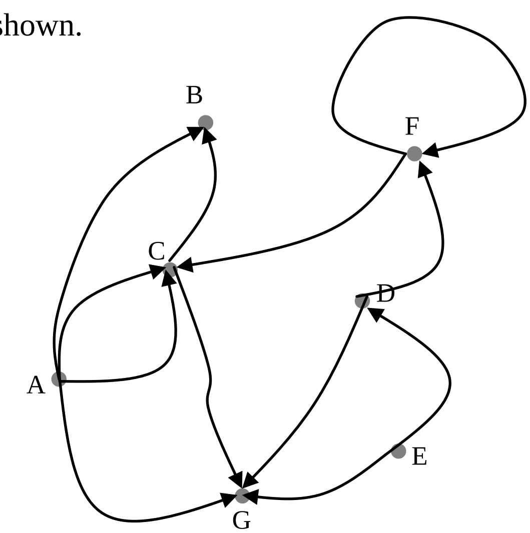


Graph	# of Vertices	# of Edges
K_2	2	1
K_3	3	3
K_4		
K_5		
⋮	⋮	⋮
K_{15}		

9. A directional graph is sometimes referred to as a **DI-GRAPH**. A di-graph is one with edges that are one directional (similar to a one way street). The vertices of di-graphs have an **INDEGREE** and an **OUTDEGREE**. The indegree is simply the number of edges coming into the vertex and the outdegree is the number of edges leaving the vertex. A vertex that only has edges coming in to the vertex is called a **RECIEVER**. A vertex that only has outgoing edges is called a **TRANSMITTER**.

A. List the indegree and outdegree of each vertex in the graph shown.

Vertex	Indegree	Outdegree	Total Degree
A	0	4	4
B			
C			
D			
E			
F			
G			



B. List all of the vertices that are TRANSMITTERS:____________________________________

C. List all of the vertices that are RECIEVERS:____

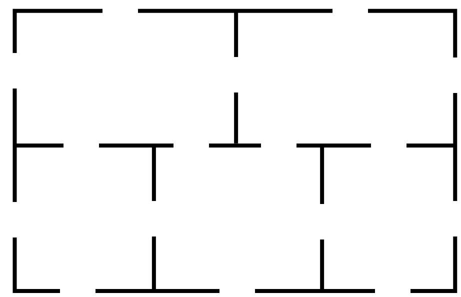
Adv. Mathematical Decision Making

Sec 6.3 – Euler Circuits & Paths
Networks & Graphs

Name:

Some brainteaser problems involving networks like the following simply cannot be done. The famous Konigsberg bridge problem is one of them. It is not possible to find a path that will enable you to cross each bridge exactly once (meaning any path that a person walks over every single bridge will require that person to walk across at least one bridge more than once). Another fairly common impossible problem is drawing a path for a person to walk through each door exactly once without going back through any door more than once.

Koenigsberg Bridge Problem

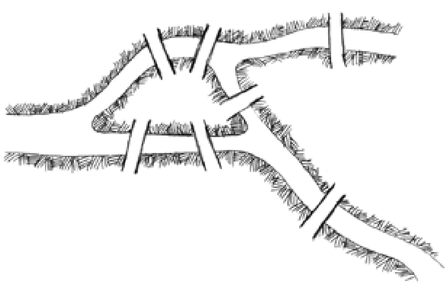


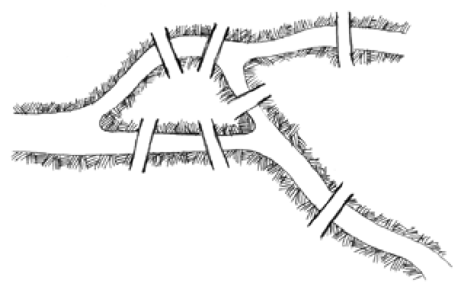
The Impossible Doorway Problem

(Can you prove Leonhard Euler wrong? Can you find a path to walk that only takes you over each <u>bridge</u> JUST ONCE?

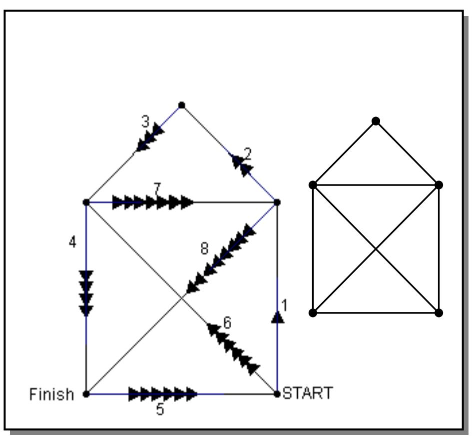
(Can you prove Leonhard Euler wrong? Can you find a path to walk that only takes you each <u>door</u> JUST ONCE?)

(Can you prove Leonhard Euler wrong? Can you find a path to walk that only takes you over each bridge JUST ONCE?)

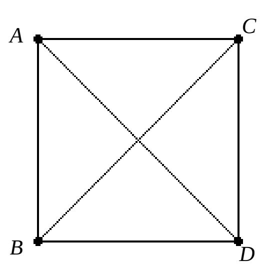


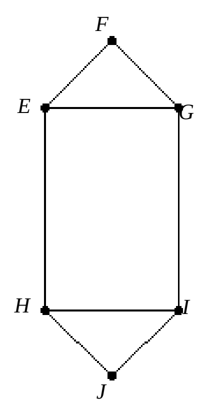


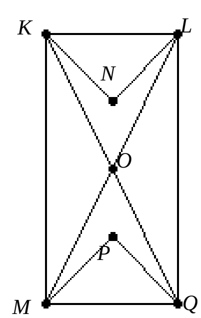
If the following graphs can be created without picking up your pencil and without ever retracing any edge, the graph is said to be traversable of these some are referred to as Euler Circuits or Euler Paths. Can you determine which are traversable? Circle the ones that are:

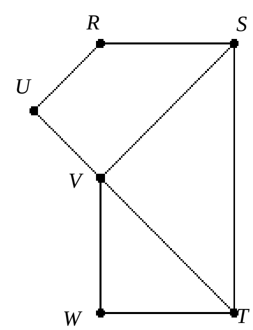


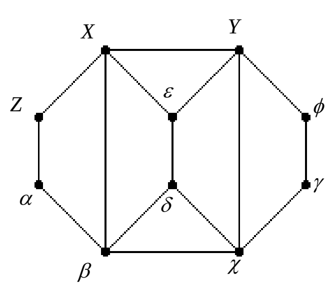
Alternately, you can label each vertex and then list the vertices in order that would complete the graph.

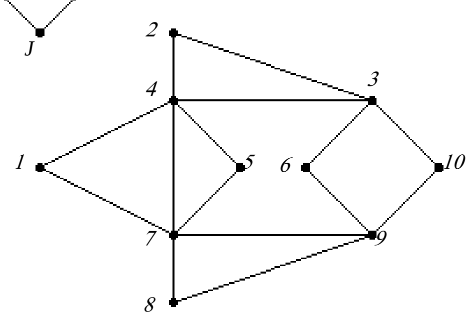






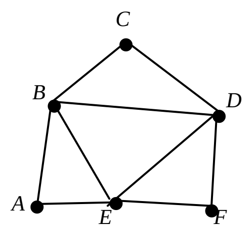


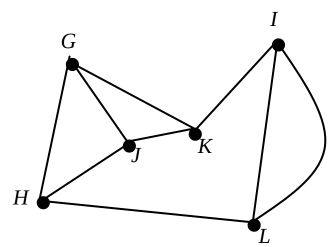


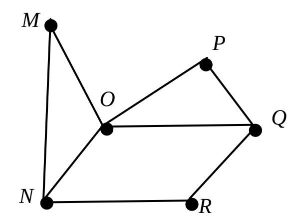


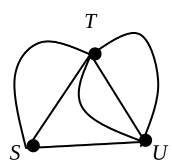
1. a. Label the degree of each vertex
   b. Put a CIRCLE around the following graphs that have an EULER CIRCUIT and list a possible circuit. Briefly explain why an Euler Circuit must have all even degree vertices.

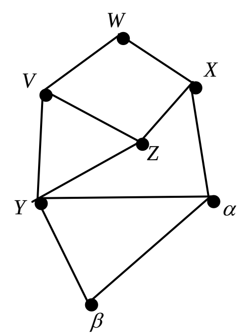
   c. Put a SQUARE around the following graphs that have an EULER PATH and list a possible path. Briefly explain why an Euler P must have exactly 2 odd vertices and the rest even.



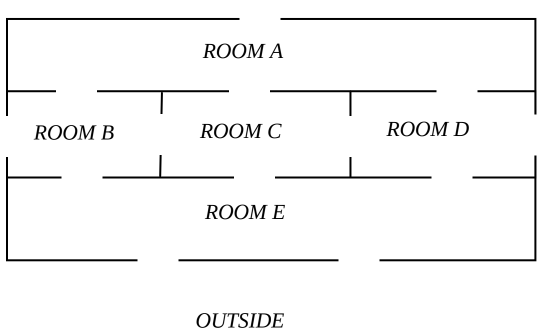


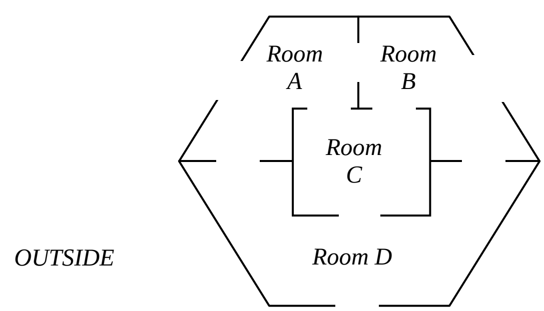






2. Create a Graph of the following map and explain whether it is impossible or possible to pass through each door exactly once.

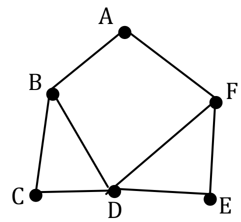


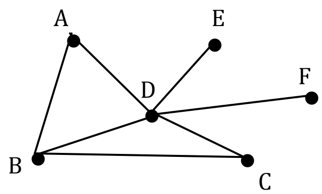


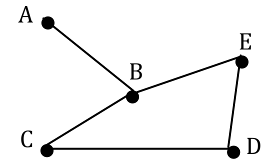
Sec 6.4 – Hamilton Circuits & Paths
Networks & Graphs

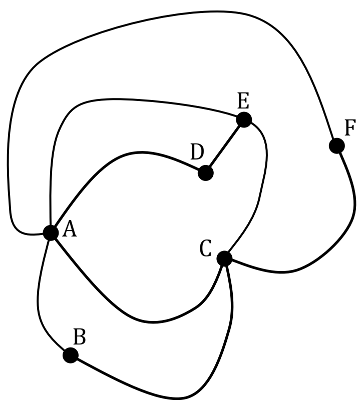
Name:

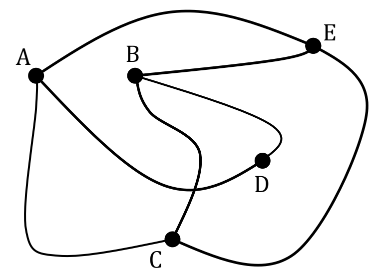
1. Hamilton looked at the ‘graph puzzle’ from a slightly different perspective. He asked if it was possible to traverse the graph and pass through each vertex only once. Again similar to Euler, the graph is consider to have a Hamilton Circuit if you can end at the same vertex you started with and a Hamilton Path if you start and end on different vertices. Circle each graph below that you think has a Hamilton Circuit and put a square around each that you think has a Hamilton Path.



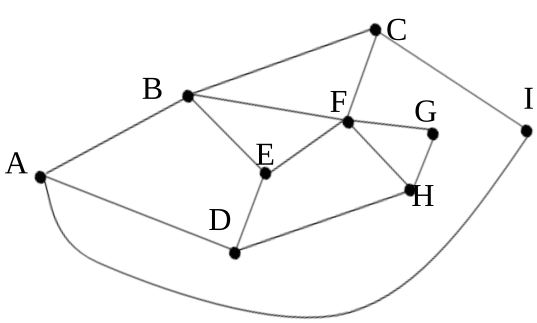




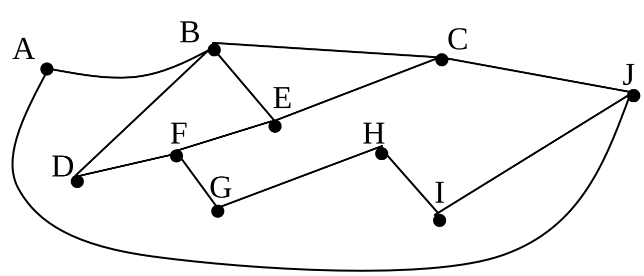




2. Find a **HAMILTONIAN CIRCUIT** of the graph below (Give a sequence of letters to describe the path (e.g. A, D, E, B, ……….. etc.))

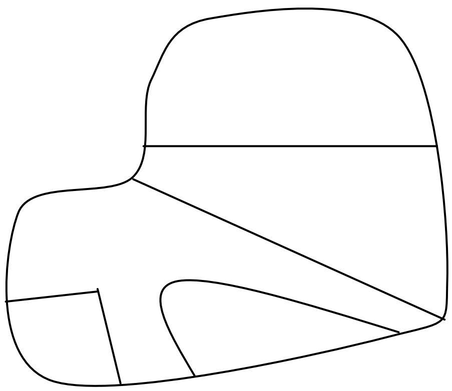


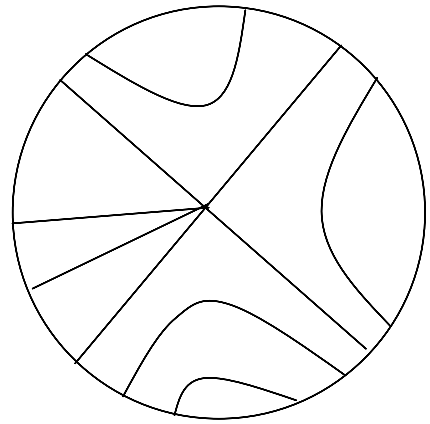
3. Find a **HAMILTONIAN PATH** of the graph below (Give a sequence of letters to describe the path (e.g. A, D, E, B, ……….. etc.))



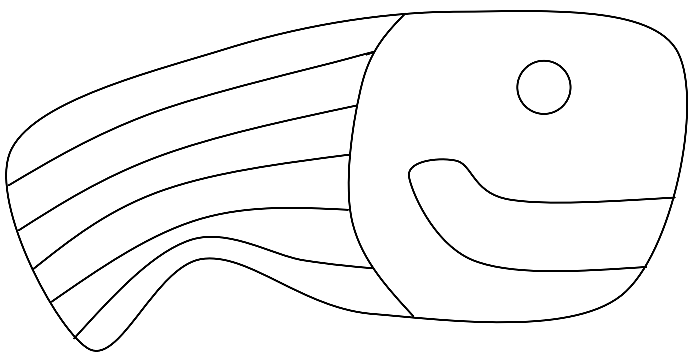
Name:

Color in each of the following maps using **2** colors such that no two adjacent regions share the same color (each map has a chromatic number of 2)

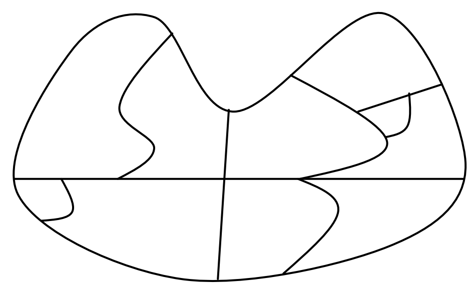




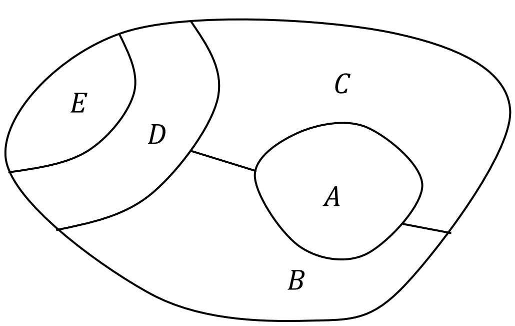
Color in the following map using **3** colors such that no two adjacent regions share the same color. (The graph has a chromatic number of 3)



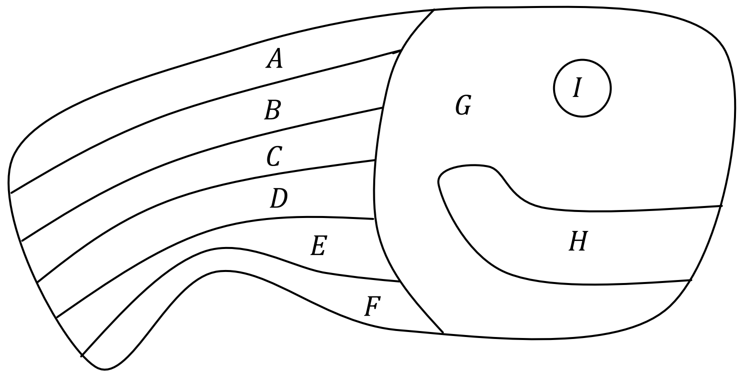
Color in the following map using **4** colors such that no two adjacent regions share the same color. (The graph has a chromatic number of 4)



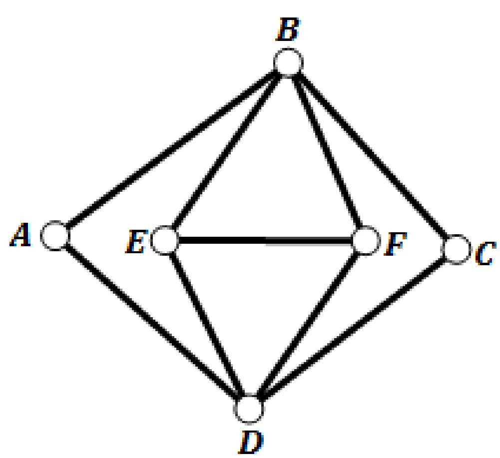
You should never need more than 4 colors to color in a 2-dimensional (planar) map. Convert the following map in to a graph (regions become vertices) and determine the chromatic number of both the graph and the map.

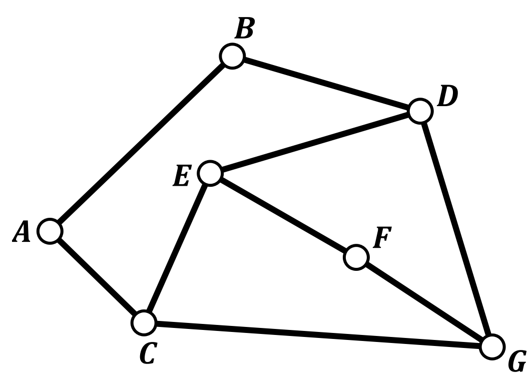


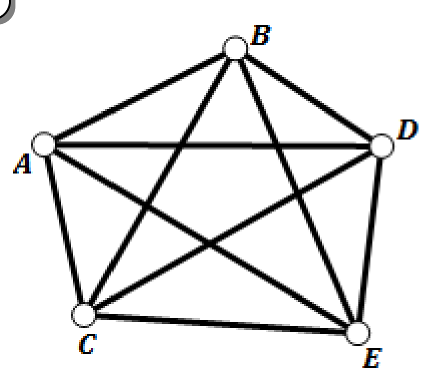
Convert the following map in to a graph (regions become vertices).



Determine the Chromatic number of each of the following graphs.







Transportation Dilemma

You are planning a trip to Elitch Gardens for your eight friends There are several people who, for various reasons, cannot ride in the same car.

Part one: Using the following information, create a graph which illustrates those people who cannot ride together. (Using Chromatic Numbers)

- Amy stole Cara's boyfriend at the last party.
- Amy wrecked Becky's car and is refusing to pay for the damages.
- Fred has been dating Amy and Ellena and telling each of them that the other girl is chasing him. Additionally, he has been telling his fellas that he has them both conned. Recently they found out about his little scam. (Amy and Ellena are still best friends).
- Cara stole Becky's cell phone and is refusing to give it back.
- Cara has been telling everyone that Ellena and Amy will talk about you behind your back if they have the chance and doesn’t like them.
- David never forgave Cara for stealing his clothes out of the boy's locker room in the seventh grade.
- David told everyone that Ellena is BORING. (Obviously this did not go over well with Ellena).
- Becky left Gary in Colorado Springs last weekend when they were camping, and Gary had to hitch-hike the entire way back to Denver.
- Ellena accidentally forgot to feed Gary's goldfish, Moby, while he was away, and it died.
- Heather has always sided with Amy and Ellena, so she is always mad at anyone who is feuding with either of the girls - EXCEPT David . They are siblings, and they always back each other up.

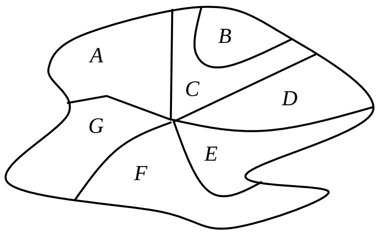
Graph

Part two: Determine the minimum number of cars necessary to transport everyone to the park. Assume that the seating capacity of the cars is not a problem, and you can ride in any car.

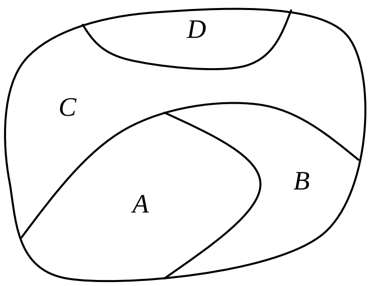
Sec 6.7 – Planar Graphs
Networks & Graphs

Name:

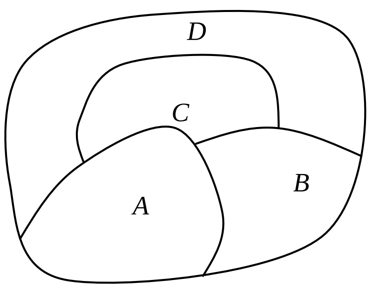
1. Create a Graph of each MAP such that NO 2 EDGES intersect.



Chromatic Number: 2



Chromatic Number: 3



Chromatic Number: 4

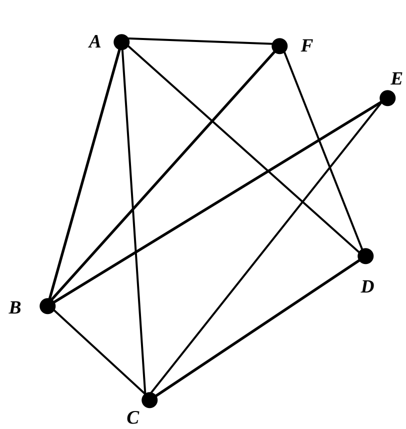
3. Create example of the Following Graphs in a planar configuration if possible:
Complete Graphs:

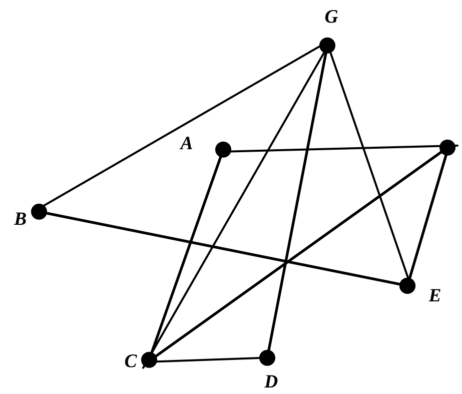
K_4 K_5

Complete Bipartite Graphs:

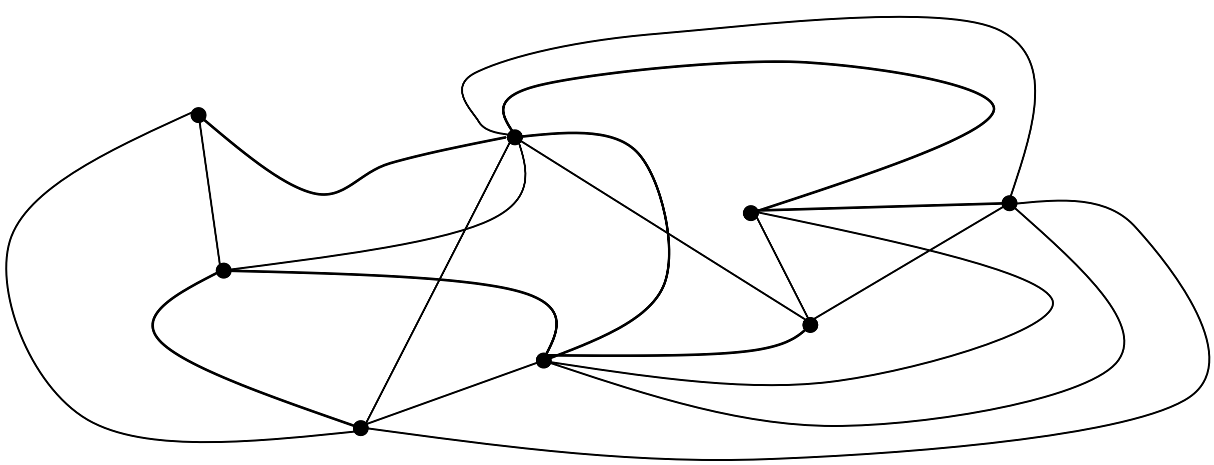
** $K_{3,3}$ $K_{5,2}$

4. The following GRAPHS are PLANAR Redraw them so that none of the edges intersect (Remember adjacent vertices must remain adjacent in the new graph).





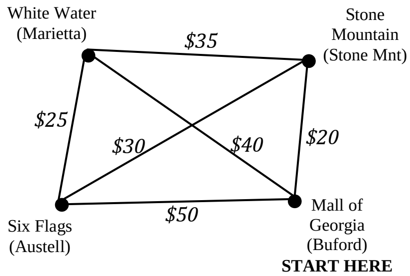
7. This Graph is not planar because it contains a K_5. Highlight the K_5 in the following graph.



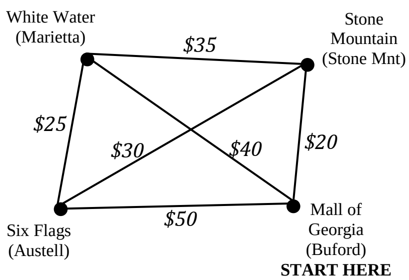
Sec 6.8 – Traveling Sales Person Graphs
Networks & Graphs

Name:

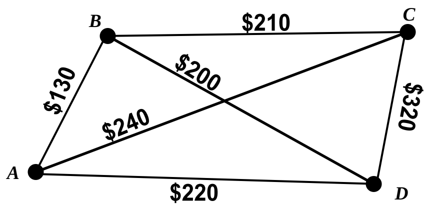
1. Using the "Nearest Neighbor Algorithm", determine a possible cheapest Hamiltonian Circuit starting from the "Mall of Georgia".



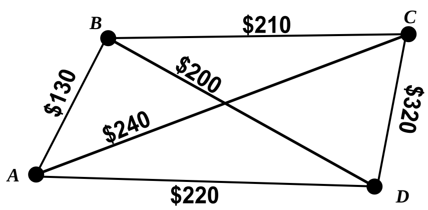
2. Using the "Brute Force Algorithm", determine the cheapest Hamiltonian Circuit starting from the "Mall of Georgia".



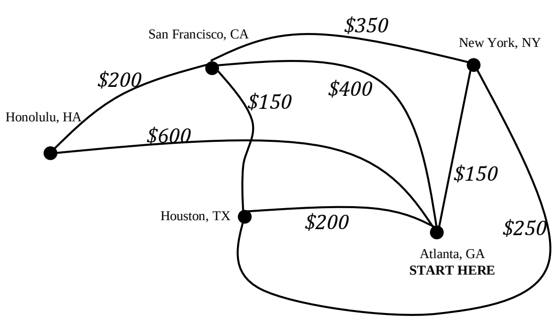
3. Using the "Nearest Neighbor Algorithm", determine a possible cheapest Hamiltonian Circuit starting from point A.



4. Using the "Brute Force Algorithm", determine the cheapest Hamiltonian Circuit starting from point A.



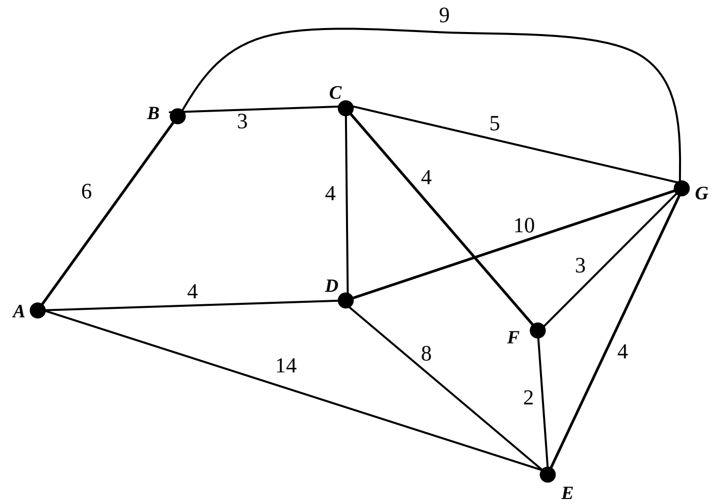
5. Using the "Nearest Neighbor Algorithm", determine a possible cheapest Hamiltonian Circuit starting from Atlanta Georgia.



Sec 6.9 – Shortest Route Algorithm
Networks & Graphs

Name:

1. Find the shortest path to each vertex from Point A.



Shortest Distance From:

AB=

AE=

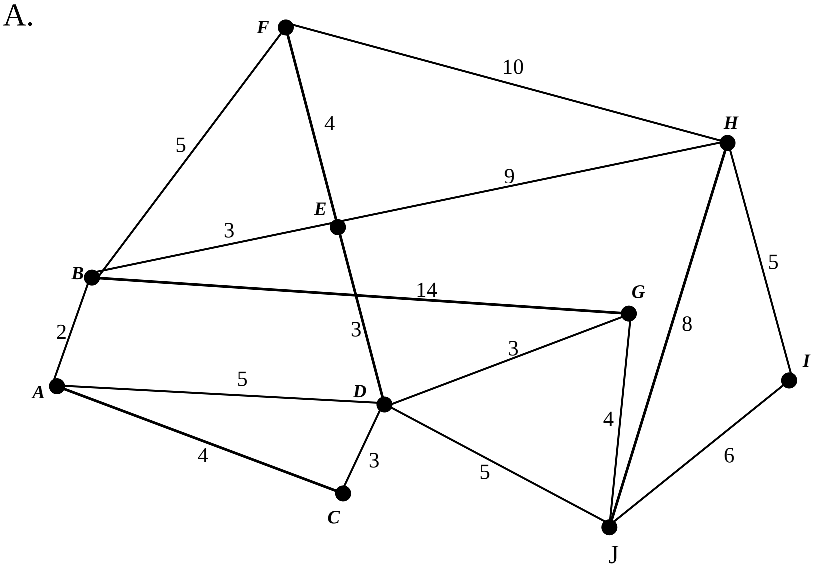
AC=

AF=

AD=

AG=

2. Find the shortest path to each vertex from Point A.



Shortest Distance From:

AB=

AF=

AC=

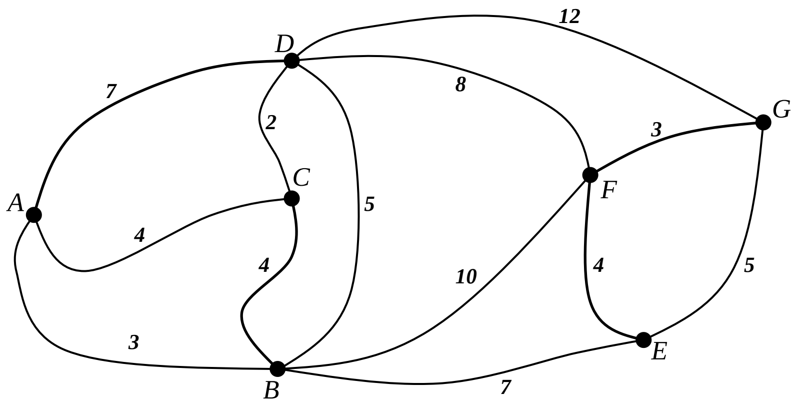
AG=

AD=

AH=

AE=

AI =



Shortest Distance From:

AB=

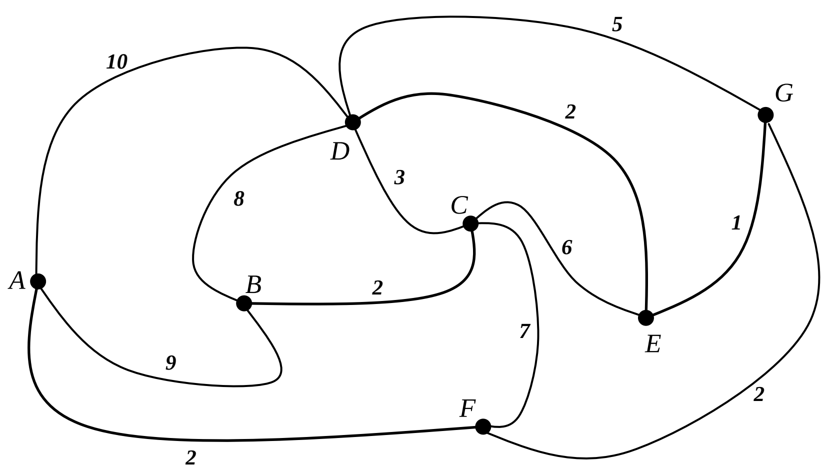
AC=

AD=

AE=

AF=

AG=



Shortest Distance From:

AB=

AC=

AD=

AE=

AF=

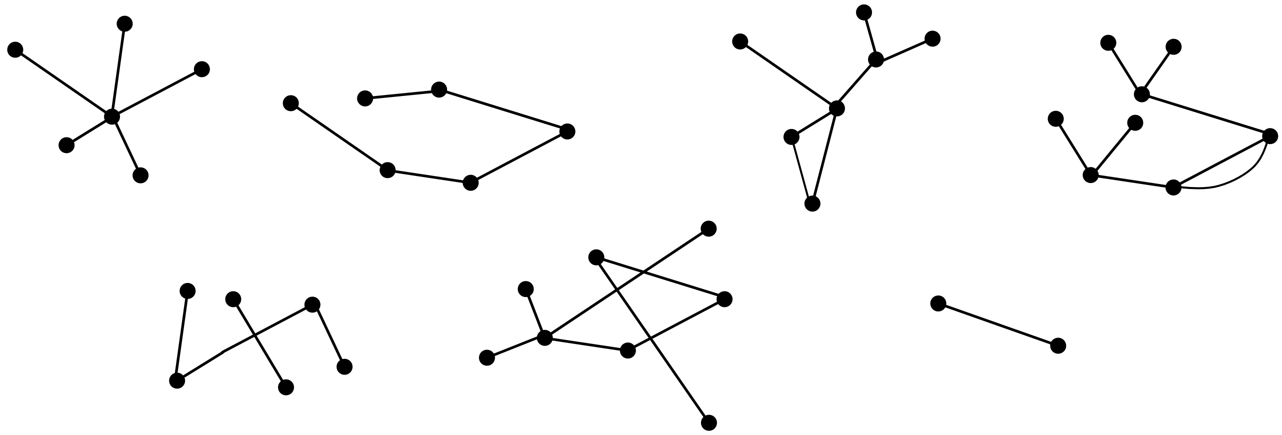
AG=

Sec 6.10 – Tree Graphs

Name:

A **tree graph** is a graph that is **CONNECTED** and **doesn't contain any CYCLES**.

1. Which of the following are trees? (If it is not a tree explain why.)



2. Show every possible way of drawing a tree with 5 vertices.

What is the sum of the degrees of each vertex in each on of your diagrams above?

3. Show every possible way of drawing a tree with 6 vertices.

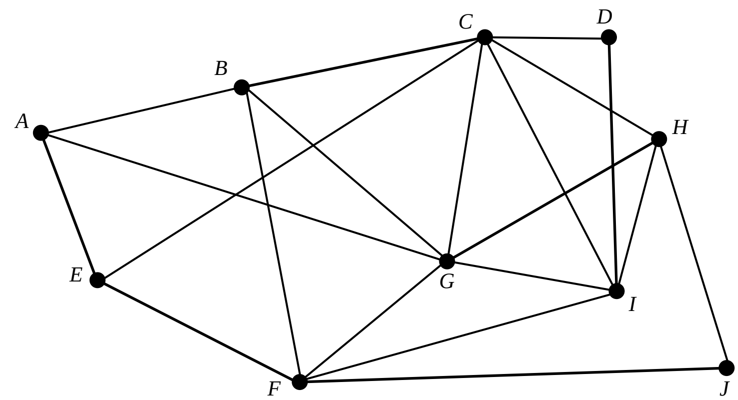
What is the sum of the degrees of each vertex in each on of your diagrams above?

4. What is the chromatic number of any tree graph?

Sec 6.11 – Minimum Spanning Trees
Networks & Graphs

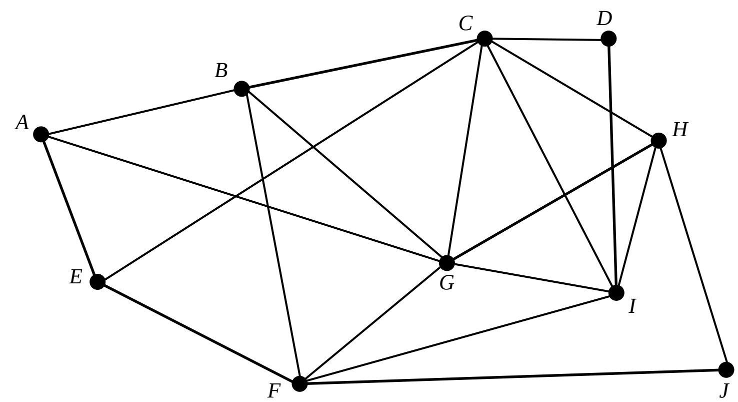
Name:

1. Create a spanning tree using the breadth-first search algorithm. Start at **A** (i.e. 0) and label each vertex with the correct number after A and show your path.



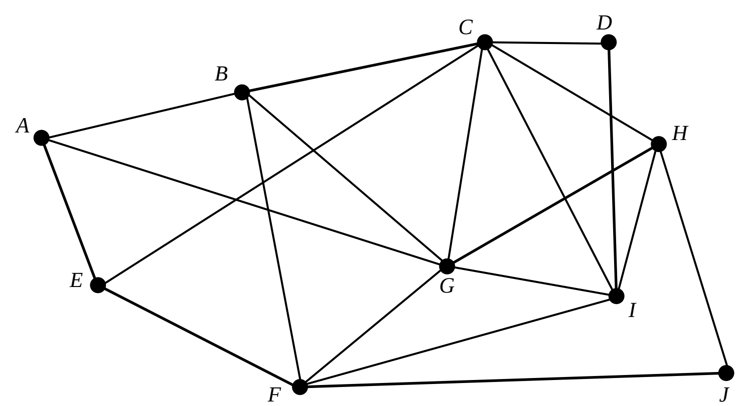
How many edges were used to create a spanning tree?

2. Create a spanning tree using the breadth-first search algorithm. Start at **G** (i.e. 0) and label each vertex with the correct number after A and show your path.



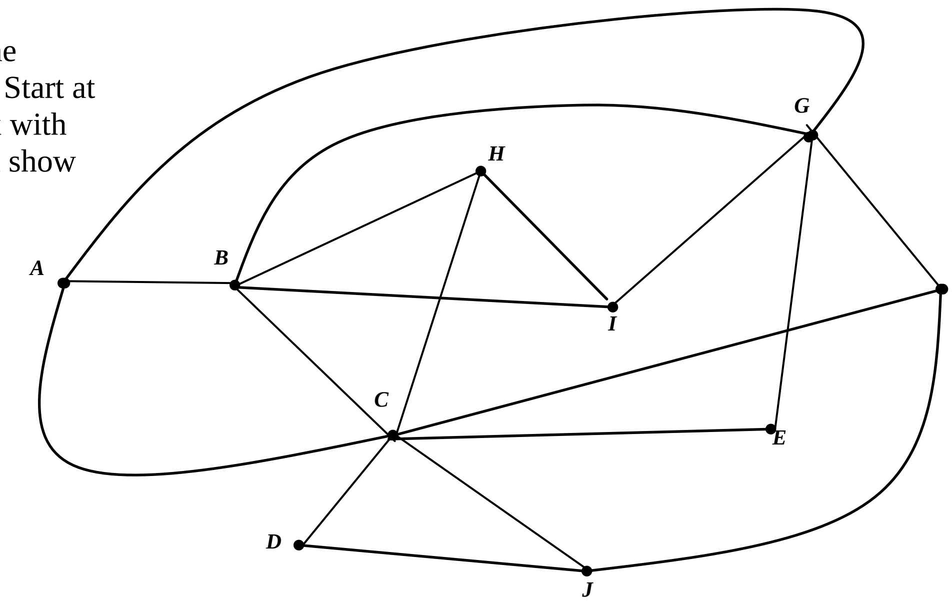
How many edges were used to create a spanning tree?

3. Create a spanning tree using the breadth-first search algorithm. Start at **J** (i.e. 0) and label each vertex with the correct number after J and show your path.

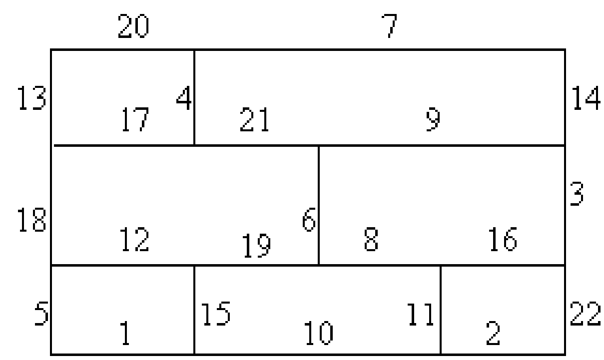


How many edges were used to create a spanning tree?

4. Create a spanning tree using the breadth-first search algorithm. Start at **A** (i.e. 0) and label each vertex with the correct number after A and show your path.



5. The minimum cost spanning tree found using Kruskal's algorithm for the following graph has a cost of____.



6. Create a minimum spanning tree using the Kruskal's algorithm. What is the total minimum length of the spanning tree? ___________